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MATHEMATICS

Class 10th (KPK)

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Chapter # 3

UNIT # 3

VARIATIONS

Ex # 3.1

Ratio

The comparison between two quantities of the same kind (same units) is called ratio.

Example

$If\ a\ and\ b\ are\ two\ quantities\ of\ the\ same\ kind\ then\ ration\ is\ written\ as\ a : b\ or\ in\ fraction\ \frac{a}{b}$

Example # 1

Write the following ration in simplified form:

(i) $\mathbf{3 : 12}$

The simplified form is $1 : 4$

(ii) $\mathbf{6a : 18b}$

The simplified form is $a : 3b$

Example # 2

Divide Rs. 5070 among three persons in the ratio $\mathbf{2 : 5 : 6}$

Solution:

$$\text{Amount} = Rs. 5070$$
$$Ratio = 2 : 5 : 6$$
$$Sum\ the\ Ratio = 2 + 5 + 6$$
$$= 13$$
$$\text{Share of 1st Person} = \frac{2}{13} \times 5070$$
$$= 2 \times 390$$
$$= \text{Rs.} 780$$
$$\text{Share of 2nd Person} = \frac{5}{13} \times 5070$$
$$= 5 \times 390$$
$$= \text{Rs.} 1950$$
$$\text{Share of 3rd Person} = \frac{6}{13} \times 5070$$
$$= 6 \times 390$$
$$= \text{Rs.} 2340$$

Proportion

A proportion is an equation that states that two ratios are equivalent.

Explanation

If a, b, c, d are four quantities then

Ex # 3.1

$$a : b :: c : d$$
$$a : b = c : d$$
$$\frac{a}{b} = \frac{c}{d}$$
$$\text{Product of mean} = \text{Product of extreme}$$
$$a \times d = b \times c$$

Example # 3

$\boldsymbol{a^3 - b^3}$, $\boldsymbol{a^2 - b^2, a^2 + ab + b^2}$ and $\boldsymbol{x}$ are in a proportion. Find the value of $\boldsymbol{x}$

Solution:

$$a^3 - b^3 , a^2 - b^2, a^2 + ab + b^2 \text{ and } x$$

As these are in proportion

$$a^3 - b^3 : a^2 - b^2 = a^2 + ab + b^2 : x$$

As we have

$$\text{Product of mean} = \text{Product of extreme}$$
$$(a^3 - b^3) \times x = (a^2 - b^2)(a^2 + ab + b^2)$$
$$(a - b)(a^2 + ab + b^2)x = (a + b)(a - b)(a^2 + ab + b^2)$$
$$\text{Divide B. S } (a - b)(a^2 + ab + b^2)$$
$$\frac{(a - b)(a^2 + ab + b^2)x}{(a - b)(a^2 + ab + b^2)} = \frac{(a + b)(a - b)(a^2 + ab + b^2)}{(a - b)(a^2 + ab + b^2)}$$
$$x = a + b$$

Variable quantity

If the value of a quantity changes under different situations, it is called a variable.

Example

Speed of train
Demand of a commodity
Population of a town

Variation

The change of variable parameters is called as variation

Example

If one quantity increase or decrease than what is its effect on other quantity.

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Chapter # 3

Ex # 3.1

Direct variation

Direct variation is the relationship between two quantities, whereby if one quantity increases the other also increases or if one quantity decreases the other also decreases.

Explanation

If y varies directly with x

Then

x increases, y also increase

x decreases, y also decreases

Equation:

$$y \propto x$$
$$y = kx$$

Example

If absence fine per day is 5.

Then the fine for One day is 5 and the fine for four days is 20. So, it means if absentee increases fine also increases and when decreases then fine also decreases.

Inverse variation

If one quantity increases, the other decreases or if one quantity decreases the other increases, it is called inverse variation.

Explanation

If y varies inversely with x

Then

x increases, y also decrease

x decreases, y also increases

Equation:

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \quad OR \quad xy = k$$

Example

If workers increase to complete the work, then it will reduce the time

Constant quantities

If the value of a quantity remains unchanged under different situations, it is called a constant.

Example

$$3, \quad 4.45, \quad \frac{22}{7}$$

Ex # 3.1

Example # 4

Given that y varies directly with x and y and $y = 27$ when $x = 3$. Find

An equation connecting x and y

The value of y when $x = 11$

Solution:

As there is direct variation

$y \propto x$

$y = kx$ ……equ(i)

Put $x = 3$ and $y = 27$ in equ(i)

$27 = k(3)$

$\frac{27}{3} = \frac{k(3)}{3}$

$9 = k$

$k = 9$

So equ (i) becomes

$y = 9x$

Thus the equation connecting x and y is $y = 9x$

Now

To Find:

y when $x = 11$

$y = ?, x = 11$

Put $x = 11$ and $k = 9$ in equ(i)

$y = 9(11)$

$y = 99$

Example # 5

If $y \propto x$, then complete the following table.

x	4	5	8		
y	6			18	22.5

Solution:

As there is direct variation

$y \propto x$

$y = kx$ ……equ(i)

Put $x = 4$ and $y = 6$ in equ(i)

$6 = k(4)$

$\frac{6}{4} = \frac{k(4)}{4}$

$\frac{3}{2} = k$

$k = \frac{3}{2}$

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Ex # 3.1

Now

Put $x = 5$ and $k = \frac{3}{2}$ in equ(i)

$y = \frac{3}{2}(5)$

$y = \frac{15}{2}$

$y = 7.5$

Now again

Put $x = 8$ and $k = \frac{3}{2}$ in equ(i)

$y = \frac{3}{2}(8)$

$y = \frac{24}{2}$

$y = 12$

Now again

Put $y = 18$ and $k = \frac{3}{2}$ in equ(i)

$18 = \frac{3}{2}(x)$

$\frac{2}{3} \times 18 = x$

$2 \times 6 = x$

$12 = x$

$x = 12$

Now again

Put $y = 22.5$ and $k = \frac{3}{2}$ in equ(i)

$22.5 = \frac{3}{2}(x)$

$\frac{2}{3} \times 22.5 = x$

$\frac{45}{3} = x$

$15 = x$

$x = 15$

x	4	5	8	12	15
y	6	7.5	12	18	22.5

Example # 6

If x varies inversely to y and $x = 3$, when $y = 12$. Find the value of y when $x = 6$

Solution:

As there is Inverse variation

$y \propto \frac{1}{x}$

Ex # 3.1

$y = \frac{k}{x}$ ……equ(i)

Put $x = 3$ and $y = 12$ in equ(i)

$12 = \frac{k}{3}$

$12 \times 3 = k$

$36 = k$

$k = 36$

Now

To Find:

$y\ when\ x = 6$

$y = ?, x = 6$

Put $x = 6$ and $k = 36$ in equ(i)

$y = \frac{36}{6}$

$y = 6$

Example # 7

Given that pressure 'P' on the quantity of gas in a container varies inversely as volume of gas 'V'. When pressure on gas is $10\ \mathrm{N/m^2}$ its volume is 25 m^3. Find pressure when volume is 20 m^3.

Solution:

As there is Inverse variation

$\mathrm{P} \propto \frac{1}{\mathrm{V}}$

$P = \frac{k}{V}$ ……equ(i)

Put $\mathrm{P} = 10$ and $\mathrm{V} = 25$ in equ(i)

$10 = \frac{k}{25}$

$10 \times 25 = k$

$250 = k$

$k = 250$

Now

To Find:

$P\ when\ V = 20$

$P = ?, V = 20$

Put $V = 20$ and $k = 250$ in equ(i)

$P = \frac{250}{20}$

$P = 12.5$

Thus Pressure $= 12.5\ \mathrm{N/m^2}$

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Ex # 3.1

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Q1: Which is the greater ratio, 5 : 7 or 151: 208 ?

Solution:

As we have

5 : 7 or 151: 208

Now

$$5:7=\frac{5}{7}=0.714285$$

Also

$$151:208=\frac{151}{208}=0.725961$$

Hence 151: 208 is greater ratio.

Q2: Gold and silver are mixed in the ratio 7 : 4. If 36 grams of silver is used. How much gold is used?

Solution:

Let gold used $= x$

$\text{Ratio of Gold and Silver} = 7:4$

$Silver\ used = 36\ grams$

Now the ratio Gold and Silver

$7:4 = x:36$

As we have

$\text{Product of mean} = \text{Product of extreme}$

$4 \times x = 7 \times 36$

$$x=\frac{7\times 36}{4}$$

$x = 63$

Thus 63 grams of Gold is used

Q3: Divide the annual profit of Rs. 40,000 of a factory among 3 partners in the ratio of 5 : 8 : 12

Solution:

$\text{Annual Profit} = Rs. 40,000$

$Ratio = 5:8:12$

$Sum\ the\ Ratio = 5+8+12$

$= 25$

$$\text{Share of 1st Partner} = \frac{5}{25}\times 40000$$

$= 5 \times 1600$

$= \text{Rs.} 8000$

$$\text{Share of 2nd Partner} = \frac{8}{25}\times 40000$$

$= 8 \times 1600$

$= \text{Rs.} 12800$

Ex # 3.1

$$\text{Share of 3rd Partner} = \frac{12}{25}\times 40000$$

$= 12 \times 1600$

$= \text{Rs.} 19200$

Q4: If $11:x-1=22:27$, find the value of x

Solution:

$$11:x-1=22:27$$

As we have

Product of mean=Product of extreme

$22(x-1) = 11\times 27$

$22x - 22 = 297$

Add 22 on B.S

$22x - 22 + 22 = 297 + 22$

$22x = 319$

Divide B.S by 22

$$\frac{22x}{22}=\frac{319}{22}$$

$x = 14.5$

Q5: There is a direct variation between x^2 and y. When $x = 7$, $y = 49$. Find:

(i) $y\ when\ x = 9$

(ii) $x\ when\ y = 100$

Solution:

As there is direct variation

$y \propto x^2$

$y = kx^2 \quad \ldots\ldots \text{equ(i)}$

Put $x = 7$ and $y = 49$ in equ(i)

$49 = k(7)^2$

$49 = k(49)$

$$\frac{49}{49}=\frac{k(49)}{49}$$

$1 = k$

$k = 1$

Now

To Find:

$y\ when\ x = 9$

$y = ?,\ x = 9$

Put $x = 9$ and $k = 1$ in equ(i)

$y = 1(9)^2$

$y = 1(81)$

$y = 81$

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Ex # 3.1

Now again

To Find:

x when $y = 100$

$x = ?, y = 100$

Put $y = 100$ and $k = 1$ in equ(i)

$100 = 1(x^2)$

$100 = x^2$

$x^2 = 100$

Taking Square root on B.S

$\sqrt{x^2} = \sqrt{100}$

$x = 10$

Q6: There is inverse variation between x and y. When $x = 4, y = 6$. Find:

(i) y when $x = 12$

(ii) x when $y = 24$

Solution:

As there is Inverse variation

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$ ……equ(i)

Put $x = 4$ and $y = 6$ in equ(i)

$6 = \frac{k}{4}$

$6 \times 4 = k$

$24 = k$

$k = 24$

Now To Find:

y when $x = 12$

$y = ?, x = 12$

Put $x = 12$ and $k = 24$ in equ(i)

$y = \frac{24}{12}$

$y = 2$

Now again Find:

x when $y = 24$

$x = ?, x = 24$

Put $y = 24$ and $k = 24$ in equ(i)

$24 = \frac{24}{x}$

$x = \frac{24}{24}$

$x = 1$

Ex # 3.1

Q7: $r \propto \frac{1}{p^3}$ and $p = 9$ when $r = 2$. Find:

(i) r when $p = 3$

(ii) p when $r = \frac{1}{4}$

Solution:

$r \propto \frac{1}{p^3}$

$r = \frac{k}{p^3}$ ……equ(i)

Put $p = 9$ and $r = 2$ in equ(i)

$2 = \frac{k}{(9)^3}$

$2 = \frac{k}{729}$

$2 \times 729 = k$

$1458 = k$

$k = 1458$

Now

To Find:

r when $p = 3$

$r = ?, p = 3$

Put $p = 3$ and $k = 1458$ in equ(i)

$r = \frac{1458}{(3)^3}$

$r = \frac{1458}{27}$

$r = 54$

Now again

To Find:

p when $r = \frac{1}{4}$

$p = ?, r = \frac{1}{4}$

Put $r = \frac{1}{4}$ and $k = 1458$ in equ(i)

$\frac{1}{4} = \frac{1458}{p^3}$

By Cross Multiplication

$1 \times p^3 = 1458 \times 4$

$p^3 = 18^3$

Taking Cube root on B.S

$\sqrt[3]{p^3} = \sqrt[3]{18^3}$

$p = 18$

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Ex # 3.1

Q8: If $y \propto x$, then complete the following table.

x	4	6		15
y	2		3.5	

Solution:

As there is direct variation

$y \propto x$

$y = kx \quad \dots \dots \text{equ(i)}$

Put $x = 4$ and $y = 2$ in equ(i)

$2 = k(4)$

$\frac{2}{4} = \frac{k(4)}{4}$

$\frac{1}{2} = k$

$k = \frac{1}{2}$

Now

Put $x = 6$ and $k = \frac{1}{2}$ in equ(i)

$y = \frac{1}{2}(6)$

$y = 3$

Now again

Put $y = 3.5$ and $k = \frac{1}{2}$ in equ(i)

$3.5 = \frac{1}{2}(x)$

$\frac{2}{1} \times 3.5 = x$

$7 = x$

$x = 7$

Now again

Put $x = 15$ and $k = \frac{1}{2}$ in equ(i)

$y = \frac{1}{2}(15)$

$y = \frac{15}{2}$

$y = 7.5$

x	4	6	7	15
y	2	3	3.5	7.5

Ex # 3.2

Third, fourth Mean and Continued Proportion

Continued Proportion

Three quantities are said to be in continued proportion, if the ratio between the first and the second is equal to the ratio between second and third.

Example

If a, b and c are in continued proportion then

$$a : b :: b : c$$

Product of mean=Product of extreme

$$\text{So } b^2 = ac$$

In the above example:

b is called mean proportion or geometric mean

c is called the third proportion

Fourth proportion

If four quantities a, b, c and d are:

$$a : b :: c : d$$

Here d is called fourth proportion

Example # 8

Find the mean proportional of 5 , 15

Solution:

Let the mean proportional $= x$

So $5, x, 15$ are in continued proportional

Now we write it

$5 : x = x : 15$

Product of mean=Product of extreme

$x \times x = 5 \times 15$

$x^2 = 75$

Taking square root on B.S

$\sqrt{x^2} = \sqrt{75}$

$x = \sqrt{25 \times 3}$

$x = \sqrt{25} \times \sqrt{3}$

$x = 5\sqrt{3}$

Example # 9

Find the mean proportional of a^2b^2 and abc

Solution:

Let the third proportional $= x$

So a^2b^2, abc, x are in continued proportional

Now we write it

$a^2b^2 : abc = abc : x$

Product of mean=Product of extreme

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Ex # 3.2

$abc \times abc = a^2b^2 \times x$

$a^2b^2c^2 = a^2b^2 \times x$

Divide B.S a^2b^2

$\frac{a^2b^2c^2}{a^2b^2} = \frac{a^2b^2 \times x}{a^2b^2}$

$c^2 = x$

$x = c^2$

Example # 10:

Find fourth proportion of $a^3 - b^3$, $a + b$ and $a^2 + ab + b^2$

Solution:

Let the fourth proportional $= x$

So

$a^3 - b^3, a + b, a^2 + ab + b^2, x$ are in proportional

Now we write it

$a^3 - b^3 : a + b = a^2 + ab + b^2 : x$

Product of mean=Product of extreme

$(a + b)(a^2 + ab + b^2) = (a^3 - b^3)x$

$(a + b)(a^2 + ab + b^2) = (a - b)(a^2 + ab + b^2)x$

Divide B.S $(a - b)(a^2 + ab + b^2)$

$\frac{(a + b)(a^2 + ab + b^2)}{(a - b)(a^2 + ab + b^2)} = \frac{(a - b)(a^2 + ab + b^2)x}{(a - b)(a^2 + ab + b^2)}$

$\frac{(a + b)}{(a - b)} = x$

$x = \frac{(a + b)}{(a - b)}$

Theorems on Proportion

Alternendo Property

If $a : b = c : d$ then $a : c = b : d$

It means that if the second and third term interchange their places, then also the four terms are in proportion.

Example

If $3: 5 = 6: 10$ then

$3 : 6 = \mathbf{1} : \mathbf{2} = 5 : 10$

Ex # 3.2

Invertendo Property

If $a: b = c: d$ then $b: a = d: c$

It means that if two ratios are equal, then their inverse are also equal.

Example

$6: 10 = 9: 15$ then

$10 : 6 = \mathbf{5} : \mathbf{3} = 15 : 9$

Componendo Property

If $a: b = c: d$ then $(a + b) : b = (c + d) : d$

Or

If $a: b = c: d$ then $\frac{a + b}{b} = \frac{c + d}{d}$

Example:

If $4: 5 = 8: 10$ then $(4 + 5) : 5 = (8 + 10) : 10$

Or

If $4: 5 = 8: 10$ then $\frac{4 + 5}{5} = \frac{8 + 10}{10}$

Dividendo Property

If $a: b = c: d$ then $(a - b) : b = (c - d) : d$

Or

If $a: b = c: d$ then $\frac{a + b}{b} = \frac{c + d}{d}$

Example:

If $5: 4 = 10: 8$ then $(5 - 4) : 4 = (10 - 8) : 8$

Or

If $5: 4 = 10: 8$ then $\frac{5 - 4}{4} = \frac{10 - 8}{8}$

Componendo-Dividendo Property

If $a: b :: c: d$ then

$(a + b) : (a - b) = (c + d) : (c - d)$

Or

If $a: b = c: d$ then $\frac{a + b}{a - b} = \frac{c + d}{c - d}$

Example

If $7: 3 = 14: 6$ then

$(7 + 3) : (7 - 3) = (14 + 6) : (14 - 6)$

Or

If $7: 3 = 14: 6$ then $\frac{7 + 3}{7 - 3} = \frac{14 + 6}{14 - 6}$

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Ex # 3.2

Example # 11

If $\frac{a}{b}=\frac{c}{d}$ then prove that

$$2a+3b : b = 2c+3d : d$$

Solution:

As we have

$$\frac{a}{b}=\frac{c}{d}$$

To prove

$$2a+3b : b = 2c+3d : d$$

Now

$$\frac{a}{b}=\frac{c}{d}$$

Multiply on B.S by $\frac{2}{3}$

$$\frac{2}{3}\times\frac{a}{b}=\frac{2}{3}\times\frac{c}{d}$$

$$\frac{2a}{3b}=\frac{2c}{3d}$$

By Componenendo Property

$$\frac{2a+3b}{3b}=\frac{2c+3d}{3d}$$

Multiply on B.S by 3

$$3\times\frac{2a+3b}{3b}=3\times\frac{2c+3d}{3d}$$

$$\frac{2a+3b}{b}=\frac{2c+3d}{d}$$

OR

$$2a+3b : b = 2c+3d : d$$

Hence Proved

Example # 12

If $\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$ Prove that $\frac{a}{b}=\frac{c}{d}$

Solution:

As we have

$$\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$$

To prove

$$\frac{a}{b}=\frac{c}{d}$$

Now

$$\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$$

By Componenendo – Dividendo Property

$$\frac{(3a-4b)+(3a+4b)}{(3a-4b)-(3a+4b)}=\frac{(3c-4d)+(3c+4d)}{(3c-4d)-(3c+4d)}$$

Ex # 3.2

$$\frac{3a-4b+3a+4b}{3a-4b-3a-4b}=\frac{3c-4d+3c+4d}{3c-4d-3c-4d}$$

$$\frac{3a+3a-4b+4b}{3a-3a-4b-4b}=\frac{3c+3c-4d+4d}{3c-3c-4d-4d}$$

$$\frac{6a}{-8b}=\frac{6c}{-8d}$$

Multiply on B.S by $\frac{-8}{6}$

$$\frac{-8}{6}\times\frac{6a}{-8b}=\frac{-8}{6}\times\frac{6c}{-8d}$$

$$\frac{a}{b}=\frac{c}{d}$$

Hence Proved

Example # 13

$$\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x+4)^2}=\frac{13}{12}$$

Solution:

$$\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x+4)^2}=\frac{13}{12}$$

By Componenendo – Dividendo Property

$$\frac{[(x+3)^2+(x-4)^2]+[(x+3)^2-(x+4)^2]}{[(x+3)^2+(x-4)^2]-[(x+3)^2-(x+4)^2]}=\frac{13+12}{13-12}$$

$$\frac{(x+3)^2+(x-4)^2+(x+3)^2-(x-4)^2}{(x+3)^2+(x-4)^2-(x+3)^2+(x-4)^2}=\frac{25}{1}$$

$$\frac{(x+3)^2+(x+3)^2+(x-4)^2-(x-4)^2}{(x+3)^2-(x+3)^2+(x-4)^2+(x-4)^2}=25$$

$$\frac{2(x+3)^2}{2(x-4)^2}=25$$

$$\frac{(x+3)^2}{(x-4)^2}=25$$

$$\left(\frac{x+3}{x-4}\right)^2=25$$

Taking square root on B.S

$$\sqrt{\left(\frac{x+3}{x-4}\right)^2}=\pm\sqrt{25}$$

$$\frac{x+3}{x-4}=\pm 5$$

$$\frac{x+3}{x-4}=5 \quad or \quad \frac{x+3}{x-4}=-5$$

Ex # 3.2

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Q1: Which of the following quantities are in continued proportion?

(i) $4, 12, 36$

Solution:

As $4, 12, 36$ are in continued proportional

So we can write it

$4 : 12 = 12 : 36$

Product of mean=Product of extreme

$12 \times 12 = 4 \times 36$

$144 = 144$

Thus $4, 12, 36$ are in continued proportional

(ii) $3, 12, 39$

Solution:

As $3, 12, 39$ are in continued proportional

So we can write it

$3 : 12 = 12 : 39$

Product of mean=Product of extreme

$12 \times 12 = 3 \times 39$

$144 = 117$

Thus $4, 12, 36$ are not in continued proportional

(iii) $72, 24, 8$

Solution:

As $72, 24, 8$ are in continued proportional

So we can write it

$72 : 24 = 24 : 8$

Product of mean=Product of extreme

$24 \times 24 = 72 \times 8$

$576 = 576$

Thus $72, 24, 8$ are in continued proportional

Q2: Find the mean proportional of 12 , 3

Solution:

Let the mean proportional $= x$

So $12, x, 3$ are in continued proportional

Now we write it

$12 : x = x : 3$

Product of mean=Product of extreme

$x \times x = 12 \times 3$

$x^2 = 36$

Taking square root on B.S

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

Q3: If $5 : 15 : x$ are in continued proportional, find the value of x

Solution:

As $5 : 15 : x$ are in continued proportional

So we can write it

$5 : 15 = 15 : x$

Product of mean=Product of extreme

$15 \times 15 = 5 \times x$

$225 = 5x$

Divide B.S by 5

$\frac{225}{5} = \frac{5x}{5}$

$45 = x$

$x = 45$

Q4: If $3x - 1 : 4 : 35$ are in continued proportional, find the value of x

Solution:

As $3x - 1 : 4 : 35$ are in continued proportional

So we can write it

$3x - 1 : 4 = 4 : 35$

As we have

Product of mean=Product of extreme

$4 \times 4 = 35(3x - 1)$

$16 = 105x - 35$

Add 35 on B.S

$16 + 35 = 105x - 35 + 35$

$51 = 105x$

Divide B.S by 105

$\frac{51}{105} = \frac{105x}{105}$

$\frac{19}{51} = x$

$x = \frac{19}{51}$

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Ex # 3.2

Q5: **Find the mean proportional of** $\boldsymbol{a^2-b^2}$ **and** $\boldsymbol{\frac{a+b}{a-b}}$

Solution:

Let the mean proportional $= x$

So

$a^2-b^2, x, \frac{a+b}{a-b}$ are in continued proportional

Now we write it

$$a^2-b^2 : x = x : \frac{a+b}{a-b}$$

Product of mean=Product of extreme

$$x \times x = (a^2-b^2)\left(\frac{a+b}{a-b}\right)$$

$$x^2 = (a+b)(a-b)\left(\frac{a+b}{a-b}\right)$$

$$x^2 = (a+b)(a+b)$$

$$x^2 = (a+b)^2$$

Taking square root on B.S

$$\sqrt{x^2} = \sqrt{(a+b)^2}$$

$$x = a+b$$

Q6: **If** $\boldsymbol{\frac{a}{b}=\frac{c}{d}}$ **then prove that** $\boldsymbol{\frac{ac+bd}{ac-bd}=\frac{a^2+b^2}{a^2-b^2}}$

Solution:

As we have

$$\frac{a}{b}=\frac{c}{d}$$

To prove

$$\frac{ac+bd}{ac-bd}=\frac{a^2+b^2}{a^2-b^2}$$

Now

$$\frac{a}{b}=\frac{c}{d}$$

Multiply $\frac{a}{b}$ on B.S

$$\frac{a}{b}\times\frac{a}{b}=\frac{a}{b}\times\frac{c}{d}$$

$$\frac{a^2}{b^2}=\frac{ac}{bd}$$

By Componenendo − Dividendo Property

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac+bd}$$

OR

$$\frac{ac+bd}{ac+bd}=\frac{a^2+b^2}{a^2-b^2}$$

Ex # 3.2

Q7: **Solve the following equations.**

(i) $\boldsymbol{\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}}=\frac{4}{1}}$

Solution:

$$\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}}=\frac{4}{1}$$

By Componenendo − Dividendo Property

$$\frac{\left(\sqrt{3x+2}+\sqrt{x}\right)+\left(\sqrt{3x+2}-\sqrt{x}\right)}{\left(\sqrt{3x+2}+\sqrt{x}\right)-\left(\sqrt{3x+2}-\sqrt{x}\right)}=\frac{4+1}{4-1}$$

$$\frac{\sqrt{3x+2}+\sqrt{x}+\sqrt{3x+2}-\sqrt{x}}{\sqrt{3x+2}+\sqrt{x}-\sqrt{3x+2}+\sqrt{x}}=\frac{5}{3}$$

$$\frac{\sqrt{3x+2}+\sqrt{3x+2}+\sqrt{x}-\sqrt{x}}{\sqrt{3x+2}-\sqrt{3x+2}+\sqrt{x}+\sqrt{x}}=\frac{5}{3}$$

$$\frac{2\sqrt{3x+2}}{2\sqrt{x}}=\frac{5}{3}$$

$$\frac{\sqrt{3x+2}}{\sqrt{x}}=\frac{5}{3}$$

$$\sqrt{\frac{3x+2}{x}}=\frac{5}{3}$$

Taking square on B.S

$$\left(\sqrt{\frac{3x+2}{x}}\right)^2=\left(\frac{5}{3}\right)^2$$

$$\frac{3x+2}{x}=\frac{25}{9}$$

By Cross Multiplication

$$9(3x+2)=25\times x$$

$$27x+18=25x$$

$$27x-25x=-18$$

$$2x=-18$$

Divide B.S by 2

$$\frac{2x}{2}=\frac{-18}{2}$$

$$x=-9$$

$$\boldsymbol{S.S=\{-9\}}$$

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Ex # 3.2

(ii) $\dfrac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=-\dfrac{17}{8}$

Solution:

$$\frac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=-\frac{17}{8}$$

By Componenendo – Dividendo Property

$$\frac{[(x-1)^2+(x+2)^2]+[(x-1)^2-(x+2)^2]}{[(x-1)^2+(x+2)^2]-[(x-1)^2-(x+2)^2]}=\frac{-17+8}{-17-8}$$

$$\frac{(x-1)^2+(x+2)^2+(x-1)^2-(x+2)^2}{(x-1)^2+(x+2)^2-(x-1)^2+(x+2)^2}=\frac{9}{25}$$

$$\frac{(x-1)^2+(x-1)^2+(x+2)^2-(x+2)^2}{(x-1)^2-(x-1)^2+(x+2)^2+(x+2)^2}=\frac{9}{25}$$

$$\frac{2(x-1)^2}{2(x+2)^2}=\frac{9}{25}$$

$$\frac{(x-1)^2}{(x+2)^2}=\frac{9}{25}$$

$$\left(\frac{x-1}{x+2}\right)^2=\frac{9}{25}$$

Taking square root on B.S

$$\sqrt{\left(\frac{x-1}{x+2}\right)^2}=\pm\sqrt{\frac{9}{25}}$$

$$\frac{x-1}{x+2}=\pm\frac{3}{5}$$

$$\frac{x-1}{x+2}=\frac{3}{5}\quad or\quad \frac{x-1}{x+2}=-\frac{3}{5}$$

By Cross Multiplication

$5(x-1)=3(x+2)\quad or\quad 5(x-1)=-3(x+2)$

$5x-5=3x+6\quad or\quad 5x-5=-3x-6$

$5x-3x=6+5\quad or\quad 5x+3x=-6+5$

$2x=11\quad or\quad 8x=-1$

$x=\dfrac{11}{2}\quad or\quad x=\dfrac{-1}{8}$

$$\mathbf{S.S}=\left\{\frac{11}{2}\ ,\quad \frac{-1}{8}\right\}$$

(iii) $\dfrac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\dfrac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\frac{1}{3}$$

By Componenendo – Dividendo Property

$$\frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)+\left(\sqrt{x^2+a^2}+\sqrt{x^2-a^2}\right)}{\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)-\left(\sqrt{x^2+a^2}+\sqrt{x^2-a^2}\right)}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}+\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}-\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}=\frac{4}{-2}$$

$$\frac{2\sqrt{x^2+a^2}}{-2\sqrt{x^2-a^2}}=-2$$

$$-\frac{2\sqrt{x^2+a^2}}{2\sqrt{x^2-a^2}}=-2$$

$$\frac{\sqrt{x^2+a^2}}{\sqrt{x^2-a^2}}=2$$

$$\sqrt{\frac{x^2+a^2}{x^2-a^2}}=2$$

Taking square on B.S

$$\left(\sqrt{\frac{x^2+a^2}{x^2-a^2}}\right)^2=(2)^2$$

$$\frac{x^2+a^2}{x^2-a^2}=4$$

$x^2+a^2=4(x^2-a^2)$

$x^2+a^2=4x^2-4a^2$

$a^2+4a^2=4x^2-x^2$

$5a^2=3x^2$

$3x^2=5a^2$

$x^2=\dfrac{5a^2}{3}$

Taking square on B.S

$$\sqrt{x^2}=\pm\sqrt{\frac{5a^2}{3}}$$

$$x=\pm\sqrt{\frac{5}{3}}\,a$$

$$\mathbf{S.S}=\left\{\pm\sqrt{\frac{5}{3}}\,a\right\}$$

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Joint variation

A combination of direct and inverse variation of one or more variables forms joint variation.

If y varies jointly as x and z

Then

$$y \propto xz$$

If y varies directly as x and inversely as z

Then

$$y \propto \frac{x}{z}$$

Example:

Area of a triangle $= \frac{1}{2}bh$

Here the constant k is $\frac{1}{2}$

Area of a triangle varies jointly with base 'b' and height 'h'

Example # 14 (imp)

If y varies jointly as x and z, and $y = 12$ when $x = 9$ and $z = 3$, find z when $y = 6$ and $x = 15$.

Solution:

As y varies jointly as x and z

So

$y \propto xz$

$y = kxz$ …… equ(i)

Put $y = 12, x = 9$ and $z = 3$ in equ(i)

$12 = k(9)(3)$

$\frac{12}{(9)(3)} = k$

$\frac{4}{(9)(1)} = k$

$\frac{4}{9} = k$

$k = \frac{4}{9}$

Now

To Find:

z *when* $x = 15$ and $y = 6$

$z = ?, x = 15$ and $y = 6$

Put $x = 15, y = 6$ and $k = \frac{4}{9}$ in equ(i)

$6 = \left(\frac{4}{9}\right)(15)(z)$

$6 = \left(\frac{4}{3}\right)(5)(z)$

Ex # 3.3

$6 = \left(\frac{20}{3}\right)(z)$

$6 \times \frac{3}{20} = z$

$\frac{18}{20} = z$

$\frac{9}{10} = z$

$z = \frac{9}{10}$

Ex # 3.3

Page # 62

Q1: If y varies jointly as x and z, and $y = 33$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 22$.

Solution:

As y varies jointly as x and z

So

$y \propto xz$

$y = kxz$ …… equ(i)

Put $y = 33, x = 9$ and $z = 12$ in equ(i)

$33 = k(9)(12)$

$\frac{33}{(9)(12)} = k$

$\frac{11}{3(12)} = k$

$\frac{11}{36} = k$

$k = \frac{11}{36}$

Now

To Find:

y *when* $x = 16$ and $z = 22$

$y = ?, x = 16$ and $z = 22$

Put $x = 16, z = 22$ and $k = 1$ in equ(i)

$y = \left(\frac{11}{36}\right)(16)(22)$

$y = \left(\frac{11}{9}\right)(4)(22)$

$y = \left(\frac{11}{9}\right)(88)$

$y = \frac{968}{9}$

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Ex # 3.3

Q2: If f varies jointly as g and the cube of h, and $f = 200$ when $g = 5$ and $h = 4$, find f when $g = 3$ and $h = 6$

Solution:

As f varies jointly as g and h^3

So

$f \propto gh^3$

$f = kgh^3$ ... ... equ(i)

Put $f = 200, g = 5$ and $h = 4$ in equ(i)

$200 = k(5)(4)^3$

$200 = k(5)(64)$

$\frac{200}{(5)(64)} = k$

$\frac{40}{(1)(64)} = k$

$\frac{5}{8} = k$

$k = \frac{5}{8}$

Now

To Find:

f when $g = 3$ and $h = 6$

$f = ?, g = 3$ and $h = 6$

Put $g = 3, h = 6$ and $k = \frac{5}{8}$ in equ(i)

$f = \left(\frac{5}{8}\right)(3)(6)^3$

$f = \left(\frac{5}{8}\right)(3)(216)$

$f = (5)(3)(27)$

$f = (15)(27)$

$f = 405$

Q3: Suppose a is jointly proportional to b and c. If $a = 4$ when $b = 8$ and $c = 9$, then what is a when $b = 2$ and $c = 18$?

Solution:

As a is jointly proportional to b and c

So

$a \propto bc$

$a = kbc$ ... ... equ(i)

Put $a = 4, b = 8$ and $c = 9$ in equ(i)

$4 = k(8)(9)$

$\frac{4}{(8)(9)} = k$

Ex # 3.3

$\frac{1}{2(9)} = k$

$\frac{1}{18} = k$

$k = \frac{1}{18}$

Now

To Find:

a when $b = 2$ and $c = 18$

$a = ?, b = 2$ and $c = 18$

Put $b = 2, c = 18$ and $k = \frac{1}{18}$ in equ(i)

$a = \left(\frac{1}{18}\right)(2)(18)$

$a = 2$

Q4: If p varies jointly as q and r squared, and $p = 225$ when $q = 4$ and $r = 3$, find p when $q = 6$ and $r = 8$.

Solution:

p varies jointly as q and r^2

$p \propto qr^2$

$p = kqr^2$ ... ... equ(i)

Put $p = 225, q = 4$ and $r = 3$ in equ(i)

$225 = k(4)(3)^2$

$225 = k(4)(9)$

$\frac{225}{(4)(9)} = k$

$\frac{25}{(4)(1)} = k$

$\frac{25}{4} = k$

$k = \frac{25}{4}$

To Find:

$p = ?, q = 6$ and $r = 8$

Put $q = 6, r = 8$ and $k = \frac{25}{4}$ in equ(i)

$p = \left(\frac{25}{4}\right)(6)(8)^2$

$p = \left(\frac{25}{4}\right)(6)(64)$

$p = \left(\frac{25}{1}\right)(6)(16)$

$p = (25)(6)(16)$

$p = 2400$

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Ex # 3.3

Q5: **If a varies jointly as b cubed and c, and $a = 36$ when $b = 4$ and $c = 6$, find a when $b = 2$ and $c = 14$.**

Solution:

As a is jointly proportional to b cubed and c

So

$a \propto bc$

$a = kb^3c$ ……equ(i)

Put $a = 36, b = 4$ and $c = 6$ in equ(i)

$36 = k(4)^3(6)$

$36 = k(64)(6)$

$\frac{36}{(64)(6)} = k$

$\frac{6}{64} = k$

$\frac{3}{32} = k$

$k = \frac{3}{32}$

Now

To Find:

a *when* $b = 2$ and $c = 14$

$a = ?, b = 2$ and $c = 14$

Put $b = 2, c = 14$ and $k = \frac{3}{32}$ in equ(i)

$a = \left(\frac{3}{32}\right)(2)^3(14)$

$a = \left(\frac{3}{32}\right)(8)(14)$

$a = \left(\frac{3}{4}\right)(1)(14)$

$a = \left(\frac{3}{2}\right)(1)(7)$

$a = \frac{21}{2}$

Q6: **If z varies jointly as x and y, and $z = 12$ when $x = 2$ and $y = 4$, find the constant of variation.**

Solution:

As z varies jointly as x and y

So

$z \propto xy$

$z = kxy$ ……equ(i)

Put $z = 12, x = 2$ and $y = 4$ in equ(i)

$12 = k(2)(4)$

Ex # 3.3

$12 = k(8)$

$\frac{12}{8} = k$

$\frac{3}{2} = k$

$k = \frac{3}{2}$

Q7: **f y varies jointly as x^2 and z, and $y = 6$ when $x = 4$ and $z = 9$ write y as a function of x and z and determine the value of y when $x = -8$ and $z = 12$.**

Solution:

As y varies jointly as x^2 and z

So

$y \propto x^2z$

$y = kx^2z$ ……equ(i)

Put $y = 6, x = 4$ and $z = 9$ in equ(i)

$6 = k(4)^2(9)$

$6 = k(16)(9)$

$\frac{6}{(16)(9)} = k$

$\frac{3}{(8)(9)} = k$

$\frac{1}{(8)(3)} = k$

$\frac{1}{24} = k$

$k = \frac{1}{24}$

Now

To Find:

y *when* $x = 16$ and $z = 22$

$y = ?, x = 16$ and $z = 22$

Put $x = 16, z = 22$ and $k = \frac{1}{24}$ in equ(i)

$y = \left(\frac{1}{24}\right)(16)(22)$

$y = \left(\frac{1}{24}\right)(-8)^2(12)$

$y = \left(\frac{1}{24}\right)(64)(12)$

$y = \left(\frac{1}{2}\right)(64)(1)$

$y = 32$

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Ex # 3.3

Q8: **If p varies jointly as q and r^2 and inversely as s and t^2, $p = 40$ when $q = 8$ and $r = 5, s = 3$ and $t = 2$. Find p in terms of q, r, s and t. Also find the value of when $q = -2$ and $r = 4, s = 3$ and $t = -1$.**

Solution:

As p varies jointly as q and r^2 and inversely as s and t^2

So

$$p \propto \frac{qr^2}{st^2}$$

$$p = k\frac{qr^2}{st^2} \quad \dots\dots \text{equ(i)}$$

Put $p = 40, q = 8, r = 3, s = 3$ and $t = 2$ in equ(i)

$$40 = k\frac{(8)(5)^2}{(3)(2)^2}$$

$$40 = k\frac{(8)(25)}{(3)(4)}$$

$$40 = k\frac{(2)(25)}{(3)(1)}$$

$$40 = k\frac{50}{3}$$

$$40 \times \frac{3}{50} = k$$

$$4 \times \frac{3}{5} = k$$

$$\frac{12}{5} = k$$

$$k = \frac{25}{4}$$

Now

To Find:

$q = -2$ and $r = 4, s = 3$ and $t = -1$

$p = ?, q = -2, r = 4, s = 3$ and $t = -1$

Put $q = -2, r = 4, s = 3, t = -1$ and $k = \frac{12}{5}$ in equ(i)

$$p = \left(\frac{12}{5}\right)\frac{(-2)(4)^2}{(3)(-1)^2}$$

$$p = \left(\frac{12}{5}\right)\frac{(-2)(16)}{(3)(1)}$$

$$p = \left(\frac{4}{5}\right)\frac{(-2)(16)}{(1)(1)}$$

$$p = \frac{-128}{5}$$

Ex # 3.4

K – Method

If $a : b :: c : d$ is a proportion, then putting each ratio equal to k

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k$$

$$a = kb, \qquad c = kd$$

These equations are used to evaluate certain expressions more easily. This method is called K – Method.

Example # 15:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then prove that each of the ratios is equal to $\frac{la + mc + ne}{lb + md + nf}$

Solution:

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \dots\dots \text{equ(i)}$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k, \qquad \frac{e}{f} = k$$

$$a = kb, \qquad c = kd, \qquad e = kf$$

$$\frac{la + mc + ne}{lb + md + nf} = \frac{lkb + mkd + nkf}{lb + md + nf}$$

$$\frac{la + mc + ne}{lb + md + nf} = \frac{k(lb + md + nf)}{lb + md + nf}$$

$$\frac{la + mc + ne}{lb + md + nf} = k \quad \dots\dots \text{equ(ii)}$$

Thus from equ (i) and equ (ii)

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{la + mc + ne}{lb + md + nf}$$

Example # 16:

Prove that $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a + c + e}{b + d + f}$

Solution:

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \dots\dots \text{equ(i)}$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k, \qquad \frac{e}{f} = k$$

$$a = kb, \qquad c = kd, \qquad e = kf$$

$$\frac{a + c + e}{b + d + f} = \frac{kb + kd + kf}{b + d + f}$$

$$\frac{a + c + e}{b + d + f} = \frac{k(b + d + f)}{b + d + f}$$

Ex # 3.4

$$\frac{a+c+e}{b+d+f} = k \quad \dots\dots \text{equ(ii)}$$

Thus from equ (i) and equ (ii)

$$\frac{a+c+e}{b+d+f} = \frac{a+c+e}{b+d+f}$$

Example # 17:

Prove that $\boldsymbol{\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}}$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \dots\dots \text{equ(i)}$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k, \qquad \frac{e}{f} = k$$

$$a = kb, \qquad c = kd, \qquad e = kf$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{(kb)^2+(kd)^2+(kf)^2}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{k^2b^2+k^2d^2+k^2f^2}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{k^2}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = k \quad \dots\dots \text{equ(i)}$$

Thus from equ (i) and equ (ii)

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

Example # 18:

If $\boldsymbol{\frac{a}{x} = \frac{b}{y} = \frac{c}{z}}$ **then prove that**

$$\boldsymbol{\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}}$$

Solution:

Let $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = k$

As each ratio equal to k

So

Ex # 3.4

$$\frac{a}{x} = k, \qquad \frac{b}{y} = k, \qquad \frac{c}{z} = k$$

Now take the reciprocal

$$\frac{x}{a} = \frac{1}{k}, \qquad \frac{y}{b} = \frac{1}{k}, \qquad \frac{z}{c} = \frac{1}{k} \quad \dots\dots \text{equ(i)}$$

To Prove:

$$\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Now

Take cube on equ(i)

$$\left(\frac{x}{a}\right)^3 = \left(\frac{1}{k}\right)^3, \left(\frac{y}{b}\right)^3 = \left(\frac{1}{k}\right)^3, \left(\frac{z}{c}\right)^3 = \left(\frac{1}{k}\right)^3$$

$$\frac{x^3}{a^3} = \frac{1}{k^3}, \qquad \frac{y^3}{b^3} = \frac{1}{k^3}, \qquad \frac{z^3}{c^3} = \frac{1}{k^3}$$

Now add them

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{1}{k^3} + \frac{1}{k^3} + \frac{1}{k^3}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{1+1+1}{k^3}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3}{k^3} \quad \dots\dots \text{equ(ii)}$$

Now take the product of equ (i)

$$\frac{x}{a} \cdot \frac{y}{b} \cdot \frac{z}{c} = \frac{1}{k} \cdot \frac{1}{k} \cdot \frac{1}{k}$$

$$\frac{xyz}{abc} = \frac{1}{k^3}$$

Multiply B.S by 3

$$3 \times \frac{xyz}{abc} = 3 \times \frac{1}{k^3}$$

$$\frac{3xyz}{abc} = \frac{3}{k^3} \quad \dots\dots \text{equ(iii)}$$

From equ (ii) and equ (iii)

$$\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

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Ex # 3.4

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Q1: (i) **If $\frac{a}{b}=\frac{c}{d}$ then prove that**

$$\frac{2a+3b}{2a-3b}=\frac{2c+3d}{2c-3d}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k, \quad \frac{c}{d}=k$

$a=kb, \quad c=kd$

As we have

$$\frac{2a+3b}{2a-3b}=\frac{2c+3d}{2c-3d}$$

L.H.S:

$$\frac{2a+3b}{2a-3b}=\frac{2kb+3b}{2kb-3b}$$

$$\frac{2a+3b}{2a-3b}=\frac{b(2k+3)}{b(2k-3)}$$

$$\frac{2a+3b}{2a-3b}=\frac{2k+3}{2k-3}$$

R.H.S:

$$\frac{2c+3d}{2c-3d}=\frac{2kd+3d}{2kd-3d}$$

$$\frac{2c+3d}{2c-3d}=\frac{d(2k+3)}{d(2k-3)}$$

$$\frac{2c+3d}{2c-3d}=\frac{2k+3}{2k-3}$$

L.H.S=R.H.S

Q1: (ii) **If $\frac{a}{b}=\frac{c}{d}$ then prove that $\frac{pa+qb}{ma-nb}=\frac{pc+qd}{mc-nd}$**

Solution:

Let $\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k, \quad \frac{c}{d}=k$

$a=kb, \quad c=kd$

As we have

$$\frac{pa+qb}{ma-nb}=\frac{pc+qd}{mc-nd}$$

L.H.S:

$$\frac{pa+qb}{ma-nb}=\frac{pkb+qb}{mkb-nb}$$

$$=\frac{b(pk+q)}{b(mk-n)}$$

$$=\frac{pk+q}{mk-n}$$

Ex # 3.4

R.H.S:

$$\frac{pc+qd}{mc-nd}=\frac{pkd+qd}{mkd-nd}$$

$$=\frac{d(pk+q)}{d(mk-n)}$$

$$=\frac{pk+q}{mk-n}$$

L.H.S=R.H.S

Q2: **Prove that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$**

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ ……equ(i)

$\frac{a}{b}=k, \quad \frac{c}{d}=k, \quad \frac{e}{f}=k$

$a=kb, \quad c=kd, \quad e=kf$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{p(kb)^2+q(kd)^2+(kf)^2}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{pk^2b^2+qk^2d^2+k^2f^2}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{k^2(pb^2+qd^2+f^2)}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{k^2}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=k \quad \text{……equ(ii)}$$

From equ(i) and equ(ii), we get

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$$

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Ex # 3.4

Q3: **If** $\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}$ **then prove that** $x=y=z$ **where** x, y **and** z **are non − zero numbers and** $z+y+z\neq 0$

Solution:

As we have

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}$$

Now we know that

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=\frac{x-y+y-z+z-x}{z+x+y}$$

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=\frac{x-x-y+y-z+z}{x+y+z}$$

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=0$$

$$\frac{x-y}{z}=0,\quad \frac{y-z}{x}=0,\quad \frac{z-x}{y}=0$$

$$x-y=0\times z\ ,y-z=0\times x\ ,z-x=0\times y$$

$$x-y=0,\quad y-z=0,\quad z-x=0$$

$$x=y,\quad y=z,\quad z=x$$

Now by Transitive Property

$$x=y=z$$

Hence Proved

Q4: $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$ **then prove that** $\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$

Solution:

Let $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}=k$

So

$$\frac{2y+2z-x}{a}=k$$

$$2y+2z-x=ak\ \ldots\ldots \text{equ(i)}$$

$$\frac{2z+2x-y}{b}=k$$

$$2z+2x-y=bk\ \ldots\ldots \text{equ(ii)}$$

$$\frac{2x+2y-z}{c}=k$$

$$2x+2y-z=ck\ \ldots\ldots \text{equ(iii)}$$

Arrange them

$$-x+2y+2z=ak\ \ldots\ldots \text{equ(iv)}$$

$$2x-y+2z=bk\ \ldots\ldots \text{equ(v)}$$

$$2x+2y-z=ck\ \ldots\ldots \text{equ(vi)}$$

Multiplied equ (iv) by − 1 $\quad x-2y-2z=-ak$
Multiplied equ (v) by 2 $\quad 4x-2y+4z=2bk$
Multiplied equ (vi) by 2 $\quad 4x+4y-2z=2ck$
Now Add them $\quad 9x=-ak+2bk+2ck$
$\quad 9x=k(-a+2b+2c)$

$$9x=k(2b+2c-a)$$

$$\frac{x}{2b+2c-a}=\frac{k}{9}\ \ldots\ldots \text{equ(vii)}$$

Multiplied equ (iv) by 2 $\quad -2x+4y+4z=2ak$
Multiplied equ (v) by − 1 $\quad -2x+y-2z=-bk$
Multiplied equ (vi) by 2 $\quad 4x+4y-2z=2ck$
Now Add them $\quad 9y=2ak-bk+2ck$
$\quad 9y=k(2a-b+2c)$

$$9y=k(2c+2a-b)$$

$$\frac{x}{2c+2a-b}=\frac{k}{9}\ \ldots\ldots \text{equ(viii)}$$

Multiplied equ (iv) by 2 $\quad -2x+4y+4z=2ak$
Multiplied equ (v) by 2 $\quad 4x-2y+4z=2bk$
Multiplied equ (vi) by − 1 $\quad -2x-2y+z=-ck$
Now Add them $\quad 9z=2ak+2bk-ck$
$\quad 9z=k(2a+2b-c)$

$$9y=k(2a+2b-c)$$

$$\frac{x}{2a+2b-c}=\frac{k}{9}\ \ldots\ldots \text{equ(ix)}$$

From equ (vii), (viii) and (ix), we get

$$\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$$

Q5: **Prove that each of its fraction in** $\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}$ **is equal to** $\frac{x+y+z}{a+b+c}$

Solution:

As we have

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}$$

Now we know that

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{x+y+y+z+z+x}{a+b+b+c+c+a}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{2x+2y+2z}{2a+2b+2c}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{2(x+y+z)}{2(a+b+c)}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{x+y+z}{a+b+c}$$

Hence Proved

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Ex # 3.4

Q6: If $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}$ then $(a+b+c)(x+y+z)=ax+by+cz$

Solution:

Let

$$\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}=k$$

$$\frac{bz+cy}{b-c}=k, \quad \frac{cx+az}{c-a}=k, \quad \frac{ay+bx}{a-b}=k$$

$$bz+cy=k(b-c)$$

$$bz+cy=kb-kc$$

$$cx+az=k(c-a)$$

$$cx+az=kc-ka$$

$$ay+bx=k(a-b)$$

$$ay+bx=ka-kb$$

Add equ (i), (ii) and (iii)

$$bz+cy+cx+az+ay+bx=kb-kc+kc-ka+ka-kb$$

$$bz+cy+cx+az+ay+bx=0$$

Add ax, by, cz on B.S

$$bz+cy+cx+az+ay+bx+ax+by+cz=ax+by+cz$$

Re-arrange it

$$ax+ay+az+bx+by+bz+cx+cy+cz=ax+by+cz$$

Now

$$a(x+y+z)+b(x+y+z)+c(x+y+z)=ax+by+cz$$

$$(x+y+z)(a+b+c)=ax+by+cz$$

$$(a+b+c)(x+y+z)=ax+by+cz$$

Q7: If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$ then $(b-c)x+(c-a)y+(a-b)z=0$

Solution:

Let $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}=k$

$$\frac{x}{b+c-a}=k, \quad \frac{y}{c+a-b}=k, \quad \frac{z}{a+b-c}=k$$

$$\frac{x}{b+c-a}=k$$

$$x=k(b+c-a)$$

$$x=kb+kc-ka$$

$$\frac{y}{c+a-b}=k$$

$$y=k(c+a-b)$$

$$y=kc+ka-kb$$

$$\frac{z}{a+b-c}=k$$

$$z=k(a+b-c)$$

$$z=ka+kb-kc$$

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Ex # 3.4

L.H.S:

$(b-c)x+(c-a)y+(a-b)z$

Put the values of x, y and z

$(b-c)(kb+kc-ka)+(c-a)(kc+ka-kb)+(a-b)(ka+kb-kc)$

$=kb^2+kbc-kab-kbc-kc^2+kac+kc^2+kac-kbc-kac-ka^2+kab+ka^2+kab-kac-kab-kb^2+kbc$

$=0$

$R.H.S$

Q8: **If $2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$ then $7x+6y+8z=0$**

Solution:

As we know that $a:b:c=x:y:z$ Then $a+b+c=x+y+c$

So

$2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$

$2x+3y+3y+4z+4z+5x=4a-5b+3b-a+2b-3a$

$2x+5x+3y+3y+4z+4z=4a-a-3a-5b+3b+2b$

$7x+6y+8z=3a-3a-5b+5b$

$7x+6y+8z=0$

Q9: **If $\frac{a-b}{d-e}=\frac{b-c}{e-f}$ then each of them is equal to $\frac{b\{(f-d)+(cd-af)\}}{e(f-d)}$**

Solution:

Let $\frac{a-b}{d-e}=\frac{b-c}{e-f}=k$

$\frac{a-b}{d-e}=k, \qquad \frac{b-c}{e-f}=k$

$a-b=k(d-e)$

$a-b=dk-ek$

Multiply B.S by f

$f(a-b)=f(dk-ek)$

$af-bf=dfk-efk$ ... ... equ(i)

Also $b-c=k(e-f)$

$b-c=ek-fk$

Multiply B.S by d

$d(b-c)=d(ek-fk)$

$bd-cd=dek-dfk$ ... ... equ(ii)

Add equ(i) and equ(ii)

$af-bf+bd-cd=dfk-efk+dek-dfk$

$af-bf+bd-cd=-efk+dek$

Multiply B.S by -1

$-1(af-bf+bd-cd)=-1(-efk+dek)$

$-af+bf-bd+cd=efk-dek$

$bf-bd+cd-af=k(ef-de)$

$\frac{b(f-d)+cd-af}{ef-de}=k$

$\frac{b(f-d)+(cd-af)}{e(f-d)}=k$

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Ex # 3.5

Example 19:

A stone is dropped from the top of a hill. The distance it falls is proportional to the square of the time of fall. The stone falls 19.6 m after 2 seconds, how far does it fall after 3 seconds?

Solution:

As there is direct variation. Thus

$d \propto t^2$

$d \propto t^2$ ... ... equ(i)

Put $d = 19.6$ and $t = 2$ in equ(i)

$19.6 = k(2)^2$

$19.6 = k(4)$

$\frac{19.6}{4} = k$

$4.9 = k$

$k = 4.9$

Now

To Find:

d *when* $t = 3$

$d = ?$ and $t = 3$

Put $t = 3$ and $k = 4.9$ in equ(i)

$d = (4.9)(3)^2$

$d = (4.9)(9)$

$d = 44.1$

Thus it has fallen 44.1 m after 3 seconds

Example 20:

Height of an image y on a screen varies directly as distance x of the projector from the screen. Height of the image is 20 cm when distance of the projector from the screen is 100 cm. At what distance should the projector kept from the screen so that the height of an image on the screen be 15 cm.

Solution:

As Height of an image $= y$

And Distance of projector $= x$

As there is direct variation. Thus

$y \propto x$

$y = kx$ ... ... equ(i)

Put $x = 100$ and $y = 20$ in equ(i)

$20 = k(100)$

$\frac{20}{100} = \frac{k(100)}{100}$

Ex # 3.5

$\frac{1}{5} = k$

$k = \frac{1}{5}$

Now

Put $y = 15$ and $k = \frac{1}{5}$ in equ(i)

$15 = \frac{1}{5}(x)$

$5 \times 15 = x$

$75 = x$

$x = 75$

Thus Distance of projectro from screen $= 75$ cm

Example 21:

The ratio of the mass of sand to cement in a particular type of concrete is $4.8 : 2$. If 6 kg of sand are used, how much cement is needed?

Solution:

Let the cement required $= x\ kg$

Now the ratio between sand and cement is given by:

$sand : cement$

$4.8 : 2$

$6 : x$

As there is direct variation

So

$\frac{4.8}{6} = \frac{2}{x}$

By cross multiplication

$4.8 \times x = 2 \times 6$

$4.8x = 12$

Divide B.S by 4.8

$\frac{4.8x}{4.8} = \frac{12}{4.8}$

$x = 2.5$

Thus the cement required $= 2.5\ kg$

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Ex # 3.5

Example 22:

4 people can paint a fence in 3 hours.
How long will it take 6 people to paint it?
How many people are needed to complete the job in half an hour?

Solution:

As Number of people = P
And Time to complete work = T
As there is Inverse variation

$T \propto \frac{1}{P}$

$T = \frac{k}{P}$ ... ... equ(i)

Put $T = 3$ and P = 4 in equ(i)

$3 = \frac{k}{4}$

$3 \times 4 = k$

$12 = k$

$k = 12$

Now
To Find:

$T \; when \; P = 10$

$T = ?, P = 6$

Put $P = 6$ and $k = 12$ in equ(i)

$T = \frac{12}{6}$

$T = 2$

Thus Time to complete work = 2 hrs

Now again
To Find:

$P \; when \; T = \frac{1}{2}$

$P = ?, T = 6$

Put $T = 6$ and $k = 12$ in equ(i)

$\frac{1}{2} = \frac{12}{P}$

By Cross Multiplication

$1 \times P = 12 \times 2$

$P = 24$

Thus Number of people required = 24

Ex # 3.5

Page # 68

Q1: A hedge is made of wooden planks. The thickness (T) of the hedge varies directly as number of planks (N). 4 planks make 12 cm thick edge. Find

(i) Thickness of the hedge when number of planks is 6.
(ii) Number of planks when thickness of the hedge is 9cm

Solution:

As thickness of the hedge = T
And number of planks = N
As there is direct variation. Thus

$T \propto N$

$T = kN$ ... ... equ(i)

Put T = 12 and N = 12 in equ(i)

$12 = k(4)$

Divide B.S by 4

$\frac{12}{4} = \frac{k(4)}{4}$

$3 = k$

$k = 3$

Now
To Find:

$T \; when \; N = 6$

$T = ?, N = 6$

Put $N = 6$ and $k = 3$ in equ(i)

$T = 3(6)$

$T = 18$

Thus thickness of the hedge = 18 cm

Now again
To Find:

$N \; when \; T = 9$

$N = ?, T = 9$

Put $T = 9$ and $k = 3$ in equ(i)

$9 = (3)N$

Divide B.S by 3

$\frac{9}{3} = \frac{(3)N}{3}$

$3 = N$

$N = 3$

Also number of planks = 3

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Ex # 3.5

Q2: In a fountain, the pressure "P" of water at any internal point varies directly as depth 'd' from the surface. Pressure is $51\ \text{Newton/cm}^2$ when depth is 3cm. find pressure when depth is 7cm.

Solution:

$\text{As Pressure} = \text{P}$

$\text{And depth} = \text{d}$

As there is direct variation. Thus

$P \propto d$

$P = kd \quad \dots\dots \text{equ(i)}$

$\text{Put P} = 51 \text{ and d} = 3 \text{ in equ(i)}$

$51 = k(3)$

Divide B.S by 3

$\frac{51}{3} = \frac{k(3)}{3}$

$17 = k$

$k = 17$

Now

To Find:

$\text{P } when\ d = 7$

$P = ?, d = 7$

$\text{Put } d = 7 \text{ and } k = 17 \text{ in equ(i)}$

$P = 17(7)$

$P = 119$

$\text{Thus Pressure} = 119\ \text{Newton/cm}^2$

Q3: Pressure P of gas in a container varies directly as temperature T. When pressure is $50\ \text{Newton/m}^2$, temperature is 75 ^{0}C. Find the pressure when temperature is 150 ^{0}C.

Solution:

$\text{As Pressure} = \text{P}$

$\text{And Temperature} = \text{T}$

As there is direct variation. Thus

$P \propto T$

$P = kT \quad \dots\dots \text{equ(i)}$

$\text{Put P} = 50 \text{ and T} = 75 \text{ in equ(i)}$

$50 = k(75)$

Divide B.S by 75

$\frac{50}{75} = \frac{k(75)}{75}$

$\frac{2}{3} = k$

$k = \frac{2}{3}$

Ex # 3.5

Now

To Find:

$\text{P } when \text{ T} = 150$

$P = ?, \text{T} = 150$

$\text{Put T} = 150 \text{ and } k = \frac{2}{3} \text{ in equ(i)}$

$P = \frac{2}{3}(150)$

$P = 2(50)$

$P = 100$

$\text{Thus Pressure} = 100\ \text{Newton/m}^2$

Q4: If 8 persons complete a work in 10 days then how many days would 10 persons take to complete same work?

Solution:

$\text{As Number of persons} = \text{P}$

$\text{And number of Days} = \text{N}$

As there is Inverse variation

$N \propto \frac{1}{P}$

$N = \frac{k}{P} \quad \dots\dots \text{equ(i)}$

$\text{Put } N = 10 \text{ and P} = 8 \text{ in equ(i)}$

$8 = \frac{k}{10}$

$8 \times 10 = k$

$80 = k$

$k = 80$

Now

To Find:

$N\ when\ P = 10$

$N = ?, P = 10$

$\text{Put } P = 10 \text{ and } k = 80 \text{ in equ(i)}$

$N = \frac{80}{10}$

$N = 8$

$\text{Thus Number of days} = 8$

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Ex # 3.5

Q5: **Volume of gas 'V' varies inversely as pressure 'P'. $P = 300\ \text{N/m}^2$ when $V = 4m^3$. Find pressure when $V = 3m^3$.**

Solution:

As Volume = V
And Pressure = P
As there is Inverse variation

$P \propto \frac{1}{V}$

$P = \frac{k}{V}$ ……equ(i)

Put P = 300 and V = 4 in equ(i)

$300 = \frac{k}{4}$

$300 \times 4 = k$

$1200 = k$

$k = 1200$

Now
To Find:

P when $V = 3$

$P = ?, V = 3$

Put $V = 3$ and $k = 1200$ in equ(i)

$P = \frac{1200}{3}$

$P = 400$

Thus Pressure = $400\ \text{N/m}^2$

Q6: **Attraction force 'F' between two magnets vary inversely as square of the distance 'd' between them. F is 18 Newton when d is 2cm. find the distance when attraction force is 2 Newton.**

Solution:

As Force = F
And Distance = d
As there is Inverse variation

$F \propto \frac{1}{d^2}$

$F = \frac{k}{d^2}$ ……equ(i)

Put F = 18 and d = 2 in equ(i)

$18 = \frac{k}{(2)^2}$

$18 = \frac{k}{4}$

$18 \times 4 = k$

Ex # 3.5

$72 = k$

$k = 72$

Now
To Find:

d when $F = 2$

$d = ?, F = 2$

Put $F = 2$ and $k = 72$ in equ(i)

$2 = \frac{72}{d^2}$

$d^2 = \frac{72}{2}$

$d^2 = 36$

Taking square on B.S

$\sqrt{d^2} = \sqrt{36}$

$d = 6$

Thus Distance between magnets = 6cm

Q7: **The volume of a right circular cylinder varies jointly as the height and the square of the radius. The volume of a right circular cylinder, with radius 4centimeters and height 7centimeters, is 352 cm^3. Find the volume of another cylinder with radius 8 centimeters and height 14centimeters.**

Solution:

Let Volume of right circular cylinder = V
And Height of right circular cylinder = h
Radius of right circular cylinder = r

According to condition

V varies jointly as h and r^2

So

$V \propto hr^2$

$V = khr^2$ ……equ(i)

Put $V = 352, h = 7$ and $r = 4$ in equ(i)

$352 = k(7)(4)^2$

$352 = k(7)(16)$

$\frac{352}{(7)(16)} = k$

$\frac{22}{7} = k$

$k = \frac{22}{7}$

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Now
To Find:
$V \; when \; h = 14$ and $r = 8$
$V = ?, h = 14$ and $r = 8$
Put $h = 14, r = 8$ and $k = \frac{22}{7}$ in equ(i)
$V = \left(\frac{22}{7}\right)(14)(8)^2$
$V = \left(\frac{22}{7}\right)(14)(64)$
$V = (22)(2)(64)$
$V = 2816$
Thus volume of a right circular cylinder = 2816cm^3

Review Ex # 3

Page # 69-70

Q2: Find the constant of variation when $s \propto t^2$ and $t = 10$ when $s = 5$

Solution:
As there is direct variation
$s \propto t^2$
$s = kt^2$ ...... equ(i)
Put $s = 5$ and $t = 10$ in equ(i)
$5 = k(10)^2$
$5 = k(100)$
$\frac{5}{100} = \frac{k(100)}{100}$
$\frac{1}{20} = k$
$k = \frac{1}{20}$

Q3: $y \propto \frac{1}{x^2}$ and $y = 4$ When $x = 3$. Find x when $y = 9$

Solution:
As there is Inverse variation
$y \propto \frac{1}{x^2}$
$y = \frac{k}{x^2}$ ...... equ(i)
Put $x = 3$ and $y = 4$ in equ(i)
$4 = \frac{k}{(3)^2}$

$4 = \frac{k}{9}$
$4 \times 9 = k$
$36 = k$
$k = 36$
Now
To Find:
$x \; when \; y = 9$
$x = ?, y = 9$
Put $y = 9$ and $k = 36$ in equ(i)
$9 = \frac{36}{x^2}$
$x^2 = \frac{36}{9}$
$x^2 = 4$
Taking square root on B.S
$\sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 2$

Q4: Pressure of gas in the closed vessel varies directly with the temperature. If pressure is 150 unit the temperature is 70 units. What will be the pressure if temperature is 140 units?

Solution:
As Pressure = P
And Temperature = T
As there is direct variation. Thus
$P \propto T$
$P = kT$ ...... equ(i)
Put P = 150 and T = 70 in equ(i)
$150 = k(70)$
Divide B.S by 70
$\frac{150}{70} = \frac{k(70)}{70}$
$\frac{15}{7} = k$
$k = \frac{15}{7}$
Now To Find:
$P = ?, T = 140$
Put T = 140 and $k = \frac{15}{7}$ in equ(i)
$P = \frac{15}{7}(140)$
$P = 15(20)$
$P = 300$
Thus Pressure = 300 units

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Review Ex # 3

5: In an electric circuit, current varies inversely as resistance. When current is 44 amp, the resistance is 30 ohm. How much current will flow if resistance becomes 22 ohm.

Solution:

As Electric current $= \mathrm{I}$

And Resistance $= \mathrm{R}$

As there is Inverse variation

$I \propto \frac{1}{R}$

$I = \frac{k}{R}$ ... ... equ(i)

Put $I = 44$ and $R = 30$ in equ(i)

$44 = \frac{k}{30}$

$44 \times 30 = k$

$1320 = k$

$k = 1320$

Now

To Find:

I when $R = 22$

$I = ?, R = 22$

Put $R = 22$ and $k = 1320$ in equ(i)

$\boldsymbol{I = \frac{1320}{22}}$

$I = 60$

Thus Electric current $= 60$ amp

6: If a varies jointly as b and square root of c. If $a = 21$ when $b = 5$ and $c = 36$, Find a when $b = 9$ and $c = 225$

Solution:

As a varies jointly as b and square root of c

So

$$a \propto b\sqrt{c}$$

$$a = kb\sqrt{c} \quad \text{... ... equ(i)}$$

Put $a = 21, b = 5$ and $c = 36$ in equ(i)

$$21 = k(5)\sqrt{36}$$

$$21 = k(5)(6)$$

$$\frac{21}{(5)(6)} = k$$

$$\frac{7}{(5)(2)} = k$$

$$\frac{7}{10} = k$$

Review Ex # 3

$k = \frac{7}{10}$

Now

To Find:

a when $b = 9$ and $c = 225$

$a = ?, b = 9$ and $c = 225$

Put $b = 9, c = 225$ and $k = \frac{7}{10}$ in equ(i)

$a = \left(\frac{7}{10}\right)(9)\sqrt{225}$

$a = \left(\frac{7}{10}\right)(9)(15)$

$a = \frac{945}{10}$

$a = 94.5$

Q7: What number should be added to each of number 3, 8, 11 and 20 to make them in proportion?

Solution:

Suppose the number $= x$

As x is added to each of number

So according to condition

$3 + x : 8 + x = 11 + x : 20 + x$

Product of mean $=$ Product of extreme

$(8 + x)(11 + x) = (3 + x)(20 + x)$

$88 + 8x + 11x + x^2 = 60 + 3x + 20x + x^2$

$88 + 19x + x^2 = 60 + 23x + x^2$

$x^2 + 19x + 88 = x^2 + 23x + 60$

$x^2 - x^2 + 19x - 23x + 88 - 60 = 0$

$-4x + 28 = 0$

$-4x = -28$

Divide B.S by -4

$\frac{-4x}{-4} = \frac{-28}{-4}$

$x = 7$

Thus 7 should be added to each number

So, number becomes

$3 + 7 : 8 + 7 = 11 + 7 : 20 + 7$

$10 : 15 = 18 : 27$

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Review Ex # 3

8: What number should be subtracted to each f the number 6, 8, 7 and 11 so that the remaining numbers are in proportion?

Solution:

Suppose the number $= x$

As x is subtracted to each of number

So according to condition

$6 - x : 8 - x = 7 - x : 11 - x$

Product of mean = Product of extreme

$(8 - x)(7 - x) = (6 - x)(11 - x)$

$56 - 8x - 7x + x^2 = 66 - 6x - 11x + x^2$

$56 - 15x + x^2 = 66 - 17x + x^2$

$x^2 - 15x + 56 = x^2 - 17x + 66$

$x^2 - x^2 - 15x + 17x + 56 - 66 = 0$

$2x - 10 = 0$

$2x = 10$

Divide B.S by 2

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Thus 5 should be subtracted to each number

So, number becomes

$6 - 5 : 8 - 5 = 7 - 5 : 11 - 5$

$1 : 3 = 2 : 6$

9: The ratio between two numbers is 8 : 3 and their difference is 20. Find the numbers.

Solution:

Let the two numbers are x and y

According to first condition

$x : y = 8 : 3$

$\frac{x}{y} = \frac{8}{3}$

By cross multiplication

$3x = 8y$ ……. equ(i)

Now according to second condition

$x - y = 20$

$x = 20 + y$ ……. equ(ii)

Put the value of x in equ(i)

$3(20 + y) = 8y$

$60 + 3y = 8y$

$60 = 8y - 3y$

$60 = 5y$

$\frac{60}{5} = y$

$12 = y$

$y = 12$

Put the value of y in equ(ii)

$x = 20 + 12$

$x = 32$

Thus the required numbers are 32 and 12 is 8 : 3 and their difference is 20.

OR

$Let\ first\ number = 8x$

$Second\ number = 3x$

$According\ to\ condition$

$8x - 3x = 20$

$5x = 20$

$x = \frac{20}{5}$

$x = 4$

$Now\ first\ number = 8x = 8(4) = 32$

$Second\ number = 3x = 3(4) = 12$

Thus, the required numbers are 32 and 12 is 8 : 3 and their difference is 20.

10: Find the number in continued proportion such that their sum is 14 and sum of their squared is 84.

Solution:

Let x, y and z be the three numbers

As they are in continued proportion

$x : y = y : z$

$y^2 = xz$ …… equ (i)

According to conditions

$x + y + z = 14$ …… equ (ii)

$x^2 + y^2 + z^2 = 84$ …… equ (iii)

Put $y^2 = xz$ in equ (iii)

$x^2 + xz + z^2 = 84$ …… equ (iv)

From equ(ii)

$x + z = 14 - y$ …… equ (v)

Taking square on B.S

$(x + z)^2 = (14 - y)^2$

$z^2 + 2xz + z^2 = 196 - 28y + y^2$

Put $y^2 = xz$

$z^2 + 2xz + z^2 = 196 - 28y + xz$

$z^2 + 2xz - xz + z^2 = 196 - 28y$

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Review Ex # 3

$z^2 + xz + z^2 = 196 - 28y$ ... ... equ (vi)

Compare equ(iv) and equ(vi)

$84 = 196 - 28y$

$84 - 196 = -28y$

$-112 = -28y$

$4 = y$

$y = 4$

Put $y = 4$ in equ(i) and equ(ii)

$(4)^2 = xz$

$16 = xz$

$xz = 16$ ... ... equ (vii)

Now

$x + 4 + z = 14$

$x + z = 14 - 4$

$x + z = 10$

$z = 10 - x$ ... ... equ (viii)

Put equ(viii) in equ(vii)

$x(10 - x) = 16$

$10x - x^2 = 16$

$0 = x^2 - 10x + 16$

$x^2 - 10x + 16 = 0$

$x^2 - 2x - 8x + 16 = 0$

$x(x - 2) - 8(x - 2) = 0$

$(x - 2)(x - 8) = 0$

$x - 2 = 0$ or $x - 8 = 0$

$x = 2$ or $x = 8$

Now Put $x = 2$ in equ(viii)

$z = 10 - 2$

$z = 8$

Also Put $x = 8$ in equ(viii)

$z = 10 - 8$

$z = 2$

Thus the required numbers are:

2, 4, 8

OR

8, 4, 2

11: The mean proportion of two numbers is 6 and their sum is 13. Find the number.

Solution:

Let x and y be the numbers

As the mean proportion=6

$x : 6 = 6 : y$

$36 = xy$

Review Ex # 3

$xy = 36$ ... ... equ (i)

According to condition

$x + y = 13$

$y = 13 - x$ ... ... equ (ii)

Put the value of x in equ(i)

$x(13 - x) = 36$

$13x - x^2 = 36$

$0 = x^2 - 13x + 36$

$x^2 - 13x + 36 = 0$

$x^2 - 4x - 9x + 36 = 0$

$x(x - 4) - 9(x - 4) = 0$

$(x - 4)(x - 9) = 0$

$x - 4 = 0$ or $x - 9 = 0$

$x = 4$ or $x = 9$

Now Put $x = 4$ in equ(iii)

$y = 13 - 4$

$y = 9$

Also Put $x = 9$ in equ(iii)

$y = 13 - 9$

$y = 4$

Thus the required numbers are:

4 and 9

OR

9 and 4

12: Find angle of a triangle which are in ratio 3 : 4 : 5

Solution:

As triangle has three angles

Also we know that

Sum of angles=180

As ratio of given triangle=3 : 4 : 5

Sum the Ratio $= 3 + 4 + 5$

$= 12$

First Angle $= \frac{3}{12} \times 180^0$

$= 3 \times 15^0$

$= 45^0$

Second Angle $= \frac{4}{12} \times 180^0$

$= 4 \times 15^0$

$= 60^0$

Third Angle $= \frac{5}{12} \times 180^0$

$= 5 \times 15^0$

$= 75^0$

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Review Ex # 3

13: **If $\frac{a}{b} = \frac{c}{d}$ then prove that**
$$\frac{ac(a+c)}{bd(b+d)} = \frac{(a+c)^3}{(b+d)^3}$$

Solution:

$As\ \frac{a}{b} = \frac{c}{d}$

Now $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

So $\frac{a}{b} = \frac{a+c}{b+d}$ ... ... equ(i)

And $\frac{c}{d} = \frac{a+c}{b+d}$ ... ... equ(ii)

As $\frac{a}{b} = \frac{a+c}{b+d}$

Multiply B.S by $\frac{c}{d}$

$\frac{a}{b} \times \frac{c}{d} = \left(\frac{c}{d}\right)\left(\frac{a+c}{b+d}\right)$

As $\frac{c}{d} = \frac{a+c}{b+d}$

So $\frac{ac}{bd} = \left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)$

Now multiply B.S by $\left(\frac{a+c}{b+d}\right)$

$\frac{ac}{bd}\left(\frac{a+c}{b+d}\right) = \left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)$

$\frac{ac(a+c)}{bd(b+d)} = \left(\frac{a+c}{b+d}\right)^3$

14: **If a, b, c are in continued proportion then prove that $\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$**

Solution:

As a, b and c are in continued proportion

So

$a : b = b : c$

$\frac{a}{b} = \frac{b}{c}$

Let $\frac{a}{b} = \frac{b}{c} = k$

$\frac{a}{b} = k, \quad \frac{b}{c} = k$

$a = kb$ ... equ (i), $\quad b = kc$ ... equ (ii)

Put $b = kc$ in equ(i)

$a = k(kc)$

$a = k^2c$ ... ... equ (iii)

Review Ex # 3

$\frac{a}{c} = k^2$ ... ... equ (iv)

Now

$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{(k^2c)^2 + (k^2c)(kc) + (kc)^2}{(kc)^2 + (kc)c + c^2}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{k^4c^2 + k^3c^2 + k^2c^2}{k^2c^2 + kc^2 + c^2}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{k^2c^2(k^2+k+1)}{c^2(k^2+k+1)}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2} = k^2$ ... ... equ (v)

Now again

$\frac{a^2-b^2}{b^2-c^2} = \frac{(k^2c)^2 - (kc)^2}{(kc)^2 - c^2}$

$\frac{a^2-b^2}{b^2-c^2} = \frac{k^4c^2 - k^2c^2}{k^2c^2 - c^2}$

$\frac{a^2-b^2}{b^2-c^2} = \frac{k^2c^2(k^2-1)}{c^2(k^2-1)}$

$\frac{a^2-b^2}{b^2-c^2} = k^2$ ... ... equ (vi)

From equ(iv) , equ(v) and equ(vi)

$\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$

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Review Ex # 3

Q15: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then prove that

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\frac{a}{b}=k,\quad \frac{c}{d}=k,\quad \frac{e}{f}=k$

$a=kb,\quad c=kd,\quad e=kf$

To prove:

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Now L.H.S:

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{(kb)^3+(kd)^3+(kf)^3}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{k^3b^3+k^3d^3+k^3f^3}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{k^3(b^3+d^3+f^3)}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=k^3$$

Also R.H.S:

$$\frac{ace}{bdf}=\frac{(kb)(kd)(kf)}{bdf}$$

$$\frac{ace}{bdf}=\frac{k^3bdf}{bdf}$$

$$\frac{ace}{bdf}=k^3$$

Thus L.H.S=R.H.S

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Hence Proved:

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Chapter # 3

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MATHEMATICS

Class 10th (KPK)

Chapter # 4 Partial Fraction

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Exercise # 4.1

UNIT # 4

PARTIAL FRACTIONS

Partial Fraction:

A procedure which does splitting up a fraction into two or more fractions with only one factors in the denominator is called partial fraction.

In other words, a set of fractions whose algebraic sum is a given fraction is called partial fraction.

Rational Fraction:

A rational function can be written in the form of:

$$\mathrm{f}\,(x)=\frac{\mathrm{P}\,(x)}{\mathrm{Q}\,(x)}$$

$$\text{Where P }(x)\text{ and Q }(x)\text{ are polynomials, where Q }(x)\ \neq 0$$

Proper rational fraction:

A rational fraction is proper fraction, if degree of numerator P (x) is less than the degree of denominator Q (x).

Example

$$\frac{1}{x+1},\ \frac{2x}{x^2+2},\ \frac{x^2+x-3}{x^3+x^2-x+1}$$

Improper rational fraction

A rational fraction is an improper fraction, if degree of numerator P (x) is greater than or equal to the degree of denominator Q (x).

Example

$$\frac{x^3+4}{(x+1)(x+2)},\ \frac{x}{2x+2},\ \frac{x^2+x-3}{x^2-x+1},\ \frac{x^3+x^2+x-3}{x^2-x+1}$$

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Note:

Any improper rational fraction can be reduced into sum of polynomials and rational fraction by large division.

Example:

$$\frac{\mathbf{2x^2+1}}{\mathbf{x-1}}$$

Solution:

$$\begin{array}{r|l} & 2x+2 \\ \hline x-1 & 2x^2+1 \\ & \pm 2x^2 \quad \mp 2x \\ \hline & 2x+1 \\ & \pm 2x \mp 2 \\ \hline & 3 \end{array}$$

$$\frac{2x^2+1}{x-1}=2x+2+\frac{3}{x-1}$$

Exercise # 4.1

Resolution of fraction into partial fraction

Resolution of rational fraction $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$ into partial fraction depends upon the factors of denominator $Q(x)$

Case # 1:

Let proper fraction $\frac{P(x)}{Q(x)}$ given

Factorize the polynomial $Q(x)$ in the denominator if it is not factorized.

$$\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Example # 1:

Resolve $\frac{1}{(x+1)(x+2)}$ into partial fraction.

Solution:

$$\frac{1}{(x+1)(x+2)}$$

Let

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x+2)$

$$\frac{1}{(x+1)(x+2)} \times (x+1)(x+2) = \frac{A}{x+1} \times (x+1)(x+2) + \frac{B}{x+2} \times (x+1)(x+2)$$

$1 = A(x+2) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$1 = A(-1+2) + B(0)$

$1 = A(1) + 0$

$1 = A$

$A = 1$

Put $x+2=0 \Rightarrow x=-2$ in equ (ii)

$1 = A(0) + B(-2+1)$

$1 = 0 + B(-1)$

$1 = -B$

$-B = 1$

$B = -1$

Put the values of A and B in equ (i)

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$$

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

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Exercise # 4.1

Example # 2: Find partial fraction of $\dfrac{3x+2}{x^2-x-2}$

Solution:

$$\frac{3x+2}{x^2-x-2}$$

R.W

$$\frac{3x+2}{x^2-x-2}=\frac{3x+2}{(x+1)(x-2)}$$

Now

Let

$$\frac{3x+2}{(x+1)(x-2)}=\frac{\mathrm{A}}{x+1}+\frac{\mathrm{B}}{x-2} \quad \ldots\ldots \mathrm{equ(i)}$$

Multiply equ (i) by $(x+1)(x-2)$

$$\frac{3x+2}{(x+1)(x-2)}\times(x+1)(x-2)=\frac{\mathrm{A}}{x+1}\times(x+1)(x-2)+\frac{\mathrm{B}}{x-2}\times(x+1)(x-2)$$

$$3x+2=\mathrm{A}(x-2)+\mathrm{B}(x+1) \quad \ldots\ldots \mathrm{equ(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$$3(-1)+2=\mathrm{A}(-1-2)+\mathrm{B}(0)$$

$$-3+2=\mathrm{A}(-3)+0$$

$$-1=-3\mathrm{A}$$

$$\frac{-1}{-3}=\mathrm{A}$$

$$\frac{1}{3}=\mathrm{A}$$

$$\mathrm{A}=\frac{1}{3}$$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$$3(2)+2=\mathrm{A}(0)+\mathrm{B}(2+1)$$

$$6+2=0+\mathrm{B}(3)$$

$$8=3\mathrm{B}$$

$$\frac{8}{3}=\mathrm{B}$$

$$\mathrm{B}=\frac{8}{3}$$

Put the values of A and B in equ (i)

$$\frac{3x+2}{(x+1)(x-2)}=\frac{\mathrm{A}}{x+1}+\frac{\mathrm{B}}{x-2}$$

$$\frac{3x+2}{(x+1)(x-2)}=\frac{\frac{1}{3}}{x+1}+\frac{\frac{8}{3}}{x-2}$$

$$\frac{3x+2}{(x+1)(x-2)}=\frac{1}{3(x+1)}+\frac{8}{2(x-2)}$$

Example # 3: Find partial fraction of $\dfrac{x}{(x+1)^2}$

Solution:

$$\frac{x}{(x+1)^2}$$

Let

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Exercise # 4.1

$$\frac{x}{(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)^2$

$$\frac{x}{(x+1)^2}\times(x+1)^2=\frac{A}{x+1}\times(x+1)^2+\frac{B}{(x+1)^2}\times(x+1)^2$$

$x=A(x+1)+B \quad \ldots\ldots \text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$-1=A(0)+B$

$-1=B$

$B=-1$

equ (ii) $\Rightarrow$

$x=A(x+1)+B$

$x=Ax+A+B$

$x=Ax+(A+B)$

By comparing the coefficients of x, we get

$A=1$

Put the values of A and B in equ (i)

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}+\frac{-1}{(x+1)^2}$$

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}-\frac{1}{(x+1)^2}$$

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Example # 4: Find partial fraction of $\frac{2x^2+1}{(x-2)^2(x+3)}$

Solution:

$$\frac{2x^2+1}{(x-2)^2(x+3)}$$

Let

$$\frac{2x^2+1}{(x-2)^2(x+3)}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x+3} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)^2$, we get

$2x^2+1=A(x-2)(x+3)+B(x+3)+C(x-2)^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$2(2)^2+1=A(0)(2+3)+B(2+3)+C(0)^2$

$2(4)+1=0+B(5)+0$

$8+1=5B$

$9=5B$

$\frac{9}{5}=B$

$B=\frac{9}{5}$

Put $x+3=0 \Rightarrow x=-3$ in equ (ii)

$2(-3)^2+1=A(-3-2)(0)+B(0)+C(-3-2)^2$

$2(9)+1=0+0+C(-3-2)^2$

$18+1=C(-5)^2$

$19=C(25)$

Exercise # 4.1

$\frac{19}{25} = C$

$C = \frac{19}{25}$

equ (ii) $\Rightarrow$

$2x^2 + 1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$

$2x^2 + 1 = A(x^2 + 3x - 2x - 6) + Bx + 3B + C(x^2 - 4x + 2)$

$2x^2 + 1 = A(x^2 + x - 6) + Bx + 3B + C(x^2 - 4x + 2)$

$2x^2 + 1 = Ax^2 + Ax - 6A + Bx + 3B + Cx^2 - 4Cx + 2C$

$2x^2 + 1 = Ax^2 + Cx^2 + Ax + Bx - 4Cx - 6A + 3B + 2C$

$2x^2 + 1 = (A + C)x^2 + (A + B - 4C)x + (-6A + 3B + 2C)$

By comparing the coefficients of x^2, we get

$A + C = 2$

Put $C = \frac{19}{25}$

$A + \frac{19}{25} = 2$

$A = 2 - \frac{19}{25}$

$A = \frac{50 - 19}{25}$

$A = \frac{31}{25}$

Put the values of A, B and C in equ (i)

$\frac{2x^2 + 1}{(x-2)^2(x+3)} = \frac{\frac{31}{25}}{x-2} + \frac{\frac{9}{5}}{(x-2)^2} + \frac{\frac{19}{25}}{x+3}$

$\frac{2x^2 + 1}{(x-2)^2(x+3)} = \frac{31}{25(x-2)} + \frac{9}{5(x-2)^2} + \frac{19}{25(x+3)}$

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Exercise # 4.1

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Resolve the following fractions into partial fraction.

(1) $\frac{3x-2}{2x^2-x}$

Solution:

$\frac{3x-2}{2x^2-x}$

$\frac{3x-2}{2x^2-x} = \frac{3x-2}{x(2x-1)}$

Let

$\frac{3x-2}{2x^2-x} = \frac{A}{x} + \frac{B}{2x-1}$ ..... equ(i)

Multiply equ (i) by $x(2x-1)$

$\frac{3x-2}{2x^2-x} \times x(2x-1) = \frac{A}{x} \times x(2x-1) + \frac{B}{2x-1} \times x(2x-1)$

$3x - 2 = A(2x-1) + Bx$ ..... equ(ii)

Put $x = 0$ in equ (ii)

Exercise # 4.1

$3(0)-2=A(2(0)-1)+B(0)$
$0-2=A(0-1)+0$
$-2=A(-1)$
$-2=-A$
$2=A$
$A=2$

Put $2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$ in equ (ii)

$3\left(\frac{1}{2}\right)-2=A(0)+B\left(\frac{1}{2}\right)$
$\frac{3}{2}-2=0+\frac{B}{2}$
$\frac{3-4}{2}=\frac{B}{2}$
$\frac{-1}{2}=\frac{B}{2}$
$-1=B$
$B=-1$

Put the values of A and B in equ (i)

$\frac{3x-2}{2x^2-x}=\frac{2}{x}+\frac{-1}{2x-1}$
$\frac{3x-2}{2x^2-x}=\frac{2}{x}-\frac{1}{2x-1}$

(2) $\boldsymbol{\frac{x-1}{x^2+6x+5}}$

Solution:

$\frac{x-1}{x^2+6x+5}$
$\frac{x-1}{x^2+6x+5}=\frac{x-1}{(x+1)(x+5)}$

R.W
$x^2+6x+5=x^2+1x+5x+5$
$x^2+6x+5=x(x+1)+5(x+1)$
$x^2+6x+5=(x+1)(x+5)$

Let

$\frac{x-1}{(x+1)(x+5)}=\frac{A}{x+1}+\frac{B}{x+5}$ .....equ(i)

Multiply equ (i) by $(x+1)(x+5)$

$\frac{x-1}{(x+1)(x+5)}\times(x+1)(x+5)=\frac{A}{x+1}\times(x+1)(x+5)+\frac{B}{x+5}\times(x+1)(x+5)$
$x-1=A(x+5)+B(x+1)$ .....equ(ii)

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$-1-1=A(-1+5)+B(0)$
$-2=A(4)+0$
$-2=4A$
$\frac{-2}{4}=A$
$\frac{-1}{2}=A$

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Exercise # 4.1

$A = \frac{-1}{2}$

Put $x + 5 = 0 \Rightarrow x = -5$ in equ (ii)

$-5 - 1 = A(0) + B(-5 + 1)$

$-6 = 0 + B(-4)$

$-6 = -4B$

$6 = 4B$

$\frac{6}{4} = B$

$\frac{3}{2} = B$

$B = \frac{3}{2}$

Put the values of A and B in equ (i)

$$\frac{x-1}{(x+1)(x+5)} = \frac{\frac{-1}{2}}{x+1} + \frac{\frac{3}{2}}{x+5}$$

$$\frac{x-1}{(x+1)(x+5)} = \frac{-1}{2(x+1)} + \frac{3}{2(x+5)}$$

OR

$$\frac{x-1}{x^2+6x+5} = \frac{-1}{2(x+1)} + \frac{3}{2(x+5)}$$

(3) $\mathbf{\frac{1}{x^2 - 1}}$

Solution:

$$\frac{1}{x^2 - 1}$$

$$\frac{1}{x^2 - 1} = \frac{1}{x^2 - 1^2}$$

$$\frac{1}{x^2 - 1} = \frac{1}{(x+1)(x-1)}$$

Now

Let

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)$, we get

$1 = A(x-1) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$1 = A(-1 - 1) + B(0)$

$1 = A(-2) + 0$

$1 = -2A$

$\frac{1}{-2} = A$

$A = \frac{1}{-2}$

Exercise # 4.1

$A = -\frac{1}{2}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A(0) + B(1 + 1)$

$1 = 0 + B(2)$

$1 = 2B$

$\frac{1}{2} = B$

$B = \frac{1}{2}$

Put the values of A and B in equ (i)

$$\frac{1}{(x+1)(x-1)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$\frac{1}{(x+1)(x-1)} = \frac{-1}{2(x+1)} + \frac{1}{2(x-1)}$$

OR

$$\frac{1}{x^2-1} = \frac{-1}{2(x+1)} + \frac{1}{2(x-1)}$$

(4) $\boldsymbol{\frac{x}{x^2+4x-5}}$

Solution:

$$\frac{x}{x^2+4x-5}$$

$$\frac{x}{x^2+4x-5} = \frac{x}{(x-1)(x+5)}$$

R.W

$x^2 + 4x - 5 = x^2 - 1x + 5x - 5$

$x^2 + 4x - 5 = x(x-1) + 5(x-1)$

$x^2 + 4x - 5 = (x-1)(x+5)$

Let

$$\frac{x}{(x-1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+5} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)(x+5)$

$$\frac{x}{(x-1)(x+5)} \times (x-1)(x+5) = \frac{A}{x-1} \times (x-1)(x+5) + \frac{B}{x+5} \times (x-1)(x+5)$$

$x = A(x+5) + B(x-1) \quad \ldots\ldots \text{equ(ii)}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A(1 + 5) + B(0)$

$1 = A(6) + 0$

$1 = 6A$

$\frac{1}{6} = A$

$A = \frac{1}{6}$

Put $x + 5 = 0 \Rightarrow x = -5$ in equ (ii)

$-5 = A(0) + B(-5 - 1)$

$-5 = 0 + B(-6)$

$-5 = -6B$

$5 = 6B$

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Exercise # 4.1

$\frac{5}{6} = \text{B}$

$\text{B} = \frac{5}{6}$

Put the values of A and B in equ (i)

$$\frac{x}{(x-1)(x+5)} = \frac{\frac{1}{6}}{x+1} + \frac{\frac{5}{6}}{x+5}$$

$$\frac{x}{(x-1)(x+5)} = \frac{1}{6(x-1)} + \frac{5}{6(x+5)}$$

OR

$$\frac{x}{x^2+4x-5} = \frac{1}{6(x-1)} + \frac{5}{6(x+5)}$$

(5) $\mathbf{\frac{4x+2}{(x+2)(2x-1)}}$

Solution:

$$\frac{4x+2}{(x+2)(2x-1)}$$

Let

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{\text{A}}{x+2} + \frac{\text{B}}{2x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+2)(2x-1)$

$$\frac{4x+2}{(x+2)(2x-1)} \times (x+2)(2x-1) = \frac{\text{A}}{x+2} \times (x+2)(2x-1) + \frac{\text{B}}{2x-1} \times (x+2)(2x-1)$$

$4x+2 = \text{A}(2x-1) + \text{B}(x+2) \quad \ldots\ldots \text{equ(ii)}$

Put $x+2=0 \Rightarrow x=-2$ in equ (ii)

$4(-2)+2 = \text{A}(2(-2)-1) + \text{B}(0)$

$-8+2 = \text{A}(-4-1)+0$

$-6 = \text{A}(-5)$

$-6 = -5\text{A}$

$6 = 5\text{A}$

$\frac{6}{5} = \text{A}$

$\text{A} = \frac{6}{5}$

Put $2x-1=0 \Rightarrow 2\text{x}=1 \Rightarrow x=\frac{1}{2}$ in equ (ii)

$4\left(\frac{1}{2}\right)+2 = \text{A}(0) + \text{B}\left(\frac{1}{2}+2\right)$

$2+2 = 0 + \text{B}\left(\frac{1+4}{2}\right)$

$4 = \text{B}\left(\frac{5}{2}\right)$

$4 \times \frac{2}{5} = \text{B}$

Exercise # 4.1

$\frac{8}{5} = \text{B}$

$\text{B} = \frac{8}{5}$

Put the values of A and B in equ (i)

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{\frac{6}{5}}{x+2} + \frac{\frac{8}{5}}{2x-1}$$

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{6}{5(x+2)} + \frac{8}{5(2x-1)}$$

(7) $\boldsymbol{\frac{x^2+5x+3}{(x^2-1)(x+1)}}$

Solution:

$$\frac{x^2+5x+3}{(x^2-1)(x+1)} = \frac{x^2+5x+3}{(x-1)(x+1)(x+1)}$$

$$\frac{x^2+5x+3}{(x^2-1)(x+1)} = \frac{x^2+5x+3}{(x-1)(x+1)^2}$$

Now

Let

$$\frac{x^2+5x+3}{(x-1)(x+1)^2} = \frac{\text{A}}{x-1} + \frac{\text{B}}{x+1} + \frac{\text{C}}{(x+1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)(x+1)^2$

$$\frac{x^2+5x+3}{(x-1)(x+1)^2} \times (x-1)(x+1)^2 = \frac{\text{A}}{x-1} \times (x-1)(x+1)^2 + \frac{\text{B}}{x+1} \times (x-1)(x+1)^2 + \frac{\text{C}}{(x+1)^2} \times (x-1)(x+1)^2$$

$x^2+5x+3 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$(1)^2 + 5(1) + 3 = A(1+1)^2 + B(0)(x+1) + C(0)$

$1+5+3 = A(2)^2 + 0 + 0$

$9 = \text{A}(4)$

$9 = 4\text{A}$

$\frac{9}{4} = \text{A}$

$\text{A} = \frac{9}{4}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$(-1)^2 + 5(-1) + 3 = A(0)^2 + B(x-1)(0) + C(-1-1)$

$1-5+3 = A(0) + B(0) + C(-2)$

$-4+3 = 0+0-2C$

$-1 = -2C$

$1 = 2C$

$\frac{1}{2} = C$

Exercise # 4.1

$C = \frac{1}{2}$

equ (ii) $\Rightarrow$

$x^2 + 5x + 3 = A(x^2 + 2x + 1) + B(x^2 - 1) + Cx - C$

$x^2 + 5x + 3 = Ax^2 + 2Ax + A + Bx^2 - B + Cx - C$

$x^2 + 5x + 3 = Ax^2 + Bx^2 + 2Ax + Cx + A - B - C$

$x^2 + 5x + 3 = (A + B)x^2 + (2A + C)x + (A - B - C)$

By comparing the coefficients of x^2, we get

$A + B = 1$

Put $A = \frac{9}{4}$

$\frac{9}{4} + B = 1$

$B = 1 - \frac{9}{4}$

$B = \frac{4 - 9}{4}$

$B = \frac{-5}{4}$

Put the values of A, B and C in equ (i)

$$\frac{x^2 + 5x + 3}{(x - 1)(x + 1)^2} = \frac{\frac{9}{4}}{x - 1} + \frac{\frac{-5}{4}}{x + 1} + \frac{\frac{1}{2}}{(x + 1)^2}$$

$$\frac{x^2 + 5x + 3}{(x - 1)(x + 1)^2} = \frac{9}{4(x - 1)} - \frac{5}{4(x + 1)} + \frac{1}{2(x + 1)^2}$$

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(8) $\frac{\boldsymbol{x^2 + 2}}{\boldsymbol{(x + 2)(x^2 + 5x + 6)}}$

Solution:

$$\frac{x^2 + 2}{(x + 2)(x^2 + 5x + 6)} = \frac{x^2 + 2}{(x + 2)(x + 3)(x + 2)}$$

$$\frac{x^2 + 2}{(x + 2)(x^2 + 5x + 6)} = \frac{x^2 + 2}{(x + 3)(x + 2)^2}$$

$R.W$
$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$
$x^2 + 5x + 6 = x(x + 2) + 3(x + 2)$
$x^2 + 5x + 6 = (x + 3)(x + 2)$

Let

$$\frac{x^2 + 2}{(x + 2)(x^2 + 5x + 6)} = \frac{A}{x + 3} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x + 3)(x + 2)^2$

$$\frac{x^2 + 2}{(x + 2)(x^2 + 5x + 6)} \times (x + 3)(x + 2)^2 = \frac{A}{x + 3} \times (x + 3)(x + 2)^2 + \frac{B}{x + 2} \times (x + 3)(x + 2)^2 + \frac{C}{(x + 2)^2} \times (x + 3)(x + 2)^2$$

$x^2 + 2 = A(x + 2)^2 + B(x + 3)(x + 2) + C(x + 3) \quad \ldots\ldots$ equ(ii)

Put $x + 3 = 0 \Rightarrow x = -3$ in equ (ii)

$(-3)^2 + 2 = A(-3 + 2)^2 + B(0)(x + 2) + C(0)$

$9 + 2 = A(-1)^2 + 0 + 0$

$11 = A(1)$

Exercise # 4.1

$11 = A$

$A = 11$

Put $x + 2 = 0 \Rightarrow x = -2$ in equ (ii)

$(-2)^2 + 2 = A(0)^2 + B(x+3)(0) + C(-2+3)$

$4 + 2 = 0 + 0 + C(1)$

$6 = C$

$C = 6$

equ (ii) $\Rightarrow$

$x^2 + 2 = A(x+2)^2 + B(x+3)(x+2) + C(x+3)$

$x^2 + 2 = A(x^2 + 2x + 1) + B(x+3)(x+2) + C(x+3)$

$x^2 + 5x + 3 = A(x^2 + 2x + 1) + B(x^2 - 1) + Cx - C$

$x^2 + 5x + 3 = Ax^2 + 2Ax + A + Bx^2 - B + Cx - C$

$x^2 + 5x + 3 = Ax^2 + Bx^2 + 2Ax + Cx + A - B - C$

$x^2 + 5x + 3 = (A+B)x^2 + (2A+C)x + (A - B - C)$

By comparing the coefficients of x^2, we get

$A + B = 1$

Put $\mathrm{A} = \dfrac{9}{4}$

$\dfrac{9}{4} + B = 1$

$B = 1 - \dfrac{9}{4}$

$B = \dfrac{4-9}{4}$

$B = \dfrac{-5}{4}$

Put the values of A, B and C in equ (i)

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{\frac{9}{4}}{x-1} + \frac{\frac{-5}{4}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2}$$

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{9}{4(x-1)} - \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}$$

OR

$$\frac{x^2 + 2}{(x+2)(x^2 + 5x + 6)} = \frac{9}{4(x-1)} - \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}$$

(8) $\dfrac{\mathbf{2x - 1}}{\boldsymbol{x(x-3)^2}}$

Solution:

$\dfrac{2x - 1}{x(5x-3)^2}$

Let

$$\frac{2x-1}{x(x-3)^2} = \frac{\mathrm{A}}{x} + \frac{\mathrm{B}}{x-3} + \frac{\mathrm{C}}{(x-3)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $x(x-3)^2$

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Exercise # 4.1

$$\frac{2x-1}{x(x-3)^2}\times x(x-3)^2=\frac{A}{x}\times x(x-3)^2+\frac{B}{x-3}\times x(x-3)^2+\frac{C}{(x-3)^2}\times x(x-3)^2$$

$2x-1=A(x-3)^2+Bx(x-3)+Cx \quad \ldots\ldots \text{equ(ii)}$

Put $x=0$ in equ (ii)

$2(0)-1=A(0-3)^2+B(0)(0-3)+C(0)$

$0-1=A(-3)^2+0+0$

$-1=A(9)$

$\frac{-1}{9}=A$

$A=\frac{-1}{9}$

Put $x-3=0 \Rightarrow x=3$ in equ (ii)

$2(3)-1=A(0)^2+B(3)(0)+C(3)$

$6-1=0+0+3C$

$5=3C$

$\frac{5}{3}=C$

$C=\frac{5}{3}$

equ (ii) $\Rightarrow$

$2x-1=A(x-3)^2+Bx(x-3)+Cx$

$2x-1=A(x^2-6x+9)+Bx^2-3Bx+Cx$

$2x-1=Ax^2-6Ax+9A+Bx^2-3Bx+Cx$

$2x-1=Ax^2+Bx^2-6Ax-3Bx+Cx+9A$

$2x-1=(A+B)x^2+(-6A-3B+C)x+9A$

By comparing the coefficients of x^2, we get

$A+B=0$

Put $A=\frac{-1}{9}$

$\frac{-1}{9}+B=0$

$B=\frac{1}{9}$

Put the values of A, B and C in equ (i)

$$\frac{2x-1}{x(x-3)^2}=\frac{\frac{-1}{9}}{x}+\frac{\frac{1}{9}}{x-3}+\frac{\frac{5}{3}}{(x-3)^2}$$

$$\frac{2x-1}{x(x-3)^2}=\frac{-1}{9x}+\frac{1}{9(x-3)}+\frac{5}{3(x-3)^2}$$

(9) $\boldsymbol{\frac{x^2}{x^2+2x+1}}$

<u>Solution</u>:

$\frac{x^2}{x^2+2x+1}$

As $\frac{x^2}{x^2+2x+1}$ is improper

Exercise # 4.1

So

$$\begin{array}{r|l} & 1 \\ x^2+2x+1 & x^2 \\ & \pm x^2 \pm 2x \pm 1 \\ \hline & -2x-1 \end{array}$$

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2x-1}{(x)^2+2(x)(1)+(1)^2}$$

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2x-1}{(x+1)^2} \quad \ldots..\text{equ}(\mathbf{A})$$

Now

Let

$$\frac{-2x-1}{(x+1)^2} = \frac{\text{A}}{x+1} + \frac{\text{B}}{(x+1)^2} \quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x+3)(x+2)^2$

$$\frac{-2x-1}{(x+1)^2} \times (x+1)^2 = \frac{\text{A}}{x+1} \times (x+1)^2 + \frac{\text{B}}{(x+1)^2} \times (x+1)^2$$

$-2x-1 = A(x+1)+B \quad \ldots..\text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$-2(-1)-1 = A(0)+B$

$2-1 = 0+B$

$1 = B$

equ (ii) $\Rightarrow$

$-2x-1 = A(x+1)+B$

$-2x-1 = Ax+A+B$

$-2x-1 = Ax+(A+B)$

By comparing the coefficients of x, we get

$A = -2$

Put the values of A and B in equ (i)

$$\frac{-2x-1}{(x+1)^2} = \frac{-2}{x+1} + \frac{1}{(x+1)^2}$$

Put the above in equ $(\mathbf{A})$

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2}{x+1} + \frac{1}{(x+1)^2}$$

$$\frac{x^2}{x^2+2x+1} = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

(10) $\boldsymbol{\frac{x^2}{(x-1)^2(x+1)}}$

Solution:

$$\frac{x^2}{(x-1)^2(x+1)}$$

Let

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{\text{A}}{x-1} + \frac{\text{B}}{(x-1)^2} + \frac{\text{C}}{x+1} \quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x-1)^2(x+1)$

Exercise # 4.1

$$\frac{x^2}{(x-1)^2(x+1)} \times (x-1)^2(x+1) = \frac{A}{x-1} \times (x-1)^2(x+1) + \frac{B}{(x-1)^2} \times (x-1)^2(x+1) + \frac{C}{x+1} \times (x-1)^2(x+1)$$

$x^2 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$(1)^2 = A(0)(1+1) + B(1+1) + C(0)^2$

$1 = 0 + B(2) + 0$

$1 = 2B$

$\frac{1}{2} = B$

$B = \frac{1}{2}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$(-1)^2 = A(-1-1)(0) + B(0) + C(-1-1)^2$

$1 = 0 + 0 + C(-2)^2$

$1 = C(4)$

$\frac{1}{4} = C$

$C = \frac{1}{4}$

equ (ii) $\Rightarrow$

$x^2 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$x^2 = A(x^2-1) + Bx + B + C(x^2-2x+1)$

$x^2 = Ax^2 - A + Bx + B + Cx^2 - 2Cx + C$

$x^2 = Ax^2 + Cx^2 + Bx - 2Cx - A + B + C$

$x^2 = (A+C)x^2 + (B-2C)x + (-A+B+C)$

By comparing the coefficients of x^2, we get

$A + C = 1$

Put $C = \frac{1}{4}$

$A + \frac{1}{4} = 1$

$A = 1 - \frac{1}{4}$

$A = \frac{4-1}{4}$

$A = \frac{3}{4}$

Put the values of A, B and C in equ (i)

$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{\frac{3}{4}}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{1}{4}}{x+1}$$

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{3}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$$

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Exercise # 4.1

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Exercise # 4.2

Example # 5: Find partial fraction of $\frac{1}{(x+1)(x^2+2)}$

Solution:

$$\frac{1}{(x+1)(x^2+2)}$$

Let

$$\frac{1}{(x+1)(x^2+2)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+2}\quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x-1)(x^2+3)$

$$\frac{1}{(x+1)(x^2+2)}\times(x+1)(x^2+2)=\frac{A}{x+1}\times(x+1)(x^2+2)+\frac{Bx+C}{x^2+2}\times(x+1)(x^2+2)$$

$1=A(x^2+2)+(Bx+C)(x+1)\quad \ldots..\text{equ(ii)}$

Put $x+1=0\Rightarrow x=-1$ in equ (ii)

$1=A((-1)^2+2)+(B(-1)+C)(0)$

$1=A(1+2)+0$

$1=A(3)$

$1=3A$

$\frac{1}{3}=A$

$A=\frac{1}{3}$

equ (ii) $\Rightarrow$

$1=A(x^2+2)+(Bx+C)(x+1)$

$1=Ax^2+2A+Bx^2+Bx+Cx+C$

$1=Ax^2+Bx^2+Bx+Cx+2A+C$

$1=(A+B)x^2+(B+C)x+(2A+C)$

Compare the coefficients of x^2, x *and constant* we get

$A+B=0\quad \ldots..\text{equ}(\boldsymbol{a})$

$B+C=0\quad \ldots..\text{equ}(\boldsymbol{b})$

$2A+C=1\quad \ldots..\text{equ}(\boldsymbol{c})$

Put $A=\frac{1}{3}$ in equ $(\boldsymbol{a})$

$\frac{1}{3}+B=0$

$B=-\frac{1}{3}$

Put $B=-\frac{1}{3}$ in equ $(\boldsymbol{b})$

$-\frac{1}{3}+C=0$

$C=\frac{1}{3}$

Put the values of A, B and C in equ (i)

$$\frac{1}{(x+1)(x^2+2)}=\frac{\frac{1}{3}}{x+1}+\frac{-\frac{1}{3}x+\frac{1}{3}}{x^2+2}$$

Exercise # 4.2

$$\frac{1}{(x+1)(x^2+2)} = \frac{\frac{1}{3}}{x+1} + \frac{\frac{-1x+1}{3}}{x^2+2}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} + \frac{-1x+1}{3(x^2+2)}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} + \frac{-(x-1)}{3(x^2+2)}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} - \frac{x-1}{3(x^2+2)}$$

Example 6: Find partial fraction of $\frac{4x^2-28}{x^4-x^2-6}$

Solution:

$$\frac{4x^2-28}{x^4-x^2-6}$$
$$\frac{4x^2-28}{x^4-x^2-6} = \frac{4x^2-28}{(x^2-3)(x^2+2)}$$

$R.W$
$x^4-x^2-6 = x^4-3x^2+2x^2-6$
$x^4-x^2-6 = x^2(x^2-3)+2(x^2-3)$
$x^4-x^2-6 = (x^2-3)(x^2+2)$

Let

$$\frac{4x^2-28}{(x^2-3)(x^2+2)} = \frac{Ax+B}{x^2-3} + \frac{Cx+D}{x^2+2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x^2-3)(x^2+2)$

$$\frac{4x^2-28}{(x^2-3)(x^2+2)} \times (x^2-3)(x^2+2) = \frac{Ax+B}{x^2-3} \times (x^2-3)(x^2+2) + \frac{Cx+D}{x^2+2} \times (x^2-3)(x^2+2)$$

$4x^2-28 = (Ax+B)(x^2+2) + (Cx+D)(x^2-3) \quad \ldots\ldots \text{equ(ii)}$

equ (ii) $\Rightarrow$

$4x^2-28 = Ax^3+2Ax+Bx^2+2B+Cx^3-3Cx+Dx^2-3D$

$4x^2-28 = Ax^3+Cx^3+Bx^2+Dx^2+2Ax-3Cx+2B-3D$

$4x^2-28 = (A+C)x^3+(B+D)x^2+(2A-3C)x+(2B-3D)$

Compare the coefficients of x^3, x^2, x *and constant* we get

$A+C=0 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$

$B+D=4 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$

$2A-3C=0 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$

$2B-3D=-28 \quad \ldots\ldots \text{equ}(\boldsymbol{d})$

From $equ(\boldsymbol{a})$

$A=-C \quad \ldots\ldots \text{equ}(\boldsymbol{e})$

Put $\boldsymbol{A}=-\boldsymbol{C}$ in equ $(\boldsymbol{c})$

$2(-C)-3C=0$

$-2C-3C=0$

$-5C=0$

$C=\frac{0}{-5}$

$C=0$

Put $C=0$ in equ $(\boldsymbol{e})$

$A=-(0)$

$A=0$

Exercise # 4.2

From equ(***b***)

$B = 4 - D \quad \ldots\ldots equ(\boldsymbol{f})$

Put $B = 4 - D$ in equ $(\boldsymbol{d})$

$2(4 - D) - 3D = -28$

$8 - 2D - 3D = -28$

$-5D = -28 - 8$

$-5D = -36$

$5D = 36$

$D = \frac{36}{5}$

Put $D = \frac{36}{5}$ in equ $(\boldsymbol{f})$

$B = 4 - \frac{36}{5}$

$B = \frac{20 - 36}{5}$

$B = \frac{-16}{5}$

Put the values of A, B , C and D in equ (i)

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{(0)x + \left(\frac{-16}{5}\right)}{x^2 - 3} + \frac{(0)x + \frac{36}{5}}{x^2 + 2}$$

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{\frac{-16}{5}}{x^2 - 3} + \frac{\frac{36}{5}}{x^2 + 2}$$

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{-16}{5(x^2 - 3)} + \frac{36}{5(x^2 + 2)}$$

Example 7: Find partial fraction of $\boldsymbol{\frac{1}{(x - 1)(x^2 + 1)^2}}$

<u>Solution</u>:

$$\frac{1}{(x - 1)(x^2 + 1)^2}$$

Let

$$\frac{1}{(x - 1)(x^2 + 1)^2} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \quad \ldots\ldots equ(i)$$

Multiply equ (i) by $(x - 1)(x^2 + 1)^2$

$$\frac{1}{(x - 1)(x^2 + 1)^2} \times (x - 1)(x^2 + 1)^2 = \frac{A}{x - 1} \times (x - 1)(x^2 + 1)^2 + \frac{Bx + C}{x^2 + 1} \times (x - 1)(x^2 + 1)^2 + \frac{Dx + E}{(x^2 + 1)^2} \times (x - 1)(x^2 + 1)^2$$

$1 = A(x^2 + 1)^2 + (Bx + C)(x - 1)(x^2 + 1) + (Dx + E)(x - 1) \quad \ldots\ldots equ(ii)$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A((1)^2 + 1)^2 + (Bx + C)(0)(x^2 + 1) + (Dx + E)(0)$

$1 = A(1 + 1)^2 + 0 + 0$

$1 = A(2)^2$

$1 = A(4)$

Exercise # 4.2

$\frac{1}{4} = A$

$A = \frac{1}{4}$

equ (ii) $\Rightarrow$

$1 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)$

$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x-x^2-1) + Dx^2 - Dx + Ex - E$

$1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 - Bx^3 - Bx + Cx^3 + Cx - Cx^2 - C + Dx^2 - Dx + Ex - E$

$1 = Ax^4 + Bx^4 - Bx^3 + Cx^3 + 2Ax^2 + Bx^2 - Cx^2 + Dx^2 - Bx + Cx - Dx + Ex + A - C - E$

$1 = (A+B)x^4 + (-B+C)x^3 + (2A+B-C+D)x^2 + (-B+C-D+E)x + (A-C-E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A + B = 0$ ..... equ($\boldsymbol{a}$)

$-B + C = 0$ ..... equ($\boldsymbol{b}$)

$2A + B - C + D = 0$ ..... equ($\boldsymbol{c}$)

$-B + C - D + E = 0$ ..... equ($\boldsymbol{d}$)

$A - C - E = 1$ ..... equ($\boldsymbol{e}$)

Put $A = \frac{1}{4}$ in equ ($\boldsymbol{a}$)

$\frac{1}{4} + B = 0$

$B = -\frac{1}{4}$

Put $B = -\frac{1}{4}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{1}{4}\right) + C = 0$

$\frac{1}{4} + C = 0$

$C = -\frac{1}{4}$

Put the values of A, B and C in equ (c)

$2\left(\frac{1}{4}\right) + \left(-\frac{1}{4}\right) - \left(-\frac{1}{4}\right) + D = 0$

$\frac{2}{4} - \frac{1}{4} + \frac{1}{4} + D = 0$

$\frac{1}{2} + D = 0$

$D = -\frac{1}{2}$

Put the values of A and C in equ ($\boldsymbol{e}$)

$A - C - E = 1$

$\frac{1}{4} - \left(-\frac{1}{4}\right) - E = 1$

$\frac{1}{4} + \frac{1}{4} - E = 1$

$\frac{1+1}{4} = 1 + E$

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Exercise # 4.2

$\frac{2}{4} - 1 = E$

$\frac{1}{2} - 1 = E$

$\frac{1-2}{2} = E$

$\frac{-1}{2} = E$

$E = \frac{-1}{2}$

Put the values of A, B , C , D and E in equ (i)

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}x + \left(-\frac{1}{4}\right)}{x^2+1} + \frac{-\frac{1}{2}x + \left(-\frac{1}{2}\right)}{(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} + \frac{\frac{-x-1}{4}}{x^2+1} + \frac{\frac{-x-1}{2}}{(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} + \frac{-(x+1)}{4(x^2+1)} + \frac{-(x+1)}{2(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} - \frac{x+1}{2(x^2+1)^2}$$

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Exercise # 4.2

Page # 82

Resolve the following fractions into partial fraction.

(1) $\frac{1}{x(x^2+1)}$

Solution:

Let

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \quad \dots\dots \text{equ(i)}$$

Multiply equ (i) by $x(x^2+1)$

$$\frac{1}{x(x^2+1)} \times x(x^2+1) = \frac{A}{x} \times x(x^2+1) + \frac{Bx+C}{x^2+1} \times x(x^2+1)$$

$1 = A(x^2+1) + (Bx+C)x \quad \dots\dots \text{equ(ii)}$

Put $x = 0$ in equ (ii)

$1 = A((0)^2+1) + (B(0)+C)(0)$

$1 = A(0+1) + 0$

$1 = A(1)$

$1 = A$

$A = 1$

equ (ii) $\Rightarrow$

$1 = A(x^2+1) + (Bx+C)x$

$1 = Ax^2 + A + Bx^2 + Cx$

$1 = Ax^2 + Bx^2 + Cx + A$

$1 = (A+B)x^2 + Cx + A$

By comparing the coefficients of x^2, x *and constant* we get

Exercise # 4.2

$A+B=0$ .....equ($\boldsymbol{a}$)
$C=0$ .....equ($\boldsymbol{b}$)
$A=1$ .....equ($\boldsymbol{c}$)
Put $A=1$ in equ ($\boldsymbol{a}$)
$1+B=0$
$B=-1$
Put the values of A, B and C in equ (i)

$$\frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-1x+0}{x^2+1}$$
$$\frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-x}{x^2+1}$$
$$\frac{1}{x(x^2+1)}=\frac{1}{x}-\frac{x}{x^2+1}$$

(2) $\boldsymbol{\frac{x^2+3x+1}{(x-1)(x^2+3)}}$

Solution:

$$\frac{x^2+3x+1}{(x-1)(x^2+3)}$$

Let

$$\frac{x^2+3x+1}{(x-1)(x^2+3)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+3} \quad \text{.....equ(i)}$$

Multiply equ (i) by $(x-1)(x^2+3)$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)}\times(x-1)(x^2+3)=\frac{A}{x-1}\times(x-1)(x^2+3)+\frac{Bx+C}{x^2+3}\times(x-1)(x^2+3)$$

$x^2+3x+1=A(x^2+3)+(Bx+C)(x-1)$ .....equ(ii)
Put $x-1=0 \Rightarrow x=1$ in equ (ii)
$(1)^2+3(1)+1=A((1)^2+3)+(B(1)+C)(0)$
$1+3+1=A(1+3)+0$
$5=A(4)$
$5=4A$
$\frac{5}{4}=A$
$A=\frac{5}{4}$
equ (ii) $\Rightarrow$
$x^2+3x+1=A(x^2+3)+(Bx+C)(x-1)$
$x^2+3x+1=Ax^2+3A+Bx^2-Bx+Cx-C$
$x^2+3x+1=Ax^2+Bx^2-Bx+Cx+3A-C$
$x^2+3x+1=(A+B)x^2+(-B+C)x+(3A-C)$
Compare the coefficients of x^2, x *and constant* we get
$A+B=1$ .....equ($\boldsymbol{a}$)
$-B+C=3$ .....equ($\boldsymbol{b}$)
$3A-C=1$ .....equ($\boldsymbol{c}$)

Exercise # 4.2

Put $A = \frac{5}{4}$ in equ $(\boldsymbol{a})$

$$\frac{5}{4} + B = 1$$

$$B = 1 - \frac{5}{4}$$

$$B = \frac{4-5}{4}$$

$$B = \frac{-1}{4}$$

Put $B = \frac{-1}{4}$ in equ $(\boldsymbol{b})$

$$-\left(\frac{-1}{4}\right) + \mathrm{C} = 3$$

$$\frac{1}{4} + \mathrm{C} = 3$$

$$C = 3 + \frac{-1}{4}$$

$$C = 3 - \frac{1}{4}$$

$$C = \frac{12-1}{4}$$

$$C = \frac{11}{4}$$

Put the values of A, B and C in equ (i)

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{\frac{5}{4}}{x-1} + \frac{\frac{-1}{4}x + \frac{11}{4}}{x^2+3}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{\frac{5}{4}}{x-1} + \frac{\frac{-1x+11}{4}}{x^2+3}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} + \frac{-1x+11}{4(x^2+3)}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} + \frac{-(1x-11)}{4(x^2+3)}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} - \frac{x-11}{4(x^2+3)}$$

(3) $\boldsymbol{\frac{2x+1}{(x^2+1)(x-1)}}$

<u>Solution</u>:

$$\frac{2x+1}{(x^2+1)(x-1)}$$

Let

$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{\mathrm{Ax+B}}{x^2+1} + \frac{\mathrm{C}}{x-1} \quad \ldots\ldots \mathrm{equ(i)}$$

Multiply equ (i) by $(x^2+1)(x-1)$

Exercise # 4.2

$$\frac{2x+1}{(x^2+1)(x-1)}\times(x^2+1)(x-1)=\frac{\mathrm{Ax+B}}{x^2+1}\times(x^2+1)(x-1)+\frac{C}{x-1}\times(x^2+1)(x-1)$$

$2x+1=(Ax+B)(x-1)+C(x^2+1)$ ......equ(ii)

Put $x-1=0\Rightarrow x=1$ in equ (ii)

$2x+1=(Ax+B)(x-1)+C(x^2+1)$

$2(1)+1=(A(1)+B)(0)+C((1)^2+1)$

$2+1=0+C(1+1)$

$3=C(2)$

$3=2C$

$\frac{3}{2}=C$

$\mathrm{C}=\frac{3}{2}$

equ (ii) $\Rightarrow$

$2x+1=(Ax+B)(x-1)+C(x^2+1)$

$2x+1=Ax^2-Ax+Bx-B+Cx^2+C$

$2x+1=Ax^2+Cx^2-Ax+Bx-B+C$

$2x+1=(A+C)x^2+(-A+B)x+(-B+C)$

Compare the coefficients of $x^2, x \text{ and constant}$ we get

$A+C=0$ .....equ($\boldsymbol{a}$)

$-\mathrm{A}+\mathrm{B}=2$ .....equ($\boldsymbol{b}$)

$-\mathrm{B}+\mathrm{C}=1$ .....equ($\boldsymbol{c}$)

Put $\mathrm{C}=\frac{3}{2}$ in equ ($\boldsymbol{a}$)

$A+\frac{3}{2}=0$

$A=-\frac{3}{2}$

Put $A=-\frac{3}{2}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{3}{2}\right)+\mathrm{B}=2$

$\frac{3}{2}+\mathrm{B}=2$

$B=2-\frac{3}{2}$

$B=\frac{4-3}{2}$

$B=\frac{1}{2}$

Put the values of A, B and C in equ (i)

$$\frac{2x+1}{(x^2+1)(x-1)}=\frac{\frac{-3}{2}x+\frac{1}{2}}{x^2+1}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{2x+1}{(x^2+1)(x-1)}=\frac{\frac{-3x+1}{2}}{x^2+1}+\frac{\frac{3}{2}}{x-1}$$

Exercise # 4.2

$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{-3x+1}{2(x^2+1)} + \frac{3}{2(x-1)}$$
$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{-(3x-1)}{2(x^2+1)} + \frac{3}{2(x-1)}$$
$$\frac{2x+1}{(x^2+1)(x-1)} = -\frac{3x-1}{2(x^2+1)} + \frac{3}{2(x-1)}$$

(4) $\mathbf{\frac{-3}{x^2(x^2+5)}}$

<u>Solution</u>:

$$\frac{-3}{x^2(x^2+5)}$$

Let

$$\frac{-3}{x^2(x^2+5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+5} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $x^2(x^2+5)$

$$\frac{-3}{x^2(x^2+5)} \times x^2(x^2+5) = \frac{A}{x} \times x^2(x^2+5) + \frac{B}{x^2} \times x^2(x^2+5) + \frac{Cx+D}{x^2+5} \times x^2(x^2+5)$$

$-3 = Ax(x^2+5) + B(x^2+5) + (Cx+D)x^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x = 0$ in equ (ii)

$-3 = A(0)(x^2+5) + B((0)^2+5) + (Cx+D)(0)^2$

$-3 = A(0) + B(0+5) + (Cx+D)(0)$

$-3 = 0 + B(5) + 0$

$-3 = 5B$

$\frac{-3}{5} = B$

$B = \frac{-3}{5}$

equ (ii) $\Rightarrow$

$-3 = Ax(x^2+5) + B(x^2+5) + (Cx+D)x^2$

$-3 = Ax^3 + 5Ax + Bx^2 + 5B + Cx^3 + Dx^2$

$-3 = Ax^3 + Cx^3 + Bx^2 + Dx^2 + 5Ax + 5B$

$-3 = (A+C)x^3 + (B+D)x^2 + 5Ax + 5B$

By comparing the coefficients of x^3, x^2, x and constant we get

$A + C = 0 \quad \ldots\ldots \text{equ}(a)$

$B + D = 0 \quad \ldots\ldots \text{equ}(b)$

$5A = 0 \quad \ldots\ldots \text{equ}(c)$

$5B = -3 \quad \ldots\ldots \text{equ}(d)$

From equ(c)

$A = \frac{0}{5}$

$A = 0$

$Put\ A = 0\ in\ equ\ (a)$

$0 + C = 0$

$C = 0$

Exercise # 4.2

Put $B = \frac{-3}{5}$ in equ (b)

$$\frac{-3}{5} + D = 0$$

$$D = \frac{3}{5}$$

Put the values of A, B , C and D in equ (i)

$$\frac{-3}{x^2(x^2+5)} = \frac{0}{x} + \frac{\frac{-3}{5}}{x^2} + \frac{0x + \frac{3}{5}}{x^2+5}$$

$$\frac{-3}{x^2(x^2+5)} = 0 + \frac{-3}{5x^2} + \frac{3}{5(x^2+5)}$$

$$\frac{-3}{x^2(x^2+5)} = \frac{-3}{5x^2} + \frac{3}{5(x^2+5)}$$

(5) $\frac{3x-2}{(x+4)(3x^2+1)}$

Solution:

$$\frac{3x-2}{(x+4)(3x^2+1)}$$

Let

$$\frac{3x-2}{(x+4)(3x^2+1)} = \frac{A}{x+4} + \frac{Bx+C}{3x^2+1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+4)(3x^2+1)$

$$\frac{3x-2}{(x+4)(3x^2+1)} \times (x+4)(3x^2+1) = \frac{A}{x+4} \times (x+4)(3x^2+1) + \frac{Bx+C}{3x^2+1} \times (x+4)(3x^2+1)$$

$$3x - 2 = A(3x^2+1) + (Bx+C)(x+4) \quad \ldots\ldots \text{equ(ii)}$$

Put $x + 4 = 0 \Rightarrow x = -4$ in equ (ii)

$$3(-4) - 2 = A(3(-4)^2+1) + (B(-4)+C)(0)$$

$$-12 - 2 = A(3(16)+1) + 0$$

$$-14 = A(48+1)$$

$$-14 = A(49)$$

$$\frac{-14}{49} = A$$

$$\frac{-2}{7} = A$$

$$A = \frac{-2}{7}$$

equ (ii) $\Rightarrow$

$$3x - 2 = A(3x^2+1) + (Bx+C)(x+4)$$

$$3x - 2 = 3Ax^2 + A + Bx^2 + 4Bx + Cx + 4C$$

$$3x - 2 = 3Ax^2 + Bx^2 + 4Bx + Cx + A + 4C$$

$$3x - 2 = (3A+B)x^2 + (4B+C)x + (A+4C)$$

Compare the coefficients of x^2, x *and constant* we get

$3A + B = 0 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$

$4B + C = 3 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$

$A + 4C = -2 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$

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Exercise # 4.2

Put $A = \frac{-2}{7}$ in equ $(\boldsymbol{a})$

$$3\left(\frac{-2}{7}\right) + B = 0$$

$$\frac{-6}{7} + B = 0$$

$$B = \frac{6}{7}$$

Put $B = \frac{6}{7}$ in equ $(\boldsymbol{b})$

$$4\left(\frac{6}{7}\right) + \mathrm{C} = 3$$

$$\frac{24}{7} + \mathrm{C} = 3$$

$$C = 3 - \frac{24}{7}$$

$$C = \frac{21 - 24}{7}$$

$$C = \frac{-3}{7}$$

Put the values of A, B and C in equ (i)

$$\frac{3x - 2}{(x + 4)(3x^2 + 1)} = \frac{\frac{-2}{7}}{x + 4} + \frac{\frac{6}{7}x + \frac{-3}{7}}{3x^2 + 1}$$

$$\frac{3x - 2}{(x + 4)(3x^2 + 1)} = \frac{\frac{-2}{7}}{x + 4} + \frac{\frac{6x - 3}{7}}{3x^2 + 1}$$

$$\frac{3x - 2}{(x + 4)(3x^2 + 1)} = \frac{-2}{7(x + 4)} + \frac{6x - 3}{7(3x^2 + 1)}$$

(6) $\boldsymbol{\frac{5x}{(x + 1)(x^2 - 2)^2}}$

<u>Solution</u>:

$$\frac{5x}{(x + 1)(x^2 - 2)^2}$$

Let

$$\frac{5x}{(x + 1)(x^2 - 2)^2} = \frac{\mathrm{A}}{x + 1} + \frac{\mathrm{B}x + \mathrm{C}}{x^2 - 2} + \frac{Dx + \mathrm{E}}{(x^2 - 2)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x + 1)(x^2 - 2)^2$

$$\frac{5x}{(x + 1)(x^2 - 2)^2} \times (x + 1)(x^2 - 2)^2 = \frac{\mathrm{A}}{x + 1} \times (x + 1)(x^2 - 2)^2 + \frac{\mathrm{B}x + \mathrm{C}}{x^2 - 2} \times (x + 1)(x^2 - 2)^2 + \frac{Dx + \mathrm{E}}{(x^2 - 2)^2} \times (x + 1)(x^2 - 2)^2$$

$$5x = A(x^2 - 2)^2 + (Bx + C)(x + 1)(x^2 - 2) + (Dx + E)(x + 1) \quad \ldots\ldots \text{equ(ii)}$$

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$$5(-1) = A((-1)^2 - 2)^2 + (B(-1) + C)(0)(x^2 - 2) + (Dx + E)(0)$$

$$-5 = A(1 - 2)^2 + 0 + 0$$

$$-5 = A(-1)^2$$

Exercise # 4.2

$-5 = A(1)$

$-5 = A$

$A = -5$

equ (ii) $\Rightarrow$

$5x = A(x^2-2)^2 + (Bx+C)(x+1)(x^2-2) + (Dx+E)(x+1)$

$5x = A(x^4-4x^2+4) + (Bx+C)(x^3-2x+x^2-2) + Dx^2+Dx+Ex+E$

$5x = Ax^4 - 4Ax^2 + 4A + Bx^4 - 2Bx^2 + Bx^3 - 2Bx + Cx^3 - 2Cx + Cx^2 - 2C + Dx^2 + Dx + Ex + E$

$5x = Ax^4 + Bx^4 + Bx^3 + Cx^3 - 4Ax^2 - 2Bx^2 + Cx^2 + Dx^2 - 2Bx - 2Cx + Dx + Ex + 4A - 2C + E$

$5x = (A+B)x^4 + (B+C)x^3 + (-4A-2B+C+D)x^2 + (-2B-2C+D+E)x + (4A-2C+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A + B = 0 \quad \ldots..\text{equ}(\boldsymbol{a})$

$B + C = 0 \quad \ldots..\text{equ}(\boldsymbol{b})$

$-4A - 2B + C + D = 0 \quad \ldots..\text{equ}(\boldsymbol{c})$

$-2B - 2C + D + E = 5 \quad \ldots..\text{equ}(\boldsymbol{d})$

$4A - 2C + E = 0 \quad \ldots..\text{equ}(\boldsymbol{e})$

Put $A = -5$ in equ $(\boldsymbol{a})$

$-5 + B = 0$

$B = 5$

Put $B = 5$ in equ $(\boldsymbol{b})$

$5 + C = 0$

$C = -5$

Put the values of A, B and C in equ (c)

$-4(-5) - 2(5) + (-5) + D = 0$

$20 - 10 - 5 + D = 0$

$10 - 5 + D = 0$

$5 + D = 0$

$D = -5$

Put the values of A and C in equ (e)

$4(-5) - 2(-5) + E = 0$

$-20 + 10 + E = 0$

$-10 + E = 0$

$E = 10$

Put the values of A, B , C , D and E in equ (i)

$$\frac{5x}{(x+1)(x^2-2)^2} = \frac{-5}{x+1} + \frac{5x+(-5)}{x^2-2} + \frac{-5x+10}{(x^2-2)^2}$$

$$\frac{5x}{(x+1)(x^2-2)^2} = \frac{-5}{x+1} + \frac{5x-5}{x^2-2} + \frac{-5x+10}{(x^2-2)^2}$$

(7) $\dfrac{\mathbf{5x^2 - 4x + 8}}{\mathbf{(x^2+1)^2(x-2)}}$

Solution:

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}$$

Let

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Exercise # 4.2

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-2} \quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(x-2)$

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}\times(x^2+1)^2(x-2)=\frac{Ax+B}{x^2+1}\times(x^2+1)^2(x-2)+\frac{Cx+D}{(x^2+1)^2}\times(x^2+1)^2(x-2)+\frac{E}{x-2}\times(x^2+1)^2(x-2)$$

$5x^2-4x+8=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)+E(x^2+1)^2 \quad \ldots..\text{equ(ii)}$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$5(2)^2-4(2)+8=(A(2)+B)(x^2+1)(0)+(C(2)+D)(0)+E((2)^2+1)^2$

$5(4)-8+8=0+0+E(4+1)^2$

$20=E(5)^2$

$20=E(25)$

$\frac{20}{25}=E$

$\frac{4}{5}=E$

$E=\frac{4}{5}$

equ (ii) $\Rightarrow$

$5x^2-4x+8=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)+E(x^2+1)^2$

$5x^2-4x+8=(Ax+B)(x^3-2x^2+x-2)+Cx^2-2Cx+Dx-2D+E(x^4+2x^2+1)$

$5x^2-4x+8=Ax^4-2Ax^3+Ax^2-2Ax+Bx^3-2Bx^2+Bx-2B+Cx^2-2Cx+Dx-2D+Ex^4+2Ex^2+E$

$5x^2-4x+8=Ax^4+Ex^4-2Ax^3+Bx^3+Ax^2-2Bx^2+Cx^2+2Ex^2-2Ax+Bx-2Cx+Dx-2B-2D+E$

$5x^2-4x+8=(A+E)x^4+(-2A+B)x^3+(A-2B+C+2E)x^2+(-2A+B-2C+D)x+(-2B-2D+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A+E=0 \quad \ldots..\text{equ}(\boldsymbol{a})$

$-2A+B=0 \quad \ldots..\text{equ}(\boldsymbol{b})$

$A-2B+C+2E=5 \quad \ldots..\text{equ}(\boldsymbol{c})$

$-2A+B-2C+D=-4 \quad \ldots..\text{equ}(\boldsymbol{d})$

$-2B-2D+E=8 \quad \ldots..\text{equ}(\boldsymbol{e})$

Put $E=\frac{4}{5}$ in equ $(\boldsymbol{a})$

$A+\frac{4}{5}=0$

$A=-\frac{4}{5}$

Put $A=-\frac{4}{5}$ in equ $(\boldsymbol{b})$

$-2\left(-\frac{4}{5}\right)+B=0$

$\frac{8}{5}+B=0$

$B=-\frac{8}{5}$

Put the values of A, B and C in equ (c)

Exercise # 4.2

$$-\frac{4}{5}-2\left(-\frac{8}{5}\right)+\mathrm{C}+2\left(\frac{4}{5}\right)=5$$

$$-\frac{4}{5}+\frac{16}{5}+\mathrm{C}+\frac{8}{5}=5$$

$$-\frac{4}{5}+\frac{16}{5}+\frac{8}{5}+\mathrm{C}=5$$

$$\frac{-4+16+8}{5}+\mathrm{C}=5$$

$$\frac{-4+16+8}{5}+\mathrm{C}=5$$

$$\frac{20}{5}+\mathrm{C}=5$$

$$4+\mathrm{C}=5$$

$$\mathrm{C}=5-4$$

$$\mathrm{C}=1$$

Put the values of A, B and E in equ (d)

$$-2\left(-\frac{4}{5}\right)+\left(-\frac{8}{5}\right)-2(1)+D=-4$$

$$\frac{8}{5}-\frac{8}{5}-2+D=-4$$

$$-2+D=-4$$

$$D=-4+2$$

$$D=-2$$

Put the values of A, B , C , D and E in equ (i)

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{-\frac{4}{5}x+\frac{-8}{5}}{x^2+1}+\frac{1x+(-2)}{(x^2+1)^2}+\frac{\frac{4}{5}}{x-2}$$

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{\frac{-4x-8}{5}}{x^2+1}+\frac{x-2}{(x^2+1)^2}+\frac{\frac{4}{5}}{x-2}$$

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{-4x-8}{5(x^2+1)}+\frac{x-2}{(x^2+1)^2}+\frac{4}{5(x-2)}$$

Important

(8) $\boldsymbol{\frac{4x-5}{(x^2+4)^2}}$

Solution:

$$\frac{4x-5}{(x^2+4)^2}$$

Let

$$\frac{4x-5}{(x^2+4)^2}=\frac{Ax+B}{x^2+4}+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+4)^2}\quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x^2+4)^2$

$$\frac{4x-5}{(x^2+4)^2}\times(x^2+4)^2=\frac{Ax+B}{x^2+4}\times(x^2+4)^2+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+4)^2}\times(x^2+4)^2$$

$$4x-5=(Ax+B)(x^2+4)+Cx+D\quad \ldots\ldots \text{equ(ii)}$$

equ (ii) $\Rightarrow$

Exercise # 4.2

$4x-5=Ax^3+4Ax+Bx^2+4B+Cx+D$

$4x-5=Ax^3+Bx^2+4Ax+Cx+4B+D$

$4x-5=Ax^3+Bx^2+(4A+C)x+(4B+D)$

Compare the coefficients of x^3, x^2, x and constant we get

$A=0$ ..... equ($\boldsymbol{a}$)

$B=0$ ..... equ($\boldsymbol{b}$)

$4A+C=4$ ..... equ($\boldsymbol{c}$)

$4B+D=-5$ ..... equ($\boldsymbol{d}$)

Put $A=0$ in equ ($\boldsymbol{c}$)

$4(0)+C=4$

$C=4$

Put $B=0$ in equ ($\boldsymbol{d}$)

$4(0)+D=-5$

$D=-5$

Put the values of A, B, C and D in equ (i)

$$\frac{4x-5}{(x^2+4)^2}=\frac{(0)x+0}{x^2+4}+\frac{4x+(-5)}{(x^2+4)^2}$$

$$\frac{4x-5}{(x^2+4)^2}=0+\frac{4x-5}{(x^2+4)^2}$$

$$\frac{4x-5}{(x^2+4)^2}=\frac{4x-5}{(x^2+4)^2}$$

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(9) $\boldsymbol{\dfrac{8x^2}{(x^2+1)(1-x^4)}}$

Solution:

$$\frac{8x^2}{(x^2+1)(1-x^4)}=\frac{8x^2}{(x^2+1)(1+x^2)(1-x^2)}$$

$$\frac{8x^2}{(x^2+1)(1-x^4)}=\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}$$

Let

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{1+x}+\frac{F}{1-x} \quad \text{..... equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(1+x)(1-x)$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}\times(x^2+1)^2(1+x)(1-x)=\frac{Ax+B}{x^2+1}\times(x^2+1)^2(1+x)(1-x)+\frac{Cx+D}{(x^2+1)^2}\times(x^2+1)^2(1+x)(1-x)+\frac{E}{1+x}\times(x^2+1)^2(1+x)(1-x)+\frac{F}{1-x}\times(x^2+1)^2(1+x)(1-x)$$

$8x^2=(Ax+B)(x^2+1)(1+x)(1-x)+(Cx+D)(1+x)(1-x)+E(x^2+1)^2(1-x)+F(x^2+1)^2(1+x)$

Put $1+x=0 \Rightarrow x=-1$ in above equation

$8(-1)^2=(Ax+B)(x^2+1)(0)(1-x)+(Cx+D)(0)(1-x)+E((-1)^2+1)^2(1-(-1))+F(x^2+1)^2(0)$

Exercise # 4.2

$8(1) = 0 + 0 + E(1+1)^2(1+1) + 0$

$8 = E(2)^2(2)$

$8 = E(4)(2)$

$8 = E(8)$

$\frac{8}{8} = E$

$1 = E$

$E = 1$

Put $1 - x = 0 \Rightarrow -x = -1 \Rightarrow x = 1$ in equ (ii)

$8(1)^2 = (Ax+B)(x^2+1)(1+x)(0) + (Cx+D)(1+x)(0) + +E(x^2+1)^2(0) + F((1)^2+1)^2(1+1)$

$8(1) = 0 + 0 + 0 + F(1+1)^2(1+1)$

$8 = F(2)^2(2)$

$8 = F(4)(2)$

$8 = F(8)$

$\frac{8}{8} = F$

$1 = F$

$F = 1$

equ (ii) $\Rightarrow$

$8x^2 = (Ax+B)(x^2+1)(1+x)(1-x) + (Cx+D)(1+x)(1-x) + E(x^2+1)^2(1-x) + F(x^2+1)^2(1+x)$

$8x^2 = (Ax+B)(x^2+1)(1-x^2) + (Cx+D)(1-x^2) + E(x^4+2x^2+1)(1-x) + F(x^4+2x^2+1)(1+x)$

$8x^2 = (Ax+B)(1-x^4) + Cx - Cx^3 + D - Dx^2 + E(x^4 - x^5 + 2x^2 - 2x^3 + 1 - x) + F(x^4 + x^5 + 2x^2 + 2x^3 + 1 + x)$

$8x^2 = Ax - Ax^5 + B - Bx^4 + Cx - Cx^3 + D - Dx^2 + Ex^4 - Ex^5 + 2Ex^2 - 2Ex^3 + E - Ex + Fx^4 + Fx^5 + 2Fx^2 + 2Fx^3 + F + Fx$

$8x^2 = -Ax^5 - Ex^5 + Fx^5 - Bx^4 + Ex^4 + Fx^4 - Cx^3 - 2Ex^3 + 2Fx^3 - Dx^2 + 2Ex^2 + 2Fx^2 + Ax + Cx - Ex + Fx + B + D + E + F$

$8x^2 = (-A-E+F)x^5 + (-B+E+F)x^4 + (-C-2E+2F)x^3 + (-D+2E+2F)x^2 + (A+C-E+F)x + (B+D+E+F)$

Compare the coefficients of x^5 , x^4 , x^3 , x^2 , x *and constant* we get

$-A - E + F = 0$ …..equ(***a***)

$-B + E + F = 0$ …..equ(***b***)

$-C - 2E + 2F = 0$ …..equ(***c***)

$-D + 2E + 2F = 8$ …..equ(***d***)

$A + C - E + F = 0$ …..equ(***e***)

$B + D + E + F = 0$ …..equ(***f***)

Put the values of E *and* F in equ (***a***)

$-A - 1 + 1 = 0$

$-A = 0$

$A = 0$

Put the values of E *and* F in equ (***b***)

$-B + 1 + 1 = 0$

$-B + 2 = 0$

$-B = -2$

$B = 2$

Put the values of E *and* F in equ (***c***)

$-C - 2(1) + 2(1) = 0$

$-C - 2 + 2 = 0$

$-C = 0$

$C = 0$

Exercise # 4.2

Put the values of $E\ and\ F$ in equ $(\boldsymbol{d})$

$-\mathrm{D}+2(1)+2(1)=8$

$-\mathrm{D}+2+2=8$

$-\mathrm{D}+4=8$

$-\mathrm{D}=8-4$

$-\mathrm{D}=4$

$\mathrm{D}=-4$

Put the values of A, B , C , D , E and F in equ (i)

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{0x+2}{x^2+1}+\frac{0x+(-4)}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{2}{x^2+1}+\frac{-4}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{2}{x^2+1}-\frac{4}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

(10) $\boldsymbol{\frac{2x^2+4}{(x^2+1)^2(x-1)}}$

Solution:

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}$$

Let

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}+\frac{\mathrm{E}}{x-1}\quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(x-1)$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}\times(x^2+1)^2(x-1)=\frac{Ax+B}{x^2+1}\times(x^2+1)^2(x-1)+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}\times(x^2+1)^2(x-1)+\frac{\mathrm{E}}{x-1}\times(x^2+1)^2(x-1)$$

$2x^2+4=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$2(1)^2+4=(A(1)+B)(x^2+1)(0)+(C(1)+D)(0)+E((1)^2+1)^2$

$2(1)+4=0+0+E(1+1)^2$

$2+4=E(2)^2$

$6=E(4)$

$\frac{6}{4}=E$

$\frac{3}{2}=E$

$E=\frac{3}{2}$

equ (ii) $\Rightarrow$

$2x^2+4=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2$

$2x^2+4=(Ax+B)(x^3-x^2+x-1)+Cx^2-Cx+Dx-D+E(x^4+2x^2+1)$

$2x^2+4=Ax^4-Ax^3+Ax^2-Ax+Bx^3-Bx^2+Bx-B+Cx^2-Cx+Dx-D+Ex^4+2Ex^2+E$

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Exercise # 4.2

$2x^2+4=Ax^4+Ex^4-Ax^3+Bx^3+Ax^2-Bx^2+Cx^2+2Ex^2-Ax+Bx-Cx+Dx-B-D+E$

$2x^2+4=(A+E)x^4+(-A+B)x^3+(A-B+C+2E)x^2+(-A+B-C+D)x+(-B-D+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A+E=0$ ....equ($\boldsymbol{a}$)

$-A+B=0$ ....equ($\boldsymbol{b}$)

$A-B+C+2E=2$ ....equ($\boldsymbol{c}$)

$-A+B-C+D=0$ ....equ($\boldsymbol{d}$)

$-B-D+E=8$ ....equ($\boldsymbol{e}$)

Put $E=\frac{3}{2}$ in equ ($\boldsymbol{a}$)

$A+\frac{3}{2}=0$

$A=-\frac{3}{2}$

Put $A=-\frac{3}{2}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{3}{2}\right)+B=0$

$\frac{3}{2}+B=0$

$B=-\frac{3}{2}$

Put the values of A, B *and* E *in equ* ($\boldsymbol{c}$)

$-\frac{3}{2}-\left(-\frac{3}{2}\right)+C+2\left(\frac{3}{2}\right)=2$

$-\frac{3}{2}+\frac{3}{2}+C+3=2$

$0+C=2-3$

$C=-1$

Put the values of A, B *and* C *in equ* ($\boldsymbol{d}$)

$-A+B-C+D=0$

$-\left(-\frac{3}{2}\right)+\left(-\frac{3}{2}\right)-(-1)+D=0$

$\frac{3}{2}-\frac{3}{2}+1+D=0$

$0+1+D=0$

$1+D=0$

$D=-1$

Put the values of A, B , C , D and E in equ (i)

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{-\frac{3}{2}x+\left(-\frac{3}{2}\right)}{x^2+1}+\frac{-1x+(-1)}{(x^2+1)^2}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{\frac{-3x-3}{2}}{x^2+1}+\frac{-x-1}{(x^2+1)^2}+\frac{\frac{3}{2}}{x-1}$$

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Exercise # 4.2

$$\frac{2x^2+4}{(x^2+1)^2(x-1)} = \frac{-3x-3}{2(x^2+1)} + \frac{-(x+1)}{(x^2+1)^2} + \frac{3}{2(x-1)}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)} = \frac{-(3x+3)}{2(x^2+1)} - \frac{x+1}{(x^2+1)^2} + \frac{3}{2(x-1)}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)} = -\frac{3x+3}{2(x^2+1)} - \frac{x+1}{(x^2+1)^2} + \frac{3}{2(x-1)}$$

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Review Exercise # 4

Q2: Resolve the following fractions into partial fraction.

(1) $\dfrac{2x^2}{(x+1)(x-1)}$

Solution:

$$\frac{2x^2}{(x+1)(x-1)}$$

As $\dfrac{2x^2}{(x+1)(x-1)}$ is improper

$$\frac{2x^2}{(x+1)(x-1)} = \frac{2x^2}{x^2-1}$$

So

$$\begin{array}{r|l} & \quad 2 \\ x^2-1 & \ 2x^2 \\ & \underline{\pm 2x^2 \mp 2} \\ & \qquad\ 2 \end{array}$$

$$\frac{2x^2}{x^2-1} = 2 + \frac{2}{x^2-1}$$

$$\frac{2x^2}{x^2-1} = 2 + \frac{2}{(x+1)(x-1)} \quad \text{.....equ}(\mathbf{A})$$

Now

Let

$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \text{.....equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)$

$$\frac{2}{(x+1)(x-1)} \times (x+1)(x-1) = \frac{A}{x+1} \times (x+1)(x-1) + \frac{B}{x-1} \times (x+1)(x-1)$$

$2 = A(x-1) + B(x+1) \quad$ .....equ(ii)

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$2 = A(-1-1) + B(0)$

$2 = A(-2) + 0$

$2 = -2A$

$\dfrac{2}{-2} = A$

$-1 = A$

$A = -1$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$2 = A(0) + B(1+1)$

$2 = 0 + B(2)$

$2 = 2B$

$\dfrac{2}{2} = B$

$1 = B$

$B = 1$

Review Exercise # 4

Put the values of A and B in equ (i)

$$\frac{2}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

Put the above in equ (**A**)

$$\frac{2x^2}{(x+1)(x-1)} = 2 + \frac{-1}{x+1} + \frac{1}{x-1}$$

$$\frac{2x^2}{(x+1)(x-1)} = 2 - \frac{1}{x+1} + \frac{1}{x-1}$$

(2) $\mathbf{\dfrac{2x^3 - 3x^2 + 9x + 8}{x^2 - 3x + 2}}$

<u>Solution</u>:

$$\frac{2x^3 - 3x^2 + 9x + 8}{x^2 - 3x + 2}$$

As $\dfrac{2x^3 - 3x^2 + 9x + 8}{x^2 - 3x + 2}$ is improper

So

$$\begin{array}{r|l} & \quad 2x + 3 \\ \hline x^2 - 3x + 2 & 2x^3 - 3x^2 + 9x + 8 \\ & \underline{\pm 2x^3 \mp 6x^2 \pm 4x} \\ & \quad 3x^2 + 5x + 8 \\ & \underline{\pm 3x^2 \mp 9x \pm 6} \\ & \quad 14x + 2 \end{array}$$

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$$\frac{2x^3 - 3x^2 + 9x + 8}{x^2 - 3x + 2} = 2x + 3 + \frac{14x + 2}{x^2 - 3x + 2}$$

$$\frac{2x^3 - 3x^2 + 9x + 8}{x^2 - 3x + 2} = 2x + 3 + \frac{14x + 2}{(x-2)(x-1)} \quad \ldots\ldots \text{equ}(\mathbf{A})$$

R.W

$x^2 - 3x + 2 = x^2 - 2x - 1x + 2$

$x^2 - 3x + 2 = x(x-2) - 1(x-2)$

$x^2 - 3x + 2 = (x-2)(x-2)$

Now

Let

$$\frac{14x + 2}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-2)(x-1)$

$$\frac{14x + 2}{(x-2)(x-1)} \times (x-2)(x-1) = \frac{A}{x-2} \times (x-2)(x-1) + \frac{B}{x-1} \times (x-2)(x-1)$$

$14x + 2 = A(x-1) + B(x-2) \quad \ldots\ldots \text{equ(ii)}$

Put $x - 2 = 0 \Rightarrow x = 2$ in equ (ii)

$14(2) + 2 = A(2-1) + B(0)$

$28 + 2 = A(1) + 0$

$30 = A$

$A = 30$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$14(1) + 2 = A(0) + B(1-2)$

$14 + 2 = 0 + B(-1)$

$16 = -B$
$-16 = B$
$B = -16$
Put the values of A and B in equ (i)
$$\frac{14x+2}{(x-2)(x-1)} = \frac{30}{x-2} + \frac{-16}{x-1}$$
$$\frac{14x+2}{(x-2)(x-1)} = \frac{30}{x-2} - \frac{16}{x-1}$$
Put the above in equ (**A**)
$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2} = 2x+3 + \frac{30}{x-2} - \frac{16}{x-1}$$

(3) $\boldsymbol{\frac{3x-1}{x^3-2x^2+x}}$

Solution:
$$\frac{3x-1}{x^3-2x^2+x}$$
$$\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x^2-2x+1)}$$
$$\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x-1)^2}$$
Let
$$\frac{3x-1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad \ldots\ldots \text{equ(i)}$$
Multiply equ (i) by $x(x-1)^2$
$$\frac{3x-1}{x(x-1)^2} \times x(x-1)^2 = \frac{A}{x} \times x(x-1)^2 + \frac{B}{x-1} \times x(x-1)^2 + \frac{C}{(x-1)^2} \times x(x-1)^2$$
$$3x-1 = A(x-1)^2 + Bx(x-1) + Cx \quad \ldots\ldots \text{equ(ii)}$$
Put $x = 0$ in equ (ii)
$3(0) - 1 = A(0-1)^2 + B(0)(0-1) + C(0)$
$0 - 1 = A(-1)^2 + 0 + 0$
$-1 = A(1)$
$-1 = A$
$A = -1$
Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)
$3(1) - 1 = A(0)^2 + B(1)(0) + C(1)$
$3 - 1 = 0 + 0 + C$
$2 = C$
$C = 2$
equ (ii) $\Rightarrow$
$3x - 1 = A(x-1)^2 + Bx(x-1) + Cx$
$3x - 1 = A(x^2 - 2x + 1) + Bx^2 - Bx + Cx$
$3x - 1 = Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$
$3x - 1 = Ax^2 + Bx^2 - 2Ax - Bx + Cx + A$
$3x - 1 = (A+B)x^2 + (-2A - B + C)x + A$

By comparing the coefficients of x^2, we get

$A + B = 0$

Put $A = -1$

$-1 + B = 0$

$B = 1$

Put the values of A, B and C in equ (i)

$$\frac{3x-1}{x(x-1)^2} = \frac{-1}{x} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

Important

(4) $\frac{\boldsymbol{x+1}}{\boldsymbol{(x-1)^2}}$

Solution:

$$\frac{x+1}{(x-1)^2}$$

Let

$$\frac{x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)^2$

$$\frac{x+1}{(x-1)^2} \times (x-1)^2 = \frac{A}{x-1} \times (x-1)^2 + \frac{B}{(x-1)^2} \times (x-1)^2$$

$x + 1 = A(x-1) + B \quad \ldots\ldots \text{equ(ii)}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 + 1 = A(0) + B$

$2 = B$

$B = 2$

equ (ii) $\Rightarrow$

$x + 1 = A(x-1) + B$

$x + 1 = Ax - A + B$

$x + 1 = Ax - A + B$

$x + 1 = Ax + (-A + B)$

By comparing the coefficients of x, we get

$A = 1$

Put the values of A and B in equ (i)

$$\frac{x+1}{(x-1)^2} = \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

(5) $\frac{\boldsymbol{2x^2}}{\boldsymbol{x^4-4}}$

Solution:

$$\frac{2x^2}{x^4-4} = \frac{2x^2}{(x^2)^2 - (2)^2}$$

Review Exercise # 4

$$\frac{2x^2}{x^4-4}=\frac{2x^2}{(x^2+2)(x^2-2)}$$

Let

$$\frac{2x^2}{(x^2+2)(x^2-2)}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2-2} \quad \text{.....equ(i)}$$

Multiply equ (i) by $(x^2+2)(x^2-2)$

$$\frac{2x^2}{(x^2+2)(x^2-2)}\times(x^2+2)(x^2-2)=\frac{Ax+B}{x^2+2}\times(x^2+2)(x^2-2)+\frac{Cx+D}{x^2-2}\times(x^2+2)(x^2-2)$$

$2x^2=(Ax+B)(x^2-2)+(Cx+D)(x^2+2)$ .....equ(ii)

equ (ii) $\Rightarrow$

$2x^2=Ax^3-2Ax+Bx^2-2B+Cx^3+2Cx+Dx^2+2D$

$2x^2=Ax^3+Cx^3+Bx^2+Dx^2-2Ax+2Cx-2B+2D$

$2x^2=(A+C)x^3+(B+D)x^2+(-2A+2C)x+(-2B+2D)$

Compare the coefficients of x^3, x^2, x *and constant* we get

$A+C=0$ .....equ(***a***)

$B+D=2$ .....equ(***b***)

$-2A+2C=0$ .....equ(***c***)

$-2B+2D=0$ .....equ(***d***)

equ (c) $\Rightarrow$

$-2(A-C)=0$

$A-C=0$

$A=C$ .....equ(***e***)

Put $A=C$ in equ (***a***)

$C+C=0$

$2C=0$

$C=\frac{0}{2}$

$C=0$

Now Put $C=0$ in equ (***e***)

$A=0$

equ (d) $\Rightarrow$

$-2(B-D)=0$

$B-D=0$

$B=D$ .....equ(***f***)

Put $B=D$ in equ (b)

$D+D=2$

$2D=2$

$D=\frac{2}{2}$

$D=1$

Now Put $D=1$ in equ (***f***)

$B=1$

Put the values of A, B, C and D in equ (i)

Review Exercise # 4

$$\frac{2x^2}{(x^2+2)(x^2-2)}=\frac{0x+1}{x^2+2}+\frac{0x+1}{x^2-2}$$
$$\frac{2x^2}{(x^2+2)(x^2-2)}=\frac{1}{x^2+2}+\frac{1}{x^2-2}$$

(6) $\frac{3x^2+3x+2}{x^4-1}$

Solution:

$$\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{(x^2)^2-(1)^2}$$
$$\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{(x^2-1)(x^2+1)}$$
$$\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}$$

Let

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)(x^2+1)$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}\times(x+1)(x-1)(x^2+1)=\frac{A}{x+1}\times(x+1)(x-1)(x^2+1)+$$
$$\frac{B}{x-1}\times(x+1)(x-1)(x^2+1)+\frac{Cx+D}{x^2+1}\times(x+1)(x-1)(x^2+1)$$

$$3x^2+3x+2=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x+1)(x-1) \quad \ldots\ldots \text{equ(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$$3(-1)^2+3(-1)+2=A(-1-1)((-1)^2+1)+B(0)(x^2+1)+(Cx+D)(0)(x-1)$$
$$3(1)-3+2=A(-2)(1+1)+0+0$$
$$3-3+2=A(-2)(2)$$
$$2=-4A$$
$$\frac{2}{-4}=A$$
$$\frac{1}{-2}=A$$
$$-\frac{1}{2}=A$$
$$A=-\frac{1}{2}$$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$$3(1)^2+3(1)+2=A(0)(x^2+1)+B(1+1)((1)^2+1)+(Cx+D)(x+1)(0)$$
$$3(1)+3+2=0+B(2)(1+1)+0$$
$$3+3+2=B(2)(2)$$
$$8=4B$$
$$\frac{8}{4}=B$$
$$2=B$$
$$B=2$$

Review Exercise # 4

equ (**ii**) ⇒

$3x^2+3x+2 = A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x+1)(x-1)$

$3x^2+3x+2 = A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+(Cx+D)(x^2-1)$

$3x^2+3x+2 = Ax^3+Ax-Ax^2-A+Bx^3+Bx+Bx^2+B+Cx^3-Cx+Dx^2-D$

$3x^2+3x+2 = Ax^3+Bx^3+Cx^3-Ax^2+Bx^2+Dx^2+Ax+Bx-Cx-A+B-D$

$3x^2+3x+2 = (A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$

Compare the coefficients of x^3 , x^2 , x *and constant* we get

$A+B+C=0$ …..equ($\boldsymbol{a}$)

$-A+B+D=3$ …..equ($\boldsymbol{b}$)

$A+B-C=3$ …..equ($\boldsymbol{c}$)

$-A+B-D$ …..equ($\boldsymbol{d}$)

Put the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ in equ ($\boldsymbol{a}$)

$-\frac{1}{2}+2+C=0$

$\frac{-1+4}{2}+C=0$

$\frac{3}{2}+C=0$

$C=-\frac{3}{2}$

Put the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{1}{2}\right)+2+D=3$

$\frac{1}{2}+2-3+D=0$

$\frac{1+4-6}{2}+D=0$

$\frac{1+4-6}{2}+D=0$

$\frac{-1}{2}+D=0$

$D=\frac{1}{2}$

Put the values of A, B , C and D in equ (i)

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)} = \frac{-\frac{1}{2}}{x+1}+\frac{2}{x-1}+\frac{-\frac{3}{2}x+\frac{1}{2}}{x^2+1}$$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)} = \frac{-1}{2(x+1)}+\frac{2}{x-1}+\frac{\frac{-3x+1}{2}}{x^2+1}$$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)} = \frac{-1}{2(x+1)}+\frac{2}{x-1}+\frac{-3x+1}{2(x^2+1)}$$

(7) $\boldsymbol{\frac{x^3+3x^2+1}{(x^2+1)^2}}$

Solution:

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Review Exercise # 4

$$\frac{x^3+3x^2+1}{(x^2+1)^2}$$

Let

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}\quad .....\text{equ(i)}$$

Multiply equ (i) by $(x^2+4)^2$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}\times(x^2+1)^2=\frac{Ax+B}{x^2+1}\times(x^2+1)^2+\frac{Cx+D}{(x^2+1)^2}\times(x^2+1)^2$$

$$x^3+3x^2+1=(Ax+B)(x^2+1)+Cx+D\quad .....\text{equ(ii)}$$

equ (ii) $\Rightarrow$

$$x^3+3x^2+1=Ax^3+Ax+Bx^2+B+Cx+D$$

$$x^3+3x^2+1=Ax^3+Bx^2+Ax+Cx+B+D$$

$$x^3+3x^2+1=Ax^3+Bx^2+(A+C)x+(B+D)$$

Compare the coefficients of x^3 , x^2 , x *and constant* we get

$A=1\quad .....\text{equ}(\boldsymbol{a})$

$B=3\quad .....\text{equ}(\boldsymbol{b})$

$A+C=0\quad .....\text{equ}(\boldsymbol{c})$

$B+D=1\ .....\text{equ}(\boldsymbol{d})$

Put $A=1$ in equ $(\boldsymbol{c})$

$1+C=0$

$C=-1$

Put $B=3$ in equ $(\boldsymbol{d})$

$3+D=1$

$D=1-3$

$D=-2$

Put the values of A, B , C and D in equ (i)

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{1x+3}{x^2+1}+\frac{-1x-2}{(x^2+1)^2}$$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{x+3}{x^2+1}+\frac{-(x+2)}{(x^2+1)^2}$$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{x+3}{x^2+1}-\frac{x+2}{(x^2+1)^2}$$

(8) $\boldsymbol{\dfrac{2x^3-1}{x^3+x^2}}$

Solution:

$$\frac{2x^3-1}{x^3+x^2}$$

As $\dfrac{2x^3-1}{x^3+x^2}$ is improper

So

$$\begin{array}{r|l} & \quad 2 \\ \hline x^2+x^2 & 2x^3-1 \\ & \pm 2x^3 \quad \pm 2x^2 \\ \hline \end{array}$$

Review Exercise # 4

$$\overline{\quad\quad\quad\quad\quad}$$
$$-2x^2-1$$

$$\frac{2x^3-1}{x^3+x^2}=2+\frac{-2x^2-1}{x^3+x^2}$$
$$\frac{2x^3-1}{x^3+x^2}=2+\frac{-(2x^2+1)}{x^2(x+1)}$$
$$\frac{2x^3-1}{x^3+x^2}=2-\frac{2x^2+1}{x^2(x+1)}\quad \text{.....equ}(\mathbf{A})$$

Now

Let

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{\text{A}}{x}+\frac{\text{B}}{x^2}+\frac{\text{C}}{x+1}\quad \text{.....equ(i)}$$

Multiply equ (i) by $x^2(x^2+5)$

$$\frac{-2x^2-1}{x^2(x+1)}\times x^2(x+1)=\frac{\text{A}}{x}\times x^2(x+1)+\frac{\text{B}}{x^2}\times x^2(x+1)+\frac{\text{C}}{x+1}\times x^2(x^2+1)$$
$$-2x^2-1=Ax(x+1)+B(x+1)+Cx^2\quad \text{.....equ(ii)}$$

Put $x=0$ in equ (ii)

$$-2(0)^2-1=A(0)(0+1)+B(0+1)+C(0)^2$$
$$0-1=0+B(1)+0$$
$$-1=B$$
$$B=-1$$

equ (ii) ⇒

$$-2x^2-1=Ax(x+1)+B(x+1)+Cx^2$$
$$-2x^2-1=Ax^2+Ax+Bx+B+Cx^2$$
$$-2x^2-1=Ax^2+Cx^2+Ax+Bx+B$$
$$-2x^2-1=(A+C)x^2+(A+B)x+B$$

By comparing the coefficients of x^3, x^2, x and constant we get

$$A+C=-2\quad \text{.....equ}(\boldsymbol{a})$$
$$A+B=0\quad \text{.....equ}(\boldsymbol{b})$$
$$B=-1\quad \text{.....equ}(\boldsymbol{c})$$

Put $B=-1$ *in equ* $(\boldsymbol{b})$

$$A+(-1)=0$$
$$A-1=0$$
$$A=1$$

Put $A=1$ *in equ* $(\boldsymbol{a})$

$$1+C=-2$$
$$C=-2-1$$
$$C=-3$$

Put the values of A, B and C in equ (i)

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{1}{x}+\frac{-1}{x^2}+\frac{-3}{x+1}$$
$$\frac{-2x^2-1}{x^2(x+1)}=\frac{1}{x}-\frac{1}{x^2}-\frac{3}{x+1}$$

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Review Exercise # 4

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(9) $\dfrac{\mathbf{4x^2+3x+14}}{\mathbf{x^3-8}}$

Solution:

$$\frac{4x^2+3x+14}{x^3-8}=\frac{4x^2+3x+14}{x^3-2^3}$$

$$\frac{4x^2+3x+14}{x^3-8}=\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}$$

Let

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}=\frac{\text{A}}{x-2}+\frac{\text{B}x+\text{C}}{x^2+2x+4}\quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-2)(x^2+2x+4)$

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}\times(x-2)(x^2+2x+4)$$

$$=\frac{\text{A}}{x-2}\times(x-2)(x^2+2x+4)+\frac{Bx+\text{C}}{x^2+2x+4}\times(x-2)(x^2+2x+4)$$

$$4x^2+3x+14=A(x^2+2x+4)+(Bx+C)(x-2)\quad \ldots\ldots \text{equ(ii)}$$

Put $x-2=0\Rightarrow x=2$ in equ (ii)

$$4(2)^2+3(2)+14=A[(2)^2+2(2)+4]+(Bx+C)(0)$$

$$4(4)+6+14=A(4+4+4)+0$$

$$16+20=A(12)$$

$$36=12A$$

$$\frac{36}{12}=A$$

$3 = A$
$A = 3$
equ (ii) $\Rightarrow$
$4x^2 + 3x + 14 = A(x^2 + 2x + 4) + (Bx + C)(x - 2)$
$4x^2 + 3x + 14 = Ax^2 + 2Ax + 4A + Bx^2 - 2Bx + Cx - 2C$
$4x^2 + 3x + 14 = Ax^2 + Bx^2 + 2Ax - 2Bx + Cx + 4A - 2C$
$4x^2 + 3x + 14 = (A + B)x^2 + (2A - 2B + C)x + (4A - 2C)$
Compare the coefficients of x^2, x *and constant* we get
$A + B = 4 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$
$2A - 2B + C = 3 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$
$4A - 2C = 14 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$
Put $A = 3$ in equ $(\boldsymbol{a})$
$3 + B = 4$
$B = 4 - 3$
$B = 1$
Put $A = 3$ in equ $(\boldsymbol{c})$
$4(3) - 2C = 14$
$12 - 2C = 14$
$-2C = 14 - 12$
$-2C = 2$
$C = \frac{2}{-2}$
$C = -1$
Put the values of A, B and C in equ (i)
$$\frac{4x^2 + 3x + 14}{(x - 2)(x^2 + 2x + 4)} = \frac{3}{x - 2} + \frac{1x + (-1)}{x^2 + 2x + 4}$$
$$\frac{4x^2 + 3x + 14}{(x - 2)(x^2 + 2x + 4)} = \frac{3}{x - 2} + \frac{x - 1}{x^2 + 2x + 4}$$

***Q3**: Resolve the following fraction into partial fraction* $\boldsymbol{\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}}$

Solution:
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}$$
Let
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \quad \ldots\ldots \text{equ(i)}$$
Multiply equ (i) by $(x + 1)(x^2 + 1)^2$
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} \times (x + 1)(x^2 + 1)^2$$
$$= \frac{A}{x + 1} \times (x + 1)(x^2 + 1)^2 + \frac{Bx + C}{x^2 + 1} \times (x + 1)(x^2 + 1)^2 + \frac{Dx + E}{(x^2 + 1)^2} \times (x + 1)(x^2 + 1)^2$$
$x^4 + 3x^2 + x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \quad \ldots\ldots \text{equ(ii)}$
Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

Review Exercise # 4

$(-1)^4+3(-1)^2+(-1)+1=A((-1)^2+1)^2+(Bx+C)(0)(x^2+1)+(Dx+E)(0)$

$1+3(1)-1+1=A(1+1)^2+0+0$

$1+3=A(2)^2$

$4=A(4)$

$\frac{4}{4}=A$

$1=A$

$A=1$

equ (ii) $\Rightarrow$

$x^4+3x^2+x+1=A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)+(Dx+E)(x+1)$

$x^4+3x^2+x+1=A(x^4+2x^2+1)+(Bx+C)(x^3+x+x^2+1)+Dx^2+Dx+Ex+E$

$x^4+3x^2+x+1=Ax^4+2Ax^2+A+Bx^4+Bx^2+Bx^3+Bx+Cx^3+Cx+Cx^2+C+Dx^2+Dx+Ex+E$

$x^4+3x^2+x+1=Ax^4+Bx^4+Bx^3+Cx^3+2Ax^2+Bx^2+Cx^2+Dx^2+Bx+Cx+Dx+Ex+A+C+E$

$x^4+3x^2+x+1=(A+B)x^4+(B+C)x^3+(2A+B+C+D)x^2+(B+C+D+E)x+(A+C+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A+B=1$ …..equ($\boldsymbol{a}$)

$B+C=0$ …..equ($\boldsymbol{b}$)

$2A+B+C+D=3$ …..equ($\boldsymbol{c}$)

$B+C+D+E=1$ …..equ($\boldsymbol{d}$)

$A+C+E=1$ …..equ($\boldsymbol{e}$)

Put $A=1$ in equ ($\boldsymbol{a}$)

$1+B=1$

$B=1-1$

$B=0$

Put $B=0$ in equ ($\boldsymbol{b}$)

$0+C=0$

$C=0$

Put the values of A, B *and* C *in equ* (c)

$2(1)+0+0+D=3$

$2+D=3$

$D=3-2$

$D=1$

Put the values of A *and* C *in equ* (e)

$1+0+E=1$

$1+E=1$

$E=1-1$

$E=0$

Put the values of A, B , C , D and E in equ (i)

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{1}{x+1}+\frac{0x+0}{x^2+1}+\frac{1x+0}{(x^2+1)^2}$$

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{1}{x+1}+\frac{x}{(x^2+1)^2}$$

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Review Exercise # 4

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MATHEMATICS

Class 10th (KPK)

Chapter # 5 Sets And Fractions

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Chapter # 5

UNIT # 5

SETS AND FUNCTIONS

Ex # 5.1

Set

The collection of well-defined and distinct objects is called set.

Some Important Sets

Set of Natural numbers =N = {1, 2, 3, 4, …….}

Set of Whole numbers = W ={0, 1, 2, 3, 4, …….}

Set of Integers = Z ={0, ±1, ±2, ±3, ±4, …….}

Set of Even Integers = Z ={0, ±2, ±4, …….}

Set of Odd Integers = Z ={±1, ±3, ±5, …….}

Set of Prime numbers = Z ={2, 3, 5, 7, 11, …….}

$Set\ of\ Rational\ numbers$

$$Q = \left\{x \mid x = \frac{p}{q}, q \neq 0 \wedge p, q \in Z\right\}$$

Operation on sets

Union of two sets

The union of two sets is a set which contains all the elements of both the sets.

اس میں دونوں Sets کے تمام elements کو ترتیب کے ساتھ لکھیں گے

لیکن ایک دفعہ

Symbol

The symbol of union is ∪

It is denoted by $A \cup B$ and read as A union B

Set Builder form

$$A \cup B = \{x \mid x \in A\ or\ x \in B\}$$

Example # 1

If $\mathbf{A = \{1, 2, 3\},\ \ B = \{3, 4, 5, 6\}}$ then find $\boldsymbol{A \cup B}$

Solution:

$A = \{1, 2, 3\},\ \ B = \{3, 4, 5, 6\}$

Now

$A \cup B = \{1, 2, 3\} \cup \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Intersection of two sets

The intersection of two sets is a set which contains all the elements that are common to both the sets.

اس میں دونوں Sets کے ایک جیسے elements لکھیں گے۔

Symbol

The symbol of union is ∩

It is denoted by $A \cap B$ and read as A intersection B

Ex # 5.1

Set Builder form

$$A \cup B = \{x \mid x \in A \wedge x \in B\}$$

Disjoint Set

The intersection of two sets have no any common element is called disjoint set.

Symbol

$$A \cap B = \varphi$$

Example # 2

If A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6, 7}

C = {5, 11, 12}, D = {8, 9, 10}

then

$A \cap B = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\}$

$A \cap B = \{3, 4, 5\}$

$B \cap C = \{3, 4, 5, 6, 7\} \cap \{5, 11, 12\}$

$B \cap C = \{5\}$

$A \cap D = \{1, 2, 3, 4, 5\} \cap \{8, 9, 10\}$

$A \cap B = \{\ \}$ or $\emptyset$

Thus, A and D are disjoint set.

Difference of two sets

A set that contains all those elements of First Set which are not in Second set.

پہلے Set کے وہ Elements لکھیں گے جو دوسرے Set میں نہ ہو۔

Symbol

It is denoted by $A \backslash B$ or $A - B$

Set Builder form

$$A \cup B = \{x \mid x \in A \wedge x \notin B\}$$

Example # 3

If $A = \{5, 6, 7, 8\}, B = \{7, 8, 9, 10\}$

then find A \B and B \ A

Solution:

$A = \{5, 6, 7, 8\}, \qquad B = \{7, 8, 9, 10\}$

To Find:

A \ B=?

B \ A =?

Now

A \ B = {5, 6, 7, 8} \{7, 8, 9, 10}

A \ B = {5, 6}

And also

B \ A = {7, 8, 9, 10} \{5, 6, 7, 8}

B \ A = {9, 10}

Chapter # 5

Ex # 5.1

Complement of two sets

If U is a universal set and A is subset of U the U \ A is called complement of the set A and is denoted by A' or A^c.

Note:

$A' = U \setminus A$
$B' = U \setminus B$
$C' = U \setminus C$
$U' = U \setminus U$
$\emptyset' = U \setminus \emptyset$
$U' = U \setminus U = \emptyset$
$\emptyset' = U \setminus \emptyset = U$

Example # 4

If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{3, 4, 5\}$, $B = \varphi$ then find:

$(i) A'$

Solution:

$U = \{1, 2, 3, 4, 5, 6\}, A = \{3, 4, 5\}$

To Find:

A'

Now

$A' = U \setminus A$
$= \{1, 2, 3, 4, 5, 6\} \setminus \{3, 4, 5\}$
$= \{1, 2, 6\}$

$(ii) B'$

Solution:

$U = \{1, 2, 3, 4, 5, 6\}, B = \varphi$

To Find:

B'

Now

$B' = U \setminus B$
$= \{1, 2, 3, 4, 5, 6\} \setminus \varphi$
$= \{1, 2, 3, 4, 5, 6\}$

Ex # 5.1

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Q1: **If $A = \{1, 2, 3\}$, $B = \{0, 1\}$ and $C = \{1, 3, 4\}$ then find:**

(i) $A \cup B$

Solution:

$A = \{1, 2, 3\}$, $B = \{0, 1\}$

To Find:

Now

Ex # 5.1

$A \cup B = \{1, 2, 3\} \cup \{0, 1\}$
$A \cup B = \{0, 1, 2, 3\}$

(ii) $A \cap B$

Solution:

$A = \{1, 2, 3\}$, $B = \{0, 1\}$

To Find:

$A \cap B$

Now

$A \cap B = \{1, 2, 3\} \cap \{0, 1\}$
$A \cap B = \{1\}$

(iii) $A \cup C$

Solution:

$A = \{1, 2, 3\}$, $C = \{1, 3, 4\}$

To Find:

$A \cup C$

Now

$A \cup C = \{1, 2, 3\} \cup \{1, 3, 4\}$
$A \cup C = \{1, 2, 3, 4\}$

(iv) $A \cap C$

Solution:

$A = \{1, 2, 3\}$, $C = \{1, 3, 4\}$

To Find:

$A \cap C$

Now

$A \cap C = \{1, 2, 3\} \cap \{1, 3, 4\}$
$A \cap C = \{1, 3\}$

(v) $B \cup C$

Solution:

$B = \{0, 1\}$ and $C = \{1, 3, 4\}$

To Find:

$B \cup C$

Now

$B \cup C = \{0, 1\} \cup \{1, 3, 4\}$
$B \cup C = \{0, 1, 3, 4\}$

(vi) $A \cap A$

Solution:

$A = \{1, 2, 3\}$

To Find:

$A \cap A$

Now

$A \cap A = \{1, 2, 3\} \cap \{1, 2, 3\}$
$A \cap A = \{1, 2, 3\}$

Chapter # 5

Ex # 5.1

Q2: Find A \ B and B \ A when:

(i) $\boldsymbol{A = \{1, 3, 5, 7\}}$, $\boldsymbol{B = \{3, 4, 5, 6, 7, 8\}}$

Solution:

$A = \{1, 3, 5, 7\}$, $B = \{3, 4, 5, 6, 7, 8\}$

To Find:

A \ B=?

B \ A =?

Now

A \ B = $\{1, 3, 5, 7\} \setminus \{3, 4, 5, 6, 7, 8\}$

A \ B = $\{1\}$

And also

B \ A = $\{3, 4, 5, 6, 7, 8\} \setminus \{1, 3, 5, 7\}$

B \ A = $\{4, 6, 8\}$

(ii) $\boldsymbol{A = \{0, \pm 1, \pm 2, \pm 3\}}$, $\boldsymbol{B = \{-1, -2, -3\}}$

Solution:

$A = \{0, \pm 1, \pm 2, \pm 3\}$, $B = \{-1, -2, -3\}$

To Find:

A \ B=?

B \ A =?

Now

A \ B = $\{0, \pm 1, \pm 2, \pm 3\} \setminus \{-1, -2, -3\}$

A \ B = $\{0, 1, 2, 3\}$

And also

B \ A = $\{-1, -2, -3\} \setminus \{0, \pm 1, \pm 2, \pm 3\}$

B \ A = $\{ \}$

(iii) $\boldsymbol{A = \{1, 2, 3, 4 \ \ldots\}}$, $\boldsymbol{B = \{1, 3, 5, 7, \ldots\}}$

Solution:

$A = \{1, 2, 3, 4 \ \ldots\}$, $B = \{1, 3, 5, 7, \ldots\}$

To Find:

A \ B=?

B \ A =?

Now

A \ B = $\{1, 2, 3, 4 \ \ldots\} \setminus \{1, 3, 5, 7, \ldots\}$

A \ B = $\{2, 4, 6, \ldots\}$

And also

B \ A = $\{1, 3, 5, 7, \ldots\} \setminus \{1, 2, 3, 4 \ \ldots\}$

B \ A = $\{ \}$

Q3: If U = $\boldsymbol{\{1, 2, 3, 4 \ \ldots 20\}}$, A = $\boldsymbol{\{2, 4, 6, \ldots 20\}}$ $\boldsymbol{B}$ =$\boldsymbol{\{1, 3, 5, \ldots 19\}}$ and $\boldsymbol{C = \varphi}$ then find:

Ex # 5.1

(i) $\boldsymbol{A'}$

Solution:

U = $\{1, 2, 3, 4 \ \ldots 20\}$, A = $\{2, 4, 6, \ldots 20\}$

To Find:

A'

Now

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{2, 4, 6, \ldots 20\}$

$= \{1, 3, 5, \ldots 19\}$

(ii) $\boldsymbol{B'}$

Solution:

U = $\{1, 2, 3, 4 \ \ldots 20\}$, B =$\{1, 3, 5, \ldots 19\}$

To Find:

B'

Now

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{1, 3, 5, \ldots 19\}$

$= \{2, 4, 6, \ldots 20\}$

(iii) $\boldsymbol{C'}$

Solution:

U = $\{1, 2, 3, 4 \ \ldots 20\}$, $C = \varphi$

To Find:

C'

Now

$C' = U \setminus C$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{ \}$

$= \{1, 2, 3, 4 \ \ldots 20\}$

(iv) $\boldsymbol{A' \cup B'}$

Solution:

U = $\{1, 2, 3, 4 \ \ldots 20\}$, A = $\{2, 4, 6, \ldots 20\}$,
B =$\{1, 3, 5, \ldots 19\}$

To Find:

$A' \cup B'$

First we find A':

A'

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{2, 4, 6, \ldots 20\}$

$= \{1, 3, 5, \ldots 19\}$

Now find B':

B'

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{1, 3, 5, \ldots 19\}$

$= \{2, 4, 6, \ldots 20\}$

Now

$A' \cup B' = \{1, 3, 5, \ldots 19\} \cup \{2, 4, 6, \ldots 20\}$

$A' \cup B' = \{1, 2, 3, 4 \ \ldots 20\}$

Chapter # 5

Ex # 5.1

(v) $\boldsymbol{A' \cap B'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}$, $A = \{2, 4, 6, \ldots 20\}$,

$B = \{1, 3, 5, \ldots 19\}$

To Find:

$A' \cap B'$

First we find A':

A'

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{2, 4, 6, \ldots 20\}$

$= \{1, 3, 5, \ldots 19\}$

Now find B':

B'

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{1, 3, 5, \ldots 19\}$

$= \{2, 4, 6, \ldots 20\}$

Now

$A' \cap B' = \{1, 3, 5, \ldots 19\} \cap \{2, 4, 6, \ldots 20\}$

$A' \cap B' = \{\ \}$

(vi) $\boldsymbol{A' \cap B}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}$, $A = \{2, 4, 6, \ldots 20\}$,

$B = \{1, 3, 5, \ldots 19\}$

To Find:

$A' \cap B$

First we find A':

A'

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{2, 4, 6, \ldots 20\}$

$= \{1, 3, 5, \ldots 19\}$

Now

$A' \cap B = \{1, 3, 5, \ldots 19\} \cap \{1, 3, 5, \ldots 19\}$

$A' \cap B = \{1, 3, 5, \ldots 19\}$

(vii) $\boldsymbol{A' \cup C'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}$, $A = \{2, 4, 6, \ldots 20\}$,

$C = \varphi$

To Find:

$$A' \cup C'$$

First we find A':

$$A'$$

$$A' = U \setminus A$$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{2, 4, 6, \ldots 20\}$

$= \{1, 3, 5, \ldots 19\}$

Ex # 5.1

Now find C':

C'

$C' = U \setminus C$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{\ \}$

$= \{1, 2, 3, 4 \ \ldots 20\}$

Now

$A' \cup C' = \{1, 3, 5, \ldots 19\} \cup \{1, 2, 3, 4 \ \ldots 20\}$

$A' \cup C' = \{1, 2, 3, 4 \ \ldots 20\}$

(viii) $\boldsymbol{A \cap C'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}$, $A = \{2, 4, 6, \ldots 20\}$,

$C = \varphi$

To Find:

$A \cap C'$

First we find C':

C'

$C' = U \setminus C$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{\ \}$

$= \{1, 2, 3, 4 \ \ldots 20\}$

Now

$A \cap C' = \{2, 4, 6, \ldots 20\} \cap \{1, 2, 3, 4 \ \ldots 20\}$

$A \cap C' = \{2, 4, 6, \ldots 20\}$

(ix) $\boldsymbol{C' \cap C}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}, C = \varphi$

To Find:

$C' \cap C$

First we find C':

C'

$C' = U \setminus C$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{\ \}$

$= \{1, 2, 3, 4 \ \ldots 20\}$

Now

$C' \cap C = \{1, 2, 3, 4 \ \ldots 20\} \cap \{\ \}$

$C' \cap C = \{\ \}$

Chapter # 5

Ex # 5.1

(x) $\boldsymbol{B' \cup C'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 20\}$, $B = \{1, 3, 5, \ldots 19\}$ and $C = \varphi$

To Find:

$B' \cup C'$

First we find B':

B'

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{1, 3, 5, \ldots 19\}$

$= \{2, 4, 6, \ldots 20\}$

Now find C':

C'

$C' = U \setminus C$

$= \{1, 2, 3, 4 \ \ldots 20\} \setminus \{\ \}$

$= \{1, 2, 3, 4 \ \ldots 20\}$

Now

$B' \cup C' = \{2, 4, 6, \ldots 20\} \cup \{1, 2, 3, 4 \ \ldots 20\}$

$B' \cup C' = \{1, 2, 3, 4 \ \ldots 20\}$

Q4: **If $\boldsymbol{U}$ = set of natural numbers upto 15 *and* $\boldsymbol{A}$ = set of even numbers upto 15 *and* $\boldsymbol{B}$ = set of odd numbers upto 15**

Then find

(i) $\boldsymbol{A' \cup B'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 15\}$, $A = \{2, 4, 6, \ldots 14\}$

$B = \{1, 3, 5, \ldots 15\}$

To Find:

$A' \cup B'$

First we find A':

A'

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{2, 4, 6, \ldots 14\}$

$= \{1, 3, 5, \ldots 15\}$

Now find B':

B'

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{1, 3, 5, \ldots 15\}$

$= \{2, 4, 6, \ldots 14\}$

Now

$A' \cup B' = \{1, 3, 5, \ldots 15\} \cup \{2, 4, 6, \ldots 14\}$

$A' \cup B' = \{1, 2, 3, 4 \ \ldots 15\}$

Ex # 5.1

(ii) $\boldsymbol{A' \cap B'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 15\}$, $A = \{2, 4, 6, \ldots 14\}$

$B = \{1, 3, 5, \ldots 15\}$

To Find:

$A' \cap B'$

First we find A':

A'

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{2, 4, 6, \ldots 14\}$

$= \{1, 3, 5, \ldots 15\}$

Now find B':

B'

$B' = U \setminus B$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{1, 3, 5, \ldots 15\}$

$= \{2, 4, 6, \ldots 14\}$

Now

$A' \cap B' = \{1, 3, 5, \ldots 15\} \cap \{2, 4, 6, \ldots 14\}$

$A' \cap B' = \{\ \}$

(iii) $\boldsymbol{U'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 15\}$

To Find:

U'

$U' = U \setminus U$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{1, 2, 3, 4 \ \ldots 15\}$

$= \{\ \}$

(iv) $\emptyset'$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 15\}$

To Find:

$\emptyset'$

$\emptyset' = U \setminus \emptyset$

$= \{1, 2, 3, 4 \ \ldots 15\} \setminus \{\ \}$

$= \{1, 2, 3, 4 \ \ldots 15\}$

(v) $\boldsymbol{B \cap A'}$

Solution:

$U = \{1, 2, 3, 4 \ \ldots 15\}$, $A = \{2, 4, 6, \ldots 14\}$

$B = \{1, 3, 5, \ldots 15\}$

To Find:

$B \cap A'$

Chapter # 5

Ex # 5.1

First we find A':
A'
$A' = U \setminus A$
$= \{1, 2, 3, 4 \ ... 15\} \setminus \{2, 4, 6, ... 14\}$
$= \{1, 3, 5, ... 15\}$
Now
$B \cap A' = \{1, 3, 5, ... 15\} \cap \{1, 3, 5, ... 15\}$
$$B \cap A' = \{1, 3, 5, ... 15\}$$

(vi) $\boldsymbol{B \cup B'}$

Solution:
$U = \{1, 2, 3, 4 \ ... 15\}$, $B = \{1, 3, 5, ... 15\}$
To Find:
$B \cup B'$
First we find B':
$B' = U \setminus B$
$= \{1, 2, 3, 4 \ ... 15\} \setminus \{1, 3, 5, ... 15\}$
$= \{2, 4, 6, ... 14\}$
Now
$B \cup B' = \{1, 3, 5, ... 15\} \cup \{2, 4, 6, ... 14\}$
$$B \cup B' = \{1, 2, 3, 4 \ ... 15\}$$

(vii) $\boldsymbol{A \cap A'}$

Solution:
$U = \{1, 2, 3, 4 \ ... 15\}$, $A = \{2, 4, 6, ... 14\}$
To Find:
$A \cap A'$
First we find A':
A'
$A' = U \setminus A$
$= \{1, 2, 3, 4 \ ... 15\} \setminus \{2, 4, 6, ... 14\}$
$= \{1, 3, 5, ... 15\}$
Now
$A \cap A' = \{2, 4, 6, ... 14\} \cap \{1, 3, 5, ... 15\}$
$$A \cap A' = \{\ \}$$

(viii) $\boldsymbol{A \cup B'}$

Solution:
$U = \{1, 2, 3, 4 \ ... 15\}$, $B = \{1, 3, 5, ... 15\}$
To Find:
$A \cup B'$
First we find B':
$B' = U \setminus B$
$= \{1, 2, 3, 4 \ ... 15\} \setminus \{1, 3, 5, ... 15\}$
$= \{2, 4, 6, ... 14\}$
$A \cup B' = \{2, 4, 6, ... 14\} \cup \{2, 4, 6, ... 14\}$
$A \cup B' = \{2, 4, 6, ... 14\}$

Ex # 5.2

Properties of Union and Intersection

Commutative Property of Union:
$$A \cup B = B \cup A$$
Commutative Property of Intersection:
$$A \cap B = B \cap A$$
Associative Property of Union:
$$A \cup (B \cup C) = (A \cup B) \cup C$$
Associative Property of Intersection:
$$A \cap (B \cap C) = (A \cap B) \cap C$$
Distributive Property of Union over Intersection:
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
Distributive Property of Intersection over Union:
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
De-Morgan's Law:
For any two sets A and B which are subsets of U then
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$
Note:
$(A \cup B)' = U \setminus (A \cup B)$
$(A \cap B)' = U \setminus (A \cap B)$

Example # 5

Verify commutative property of union for the following set.
$\boldsymbol{A = \{1, 2, 3\}, B = \{4, 5, 6\}}$

Solution:
$A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$
To Prove:
Commutative Property of Union:
Now
$A \cup B = B \cup A$
L.H.S: $A \cup B$
$A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\}$
$A \cup B = \{1, 2, 3, 4, 5, 6\}$
R.H.S: $B \cup A$
$B \cup A = \{4, 5, 6\} \cup \{1, 2, 3\}$
$B \cup A = \{1, 2, 3, 4, 5, 6\}$
Hence
$A \cup B = B \cup A$
Proved

Chapter # 5

Ex # 5.2

Example # 6

Verify commutative property of intersection for the following set.

A ={a, b, c} ,B ={ b, c, d, e}

Solution:

A ={a, b, c} ,B ={ b, c, d, e}

To Prove:

Commutative Property of Intersection:

Now

$A \cap B = B \cap A$

L.H.S: $A \cap B$

$A \cap B = \{a, b, c\} \cap \{ b, c, d, e\}$

$A \cap B = \{ b, c\}$

R.H.S: $B \cap A$

$B \cap A = \{ b, c, d, e\} \cap \{a, b, c\}$

$B \cap A = \{ b, c\}$

Hence

$A \cap B = B \cap A$

Proved

Example # 7

$A = \{3, 4, 5\}$, $B = \{5, 6, 7\}$, $C = \{8, 9, 10\}$ then prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Solution:

$A = \{3, 4, 5\}$, $B = \{5, 6, 7\}$, $C = \{8, 9, 10\}$

To Prove:

Associative Property of Union:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $A \cup (B \cup C)$

$B \cup C = \{5, 6, 7\} \cup \{8, 9, 10\}$

$B \cup C = \{5, 6, 7, 8, 9, 10\}$

Now

$A \cup (B \cup C) = \{3, 4, 5\} \cup \{5, 6, 7, 8, 9, 10\}$

$A \cup (B \cup C) = \{3, 4, 5, 6, 7, 8, 9, 10\}$

R.H.S: $(A \cup B) \cup C$

$A \cup B = \{3, 4, 5\} \cup \{5, 6, 7\}$

$A \cup B = \{3, 4, 5, 6, 7\}$

Now

$(A \cup B) \cup C = \{3, 4, 5, 6, 7\} \cup \{8, 9, 10\}$

$(A \cup B) \cup C = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Hence

$A \cup (B \cup C) = (A \cup B) \cup C$

Proved

Ex # 5.2

Example # 8

$A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$ then prove that $A \cap (B \cap C) = (A \cap B) \cap C$

Solution:

$A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$

To Prove:

Associative Property of Intersection

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S: $A \cap (B \cap C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 4, 5\}$

$B \cap C = \{ 3, 4 \}$

Now

$A \cap (B \cap C) = \{1, 2, 3\} \cap \{ 3, 4 \}$

$A \cap (B \cap C) = \{ 3 \}$

R.H.S: $(A \cap B) \cap C$

$A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\}$

$A \cap B = \{ 2, 3 \}$

Now

$(A \cap B) \cap C = \{ 2, 3 \} \cap \{3, 4, 5\}$

$(A \cap B) \cap C = \{3 \}$

Hence

$A \cap (B \cap C) = (A \cap B) \cap C$

Proved

Example # 9

$A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$, $C = \{7, 8, 9\}$ then prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

$A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$, $C = \{7, 8, 9\}$

To prove:

Distributive Property of Union over Intersection:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $A \cup (B \cap C)$

$B \cap C = \{5, 6, 7\} \cap \{7, 8, 9\}$

$B \cap C = \{ 7 \}$

Now

$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{ 7 \}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 7\}$

R.H.S: $(A \cup B) \cap (A \cup C)$

First we find $A \cup B$:

$A \cup B = \{1, 2, 3, 4\} \cup \{5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

Chapter # 5

Ex # 5.2

Now
$A \cup C = \{1, 2, 3, 4\} \cup \{7, 8, 9\}$
$A \cup C = \{1, 2, 3, 4, 7, 8, 9\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 7, 8, 9\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 7\}$
Hence
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Proved

<u>Example # 10</u>

$A = \{a, b, c\}$, $B = \{c, d, e\}$, $C = \{e, f, g\}$ then prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

<u>Solution</u>:
$A = \{a, b, c\}$, $B = \{c, d, e\}$, $C = \{e, f, g\}$
To Prove:
Distributive Property of Intersection over Union
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
L.H.S: $\boldsymbol{A \cap (B \cup C)}$
$B \cup C = \{c, d, e\} \cup \{e, f, g\}$
$B \cup C = \{c, d, e, f, g\}$
Now
$A \cap (B \cup C) = \{a, b, c\} \cap \{c, d, e, f, g\}$
$A \cap (B \cup C) = \{c\}$
R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$
First we find $A \cap B$:
$A \cap B = \{a, b, c\} \cap \{c, d, e\}$
$A \cap B = \{c\}$
Now we find $A \cap B$:
$A \cap C = \{a, b, c\} \cap \{e, f, g\}$
$A \cap C = \{\ \}$
$(A \cap B) \cup (A \cap C) = \{c\} \cup \{\ \}$
$(A \cap B) \cup (A \cap C) = \{c\}$
Hence
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Proved

<u>Example # 11</u>

If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$, $B = \{3, 4, 5\}$ then verify De-Morgan's Laws

<u>Solution</u>:

<u>De-Morgan's Law</u>:
$U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$, $B = \{3, 4, 5\}$
To Prove:
$(i)\quad (A \cup B)' = A' \cap B'$
$(ii)\quad (A \cap B)' = A' \cup B'$

Ex # 5.2

$(A \cup B)' = A' \cap B'$
L.H.S: $\boldsymbol{(A \cup B)'}$
First we find $A \cup B$:
$A \cup B = \{2, 3\} \cup \{3, 4, 5\}$
$A \cup B = \{2, 3, 4, 5\}$
$(A \cup B)' = U \setminus (A \cup B)$
$(A \cup B)' = \{1, 2, 3, 4, 5, 6\} \setminus \{2, 3, 4, 5\}$
$(A \cup B)' = \{1, 6\}$
R.H.S: $\boldsymbol{A' \cap B'}$
First we find A':
$A' = U \setminus A$
$= \{1, 2, 3, 4, 5, 6\} \setminus \{2, 3\}$
$= \{1, 4, 5, 6\}$
And Also
$B' = U \setminus B$
$= \{1, 2, 3, 4, 5, 6\} \setminus \{3, 4, 5\}$
$= \{1, 2, 6\}$
Now
$A' \cap B' = \{1, 4, 5, 6\} \cap \{1, 2, 6\}$
$A' \cap B' = \{1, 6\}$
Hence
$(A \cup B)' = A' \cap B'$
Proved
$(A \cap B)' = A' \cup B'$
L.H.S: $\boldsymbol{(A \cap B)'}$
First we find $A \cap B$:
$A \cap B = \{2, 3\} \cap \{3, 4, 5\}$
$A \cap B = \{3\}$
$(A \cap B)' = U \setminus (A \cap B)$
$(A \cap B)' = \{1, 2, 3, 4, 5, 6\} \setminus \{3\}$
$(A \cap B)' = \{1, 2, 4, 5, 6\}$
R.H.S: $\boldsymbol{A' \cup B'}$
First we find A':
$A' = U \setminus A$
$= \{1, 2, 3, 4, 5, 6\} \setminus \{2, 3\}$
$= \{1, 4, 5, 6\}$
And Also
$B' = U \setminus B$
$= \{1, 2, 3, 4, 5, 6\} \setminus \{3, 4, 5\}$
$= \{1, 2, 6\}$
Now
$A' \cup B' = \{1, 4, 5, 6\} \cup \{1, 2, 6\}$
$A' \cup B' = \{1, 2, 4, 5, 6\}$
Hence
$(A \cap B)' = A' \cup B'$
Proved

Ex # 5.2

Page # 95

Q1: Verify commutative property of union and intersection for the following sets.

(i) A ={1, 2, 3, ... 12} ,B ={2, 4, 5, 8, 10, 12}

Solution:

A ={1, 2, 3, ... 12} ,B ={2, 4, 5, 8, 10, 12}

To Prove:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Now

Commutative Property of Union:

$A \cup B = B \cup A$

L.H.S: $A \cup B$

$A \cup B = \{1, 2, 3, \dots\ 12\} \cup \{2, 4, 5, 8, 10, 12\}$

$A \cup B = \{1, 2, 3, \dots\ 12\}$

R.H.S: $B \cup A$

$B \cup A = \{2, 4, 5, 8, 10, 12\} \cup \{1, 2, 3, \dots\ 12\}$

$B \cup A = \{1, 2, 3, \dots\ 12\}$

Hence

$A \cup B = B \cup A$

Proved

Commutative Property of Intersection:

$A \cap B = B \cap A$

L.H.S: $A \cap B$

$A \cap B = \{1, 2, 3, \dots\ 12\} \cap \{2, 4, 5, 8, 10, 12\}$

$A \cap B = \{2, 4, 5, 8, 10, 12\}$

R.H.S: $B \cap A$

$B \cap A = \{2, 4, 5, 8, 10, 12\} \cap \{1, 2, 3, \dots\ 12\}$

$B \cap A = \{2, 4, 5, 8, 10, 12\}$

Hence

$A \cap B = B \cap A$

Proved

(ii) $A = N$,

$B = \{x \mid x \in N \land x$ is an even integer$\}$

Solution:

$A = N$

$A = \{1, 2, 3, 4\ \dots\}$

$B = \{x \mid x \in N \land x \text{ is an even integer}\}$

$B = \{2, 4, 6, 8, \dots\}$

To Prove:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Now

Commutative Property of Union:

$A \cup B = B \cup A$

L.H.S: $A \cup B$

$A \cup B = \{1, 2, 3, 4\ \dots\} \cup \{2, 4, 6, 8, \dots\}$

$A \cup B = \{1, 2, 3, 4\ \dots\}$

R.H.S: $B \cup A$

$B \cup A = \{2, 4, 6, 8, \dots\} \cup \{1, 2, 3, 4\ \dots\}$

$B \cup A = \{1, 2, 3, 4\ \dots\}$

Hence

$A \cup B = B \cup A$

Proved

Commutative Property of Intersection:

$A \cap B = B \cap A$

L.H.S: $A \cap B$

$A \cap B = \{1, 2, 3, 4\ \dots\} \cap \{2, 4, 6, 8, \dots\}$

$A \cap B = \{2, 4, 6, 8, \dots\}$

R.H.S: $B \cap A$

$B \cap A = \{2, 4, 6, 8, \dots\} \cap \{1, 2, 3, 4\ \dots\}$

$B \cap A = \{2, 4, 6, 8, \dots\}$

Hence

$A \cap B = B \cap A$

Proved

(iii) A = Set of first ten prime numbers.

B = Set of first ten composite numbers.

Solution:

$A =$ Set of first ten prime numbers.

A ={2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

B = Set of first ten composite numbers.

B ={4, 6, 8, 9, 10, 12, 14, 15, 16, 18}

To Prove:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Now

Commutative Property of Union:

$A \cup B = B \cup A$

L.H.S: $A \cup B$

$A \cup B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \cup \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$A \cup B = \{2, 3, 4, 5\ \dots 18, 19, 23, 29\}$

R.H.S: $B \cup A$

$B \cup A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$B \cup A = \{2, 3, 4, 5\ \dots 18, 19, 23, 29\}$

Hence

$A \cup B = B \cup A$

Proved

Chapter # 5

Ex # 5.2

Commutative Property of Intersection:

$A \cap B = B \cap A$

L.H.S: $A \cap B$

$A \cap B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \cap \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$A \cap B = \{\ \}$

R.H.S: $B \cap A$

$B \cap A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$B \cap A = \{\ \}$

Hence

$A \cap B = B \cap A$

Proved

Q2: Verify associative properties of union and intersection for the following sets.

(*i*) $A = \{a, b, c, \ldots \ldots z\}$, $B = \{a, e, i, o, u\}$, $C = \{a, d, i, l, m, n, o\}$

<u>Solution</u>:

$A = \{a, b, c, \ldots \ldots z\}$, $B = \{a, e, i, o, u\}$,

$C = \{a, d, i, l, m, n, o\}$

To Prove:

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Now

Associative Property of Union:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $\boldsymbol{A \cup (B \cup C)}$

$B \cup C = \{a, e, i, o, u\} \cup \{a, d, i, l, m, n, o\}$

$B \cup C = \{a, d, e, i, l, m, n, o, u\}$

Now

$A \cup (B \cup C) = \{a, b, c, \ldots \ldots z\} \cup \{a, d, e, i, l, m, n, o, u\}$

$A \cup (B \cup C) = \{a, b, c, \ldots \ldots z\}$

R.H.S: $\boldsymbol{(A \cup B) \cup C}$

$A \cup B = \{a, b, c, \ldots \ldots z\} \cup \{a, e, i, o, u\}$

$A \cup B = \{a, b, c, \ldots \ldots z\}$

Now

$(A \cup B) \cup C = \{a, b, c, \ldots \ldots z\} \cup \{a, d, i, l, m, n, o\}$

$(A \cup B) \cup C = \{a, b, c, \ldots \ldots z\}$

Hence

$A \cup (B \cup C) = (A \cup B) \cup C$

Proved

Ex # 5.2

Associative Property of Intersection:

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S: $\boldsymbol{A \cap (B \cap C)}$

$B \cap C = \{a, e, i, o, u\} \cap \{a, d, i, l, m, n, o\}$

$B \cap C = \{a, i, o\}$

Now

$A \cap (B \cap C) = \{a, b, c, \ldots \ldots z\} \cap \{a, i, o\}$

$A \cap (B \cap C) = \{a, i, o\}$

R.H.S: $\boldsymbol{(A \cap B) \cap C}$

$A \cap B = \{a, b, c, \ldots \ldots z\} \cap \{a, e, i, o, u\}$

$A \cap B = \{a, e, i, o, u\}$

Now

$(A \cap B) \cap C = \{a, e, i, o, u\} \cap \{a, d, i, l, m, n, o\}$

$(A \cap B) \cap C = \{a, i, o\}$

Hence

$A \cap (B \cap C) = (A \cap B) \cap C$

Proved

(ii) $A = \{1, 2, 3, \ldots \ldots 100\}$, $B = \{2, 4, 6, \ldots \ldots 100\}$, $C = \{1, 3, 5, \ldots \ldots 99\}$

<u>Solution</u>:

$A = \{1, 2, 3, \ldots \ldots 100\}$, $B = \{2, 4, 6, \ldots \ldots 100\}$

$C = \{1, 3, 5, \ldots \ldots 99\}$

To Prove:

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Now

Associative Property of Union:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $\boldsymbol{A \cup (B \cup C)}$

$B \cup C = \{2, 4, 6, \ldots \ldots 100\} \cup \{1, 3, 5, \ldots \ldots 99\}$

$B \cup C = \{1, 2, 3, 4, \ldots \ldots 100\}$

Now

$A \cup (B \cup C) = \{1, 2, 3, \ldots \ldots 100\} \cup \{1, 2, 3, 4, \ldots \ldots 100\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, \ldots \ldots 100\}$

R.H.S: $\boldsymbol{(A \cup B) \cup C}$

$A \cup B = \{1, 2, 3, 4, \ldots \ldots 100\} \cup \{2, 4, 6, \ldots \ldots 100\}$

$A \cup B = \{1, 2, 3, 4, \ldots \ldots 100\}$

Now

$(A \cup B) \cup C = \{1, 2, 3, 4, \ldots \ldots 100\} \cup \{1, 3, 5, \ldots \ldots 99\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, \ldots \ldots 100\}$

Hence

$A \cup (B \cup C) = (A \cup B) \cup C$

Proved

Chapter # 5

Ex # 5.2

Associative Property of Intersection:

$$A \cap (B \cap C) = (A \cap B) \cap C$$

L.H.S: $\boldsymbol{A \cap (B \cap C)}$

$B \cap C = \{2, 4, 6, \dots \dots 100\} \cap \{1, 3, 5, \dots \dots 99\}$

$B \cap C = \{\ \}$

Now

$A \cap (B \cap C) = \{1, 2, 3, \dots \dots 100\} \cap \{\ \}$

$A \cap (B \cap C) = \{\ \}$

R.H.S: $\boldsymbol{(A \cap B) \cap C}$

$A \cap B = \{1, 2, 3, 4, \dots \dots 100\} \cap \{2, 4, 6, \dots \dots 100\}$

$A \cap B = \{\ \}$

Now

$(A \cap B) \cap C = \{\ \} \cup \{1, 3, 5, \dots \dots 99\}$

$(A \cap B) \cap C = \{\ \}$

Hence

$A \cap (B \cap C) = (A \cap B) \cap C$

Proved

Q3: Verify distributive properties of union over intersection and intersection over union.

(i) $\boldsymbol{A = \{0, 1, 2\}, B = \{0\}, C = \varphi}$

Solution:

Distributive Property of Union over Intersection:

$A = \{0, 1, 2\}$, $B = \{0\}$, $C = \varphi$

To Prove:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $\boldsymbol{A \cup (B \cap C)}$

$B \cap C = \{0\} \cap \varphi$

$B \cap C = \{\ \}$

Now

$A \cup (B \cap C) = \{0, 1, 2\} \cup \{\ \}$

$A \cup (B \cap C) = \{0, 1, 2\}$

R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cup B$:

$A \cup B = \{0, 1, 2\} \cup \{0\}$

$A \cup B = \{0, 1, 2\}$

Now

$A \cup C = \{0, 1, 2\} \cup \varphi$

$A \cup C = \{0, 1, 2\}$

$(A \cup B) \cap (A \cup C) = \{0, 1, 2\} \cap \{0, 1, 2\}$

$(A \cup B) \cap (A \cup C) = \{0, 1, 2\}$

Hence

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

Ex # 5.2

Distributive Property of Intersection over Union:

$A = \{0, 1, 2\}, B = \{0\}, C = \varphi$

To Prove:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $\boldsymbol{A \cap (B \cup C)}$

$B \cup C = \{0\} \cup \varphi$

$B \cup C = \{0\}$

Now

$A \cap (B \cup C) = \{0, 1, 2\} \cap \{0\}$

$A \cap (B \cup C) = \{0\}$

R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cap B$:

$A \cap B = \{0, 1, 2\} \cap \{0\}$

$A \cap B = \{0\}$

Now we find $A \cap B$:

$A \cap C = \{0, 1, 2\} \cap \varphi$

$A \cap C = \{\ \}$

$(A \cap B) \cup (A \cap C) = \{0\} \cup \{\ \}$

$(A \cap B) \cup (A \cap C) = \{0, 1, 2\}$

Hence

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved

(ii) $\boldsymbol{A = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}}$,
$\boldsymbol{B = \{-1, -2, -3, -4, -5\}, C = \{-1, -2, +3, +4\}}$

Solution:

Distributive Property of Union over Intersection:

$A = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}, B = \{-1, -2, -3, -4, -5\}$

$C = \{-1, -2, +3, +4\}$

To Prove:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $\boldsymbol{A \cup (B \cap C)}$

$B \cap C = \{-1, -2, -3, -4, -5\} \cap \{-1, -2, +3, +4\}$

$B \cap C = \{-1, -2\}$

Now

$A \cup (B \cap C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2\}$

$A \cup (B \cap C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cup B$:

$A \cup B = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2, -3, -4, -5\}$

$A \cup B = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

Now

$A \cup C = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2, +3, +4\}$

$A \cup C = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

Ex # 5.2

$(A\cup B)\cap(A\cup C)=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}\cap\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}$

$(A\cup B)\cap(A\cup C)=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}$

Hence

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proved

Distributive Property of Intersection over Union:

$A=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}, B=\{-1,-2,-3,-4,-5\}$

$C=\{-1,-2,+3,+4\}$

To Prove:

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

L.H.S: $\boldsymbol{A\cap(B\cup C)}$

$B\cup C=\{-1,-2,-3,-4,-5\}\cup\{-1,-2,+3,+4\}$

$B\cup C=\{-1,-2,\pm3,\pm4,-5\}$

Now

$A\cap(B\cup C)=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}\cap\{-1,-2,\pm3,\pm4,-5\}$

$A\cap(B\cup C)=\{-1,-2,\pm3,\pm4,-5\}$

R.H.S: $\boldsymbol{(A\cup B)\cap(A\cup C)}$

First we find $A\cap B$:

$A\cap B=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}\cap\{-1,-2,-3,-4,-5\}$

$A\cap B=\{-1,-2,-3,-4,-5\}$

Now we find $A\cap B$:

$A\cap C=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}\cap\{-1,-2,+3,+4\}$

$A\cap C=\{-1,-2,+3,+4\}$

$(A\cap B)\cup(A\cap C)=\{-1,-2,-3,-4,-5\}\cup\{-1,-2,+3,+4\}$

$(A\cap B)\cup(A\cap C)=\{-1,-2,\pm3,\pm4,-5\}$

Hence

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Proved

Q4: Verify De Morgan's laws for the following sets.

(i) $\boldsymbol{U=\{x\mid x\in N\land 1\le x\le 20\}}$,

$\boldsymbol{A}$ **= {2, 3, 5, 7, 11, 12, 13, 17}**

$\boldsymbol{B}$ **= {1, 4, 6, 8, 10, 14, 17, 18}**

Solution:

De-Morgan's Law:

$U=\{x\mid x\in N\land 1\le x\le 20\}$

$\text{U}=\{1,2,3,4\ \dots 20\}$

$A=\{2, 3, 5, 7, 11, 12, 13, 17\}, B=\{1, 4, 6, 8, 10, 14, 17, 18\}$

To Prove:

$(A\cup B)'=A'\cap B'$

L.H.S: $\boldsymbol{(A\cup B)'}$

First we find $A\cup B$:

$A\cup B=\{2,3,5,7,11,12,13,17\}\cup\{1,4,6,8,10,14,17,18\}$

$A\cup B=\{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 17, 18\}$

Ex # 5.2

$(A\cup B)'=U\setminus(A\cup B)$

$(A\cup B)'=\{1,2,3,4\ \dots 20\}\setminus$

$\{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 17, 18\}$

$(A\cup B)'=\{9, 15, 16, 19, 20\}$

R.H.S: $\boldsymbol{A'\cap B'}$

First we find A':

$A'=U\setminus A$

$=\{1,2,3,4\ \dots 20\}\setminus\{2, 3, 5, 7, 11, 12, 13, 17\}$

$=\{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\}$

And Also

$B'=B\setminus A$

$=\{1,2,3,4\ \dots 20\}\setminus\{1, 4, 6, 8, 10, 14, 17, 18\}$

$=\{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

Now

$A'\cap B'=\{1,4,6,8,9,10,14,15,16,18,19,20\}\cap$

$\{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

$A'\cap B'=\{9, 15, 16, 19, 20\}$

Hence

$(A\cup B)'=A'\cap B'$

Proved

De-Morgan's Law:

$U=\{x\mid x\in N\land 1\le x\le 20\}$

$\text{U}=\{1,2,3,4\ \dots 20\}$

$A=\{2, 3, 5, 7, 11, 12, 13, 17\}$

$B=\{1, 4, 6, 8, 10, 14, 17, 18\}$

To Prove:

$(A\cap B)'=A'\cup B'$

L.H.S: $\boldsymbol{(A\cap B)'}$

First we find $A\cap B$:

$A\cap B=\{2,3,5,7,11,12,13,17\}\cap$

$\{1,4,6,8,10,14,17,18\}$

$A\cap B=\{17\}$

$(A\cap B)'=U\setminus(A\cap B)$

$(A\cap B)'=\{1,2,3,4\ \dots 20\}\setminus\{17\}$

$(A\cap B)'=\{1,2,3,4\ \dots, 15,16,18,19,20\}$

R.H.S: $\boldsymbol{A'\cup B'}$

First we find A':

$A'=U\setminus A$

$=\{1,2,3,4\ \dots 20\}\setminus\{2, 3, 5, 7, 11, 12, 13, 17\}$

$=\{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\}$

Ex # 5.2

And Also

$B' = B \setminus A$

$= \{1, 2, 3, 4\ \dots 20\} \setminus \{1, 4, 6, 8, 10, 14, 17, 18\}$

$= \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

Now

$A' \cup B' = \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\} \cup \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

$A' \cup B' = \{1, 2, 3, 4 \dots, 15, 16, 18, 19, 20\}$

Hence

$(A \cap B)' = A' \cup B'$

Proved

(ii) $\boldsymbol{U = \{1, 2, 3, \dots \dots 10\}, A = \{2, 4, 6, 8, 10\}}$

$\boldsymbol{B = \{1, 3, 5, 7, 9\}}$

<u>Solution</u>:

<u>De-Morgan's Law</u>:

$U = \{1, 2, 3, \dots \dots 10\}, A = \{2, 4, 6, 8, 10\}$

$B = \{1, 3, 5, 7, 9\}$

To Prove:

$$(A \cup B)' = A' \cap B'$$

L.H.S: $(\boldsymbol{A} \cup \boldsymbol{B})'$

First we find $A \cup B$:

$A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9\}$

$A \cup B = \{1, 2, 3, \dots \dots 10\}$

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{1, 2, 3, \dots \dots 10\} \setminus \{1, 2, 3, \dots \dots 10\}$

$(A \cup B)' = \{\ \}$

R.H.S: $\boldsymbol{A' \cap B'}$

First we find A':

$A' = U \setminus A$

$= \{1, 2, 3, \dots \dots 10\} \setminus \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

And Also

$B' = B \setminus A$

$= \{1, 2, 3, \dots \dots 10\} \setminus \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

Now

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\}$

$A' \cap B' = \{\ \}$

Hence

$(A \cup B)' = A' \cap B'$

Proved

Ex # 5.2

$\boldsymbol{U = \{1, 2, 3, \dots \dots 10\}, A = \{2, 4, 6, 8, 10\}}$

$\boldsymbol{B = \{1, 3, 5, 7, 9\}}$

<u>Solution</u>:

<u>De-Morgan's Law</u>:

$U = \{1, 2, 3, \dots \dots 10\}, A = \{2, 4, 6, 8, 10\}$

$B = \{1, 3, 5, 7, 9\}$

To Prove:

$(A \cap B)' = A' \cup B'$

L.H.S: $(\boldsymbol{A} \cap \boldsymbol{B})'$

First we find $A \cap B$:

$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$A \cap B = \{\ \}$

$(A \cap B)' = U \setminus (A \cap B)$

$(A \cap B)' = \{1, 2, 3, \dots \dots 10\} \setminus \{\ \}$

$(A \cap B)' = \{1, 2, 3, \dots \dots 10\}$

R.H.S: $\boldsymbol{A' \cup B'}$

First we find A':

$A' = U \setminus A$

$= \{1, 2, 3, \dots \dots 10\} \setminus \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

And Also

$B' = B \setminus A$

$= \{1, 2, 3, \dots \dots 10\} \setminus \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

Now

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$

$A' \cup B' = \{1, 2, 3, \dots \dots 10\}$

Hence

$(A \cap B)' = A' \cup B'$

Proved

Ex # 5.3

Overlapping Set

Two sets are overlapping set if

At least one element is common in both sets

None of them is a subset of each other

Venn Diagram

A Venn diagram is a visual way to show the relationships among or between sets that share something in common.

Representation

The Venn diagram consists of two or more overlapping circles, with each circle representing a set of elements and universal set is represented by a rectangle.

Note:

If two circles overlap, the members in the overlap belong to both sets; if three circles overlap, the members in the overlap belong to all three sets.

Example # 14

A ={1, 2, 3, 4} ,B ={3, 4, 5, 6} and C = {3, 4, 7, 8}
Then verify the following with the help of Venn Diagrams

(i) $A \cup B = B \cup A$

Solution:

A ={1, 2, 3, 4} ,B ={3, 4, 5, 6}

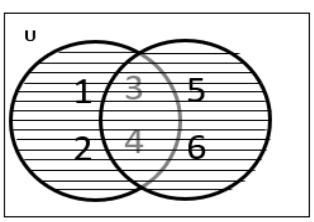
To Prove:

$A \cup B = B \cup A$

L.H.S: $A \cup B$

$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

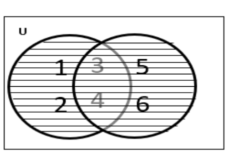
$A \cup B = \{1, 2, 3, 4, 5, 6\}$



R.H.S: $B \cup A$

$B \cup A = \{3, 4, 5, 6\} \cup \{1, 2, 3, 4\}$

$B \cup A = \{1, 2, 3, 4, 5, 6\}$



Hence$A \cup B = B \cup A$

Ex # 5.3

(ii) $A \cap B = B \cap A$

Solution:

A ={1, 2, 3, 4} ,B ={3, 4, 5, 6}

To Prove:

Commutative Property of Intersection:

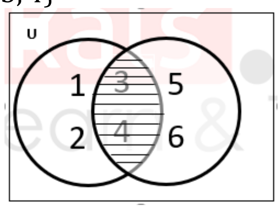
Now

$A \cap B = B \cap A$

L.H.S: $A \cap B$

$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$

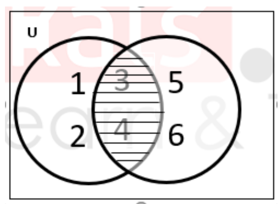
$A \cap B = \{3, 4\}$



R.H.S: $B \cap A$

$B \cap A = \{3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

$B \cap A = \{3, 4\}$



Hence

$A \cap B = B \cap A$

Proved

(iii) $A \cup (B \cup C) = (A \cup B) \cup C$

Solution:

A ={1, 2, 3, 4} ,B ={3, 4, 5, 6} and

C = {3, 4, 7, 8}

To Prove:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $A \cup (B \cup C)$

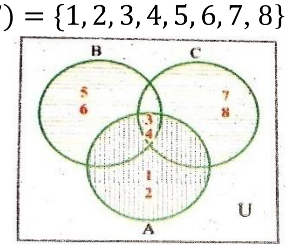
$B \cup C = \{3, 4, 5, 6\} \cup \{3, 4, 7, 8\}$

$B \cup C = \{3, 4, 5, 6, 7, 8\}$

Now

$A \cup (B \cup C) = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8\}$



Ex # 5.3

R.H.S: $(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C}$

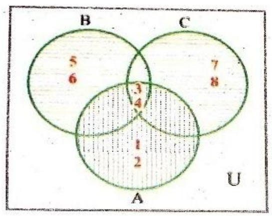
$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Now

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 7, 8\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$



Hence $A \cup (B \cup C) = (A \cup B) \cup C$

Proved

(iv) $\boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}$

Solution:

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and

$C = \{3, 4, 7, 8\}$

To Prove:

$$A \cap (B \cap C) = (A \cap B) \cap C$$

L.H.S: $\boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$

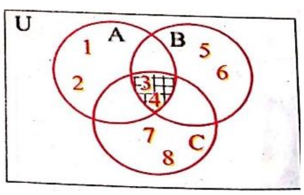
$B \cap C = \{3, 4, 5, 6\} \cap \{3, 4, 7, 8\}$

$B \cap C = \{3, 4\}$

Now

$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap \{3, 4\}$

$A \cap (B \cap C) = \{3, 4\}$



R.H.S: $(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}$

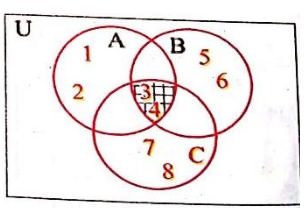
$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

Now

$(A \cap B) \cap C = \{3, 4\} \cap \{3, 4, 7, 8\}$

$(A \cap B) \cap C = \{3, 4\}$



Hence

$A \cap (B \cap C) = (A \cap B) \cap C$

Proved

Ex # 5.3

(v) $\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

Solution:

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and

$C = \{3, 4, 7, 8\}$

To prove:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

L.H.S: $\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C})$

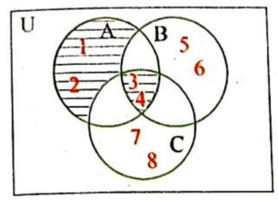
$B \cap C = \{3, 4, 5, 6\} \cap \{3, 4, 7, 8\}$

$B \cap C = \{3, 4\}$

Now

$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{3, 4\}$

$A \cup (B \cap C) = \{1, 2, 3, 4\}$



R.H.S: $(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

First we find $A \cup B$:

$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

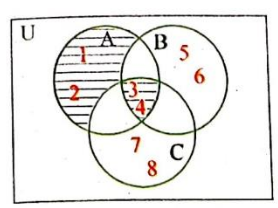
Now

$A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 7, 8\}$

$A \cup C = \{1, 2, 3, 4, 7, 8\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 7, 8\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\}$



Hence

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

Chapter # 5

Ex # 5.3

(vi) $\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

<u>Solution</u>:

A ={1, 2, 3, 4} ,B ={3, 4, 5, 6} and

C = {3, 4, 7, 8}

To Prove:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $\boldsymbol{A \cap (B \cup C)}$

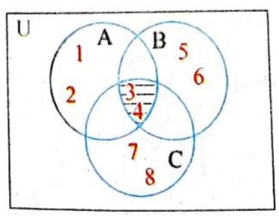
$B \cup C = \{3, 4, 5, 6\} \cup \{3, 4, 7, 8\}$

$B \cup C = \{3, 4, 5, 6, 7, 8\}$

Now

$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}$

$A \cap (B \cup C) = \{3, 4\}$



R.H.S: $(\boldsymbol{A \cup B}) \cap (\boldsymbol{A \cup C})$

First we find $A \cap B$:

$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

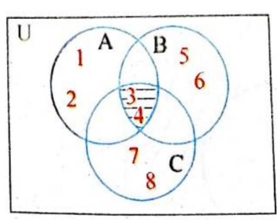
Now we find $A \cap C$:

$A \cap C = \{1, 2, 3, 4\} \cap \{3, 4, 7, 8\}$

$A \cap C = \{3, 4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \{3, 4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4\}$



Hence

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved

<u>Example # 15</u>

$\boldsymbol{U}$ **={1, 2, 3, 4, 5, 6, 7} ,**$\boldsymbol{A}$ **={2, 5, 6} and**

$\boldsymbol{B}$ **= {1, 2, 3}**

(i) $\boldsymbol{A \cup B = B \cup A}$

<u>Solution</u>:

A ={2, 5, 6} and B = {1, 2, 3}

To Find:

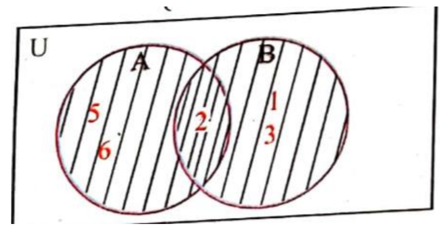
$A \cup B$

Now

Ex # 5.3

$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 5, 6$



(ii) $\boldsymbol{A \cap B}$

<u>Solution</u>:

$A = \{2, 5, 6\}$, $B = \{1, 2, 3\}$

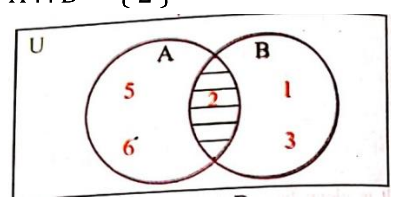
To Find:

$A \cap B$

Now

$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$

$A \cap B = \{2\}$



(iii) $\boldsymbol{A'}$

<u>Solution</u>:

U ={1, 2, 3, 4, 5, 6, 7} ,A ={2, 5, 6}

To Find:

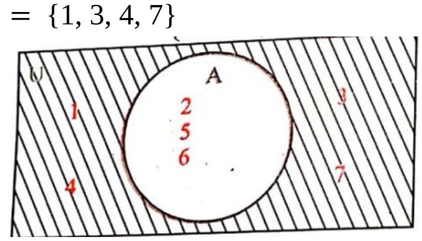
A'

Now

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$

$= \{1, 3, 4, 7\}$



(iv) $\boldsymbol{B'}$

<u>Solution</u>:

U ={1, 2, 3, 4, 5, 6, 7} and B = {1, 2, 3}

To Find:

B'

Now

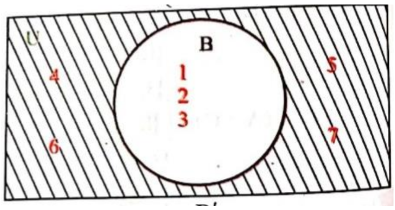
$B' = U \setminus B$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

Chapter # 5

Ex # 5.3



(v) $(\boldsymbol{A} \cup \boldsymbol{B})'$

Solution:

U ={1, 2, 3, 4, 5, 6, 7} ,A ={2, 5, 6} and

$B = \{1, 2, 3\}$

To Find:

$(A \cup B)'$

First we find $A \cup B$:

$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$

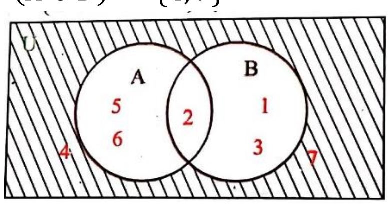
$A \cup B = \{1, 2, 3, 5, 6\}$

Now

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 5, 6\}$

$(A \cup B)' = \{4, 7\}$



(vi) $\boldsymbol{A}' \cap \boldsymbol{B}'$

Solution:

U ={1, 2, 3, 4, 5, 6, 7} ,A ={2, 5, 6} and

$B = \{1, 2, 3\}$

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$

$= \{1, 3, 4, 7\}$

Now find B'

$B' = U \setminus B$

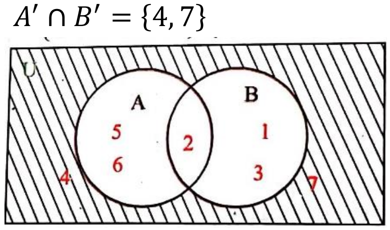
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

Now

$A' \cap B' = \{1, 3, 4, 7\} \cap \{4, 5, 6, 7\}$

$A' \cap B' = \{4, 7\}$



Ex # 5.3

(vii) $(\boldsymbol{A} \cap \boldsymbol{B})'$

Solution:

U ={1, 2, 3, 4, 5, 6, 7} ,A ={2, 5, 6} and

$B = \{1, 2, 3\}$

To Find:

$(A \cap B)'$

First we find $A \cap B$:

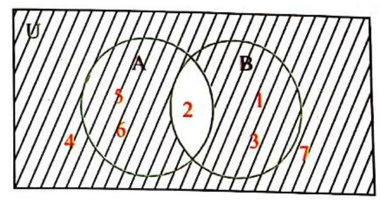
$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$

$A \cap B = \{ 2 \}$

$(A \cap B)' = U \setminus (A \cap B)$

$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7\}$



(viii) $\boldsymbol{A}' \cup \boldsymbol{B}'$

Solution:

U ={1, 2, 3, 4, 5, 6, 7} ,A ={2, 5, 6} and B = {1, 2, 3}

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$

$= \{1, 3, 4, 7\}$

Now find B'

$B' = U \setminus B$

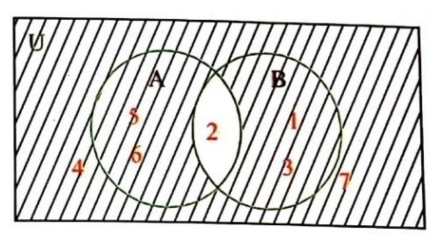
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

Now

$A' \cup B' = \{1, 3, 4, 7\} \cup \{4, 5, 6, 7\}$

$A' \cup B' = \{1, 3, 4, 5, 6, 7\}$



Ex # 5.3

Page # 106

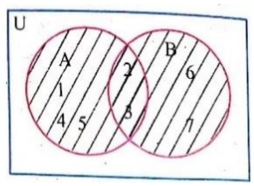
Q1: **If A = $\{1,2,3,4,5\}$ and B = $\{2,3,6,7\}$ then draw Venn diagrams for the following**

(i) $\boldsymbol{A \cup B}$

Solution:

$A \cup B = \{1,2,3,4,5\} \cup \{2,3,6,7\}$

$A \cup B = \{1,2,3,4,5,6,7\}$



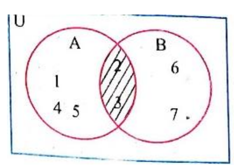
(ii) $\boldsymbol{A \cap B}$

Solution:

$A \cap B$

$A \cap B = \{1,2,3,4,5\} \cap \{2,3,6,7\}$

$A \cap B = \{2,3\}$



Q2: **If A ={1, 2, 3, 4, 5, 6} ,B ={3, 4, 5, 6, 7, 8} and C ={5, 6, 9, 10} then verify with the help of Venn diagrams.**

(i) $\boldsymbol{A \cup (B \cup C) = (A \cup B) \cup C}$

Solution:

A ={1, 2, 3, 4, 5, 6} ,B ={3, 4, 5, 6, 7, 8} and C ={5, 6, 9, 10}

To Prove:

Associative Property of Union:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $\boldsymbol{A \cup (B \cup C)}$

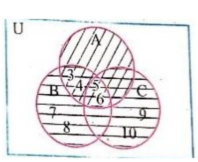
$B \cup C = \{3,4,5,6,7,8\} \cup \{5, 6, 9, 10\}$

$B \cup C = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Now

$A \cup (B \cup C) = \{1,2,3,4,5,6\} \cup \{3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = \{1,2,3,4, \dots \dots 10\}$



Ex # 5.3

R.H.S: $\boldsymbol{(A \cup B) \cup C}$

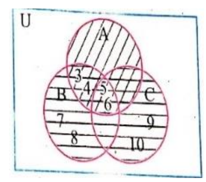
$A \cup B ==\{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 5, 6, 7, 8\}$

$A \cup B =\{1, 2, 3, 4, 5, 6, 7, 8\}$

Now

$(A \cup B) \cup C = \{1,2,3,4,5,6,7,8\} \cup \{5,6,9,10\}$

$(A \cup B) \cup C = \{1,2,3,4, \dots \dots 10\}$



Hence $A \cup (B \cup C) = (A \cup B) \cup C$

Proved

(ii) $\boldsymbol{A \cap (B \cap C) = (A \cap B) \cap C}$

Solution:

A ={1, 2, 3, 4, 5, 6} ,B ={3, 4, 5, 6, 7, 8} and C ={5, 6, 9, 10}

To Prove:

Associative Property of Intersection:

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S: $\boldsymbol{A \cap (B \cap C)}$

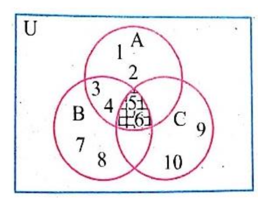
$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,9,10\}$

$B \cap C = \{5,6\}$

Now

$A \cap (B \cap C) = \{1,2,3,4,5,6\} \cap \{5,6\}$

$A \cap (B \cap C) = \{5,6\}$



R.H.S: $\boldsymbol{(A \cap B) \cap C}$

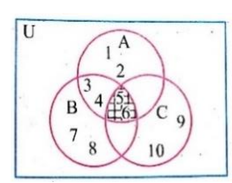
$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{3,4,5,6,7,8\}$

$A \cap B = \{3, 4, 5, 6\}$

Now

$(A \cap B) \cap C = \{3,4,5,6\} \cup \{5,6,9,10\}$

$(A \cap B) \cap C = \{5,6\}$



Ex # 5.3

(iii) $\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

Solution:

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{5, 6, 9, 10\}$

To Prove:

Distributive Property of Union over Intersection:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $\boldsymbol{A \cup (B \cap C)}$

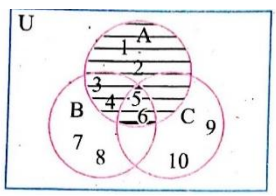
$B \cap C = \{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 9, 10\}$

$B \cap C = \{5, 6\}$

Now

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$



R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cup B$:

$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 5, 6, 7, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

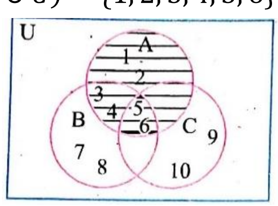
Now

$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 9, 10\}$

$A \cup C = \{1, 2, 3, 4, 5, 6, 9, 10\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{1, 2, 3, 4, 5, 6, 9, 10\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\}$



Hence

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

Ex # 5.3

(iv) $\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Solution:

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{5, 6, 9, 10\}$

To Prove:

Distributive Property of Intersection over Union:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $\boldsymbol{A \cap (B \cup C)}$

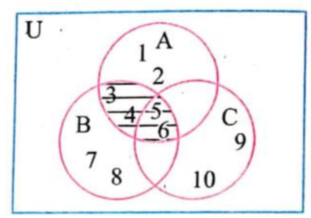
$B \cup C = \{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 9, 10\}$

$B \cup C = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Now

$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cap (B \cup C) = \{3, 4, 5, 6\}$



R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cap B$:

$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8\}$

$A \cap B = \{3, 4, 5, 6\}$

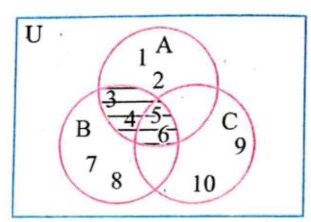
Now we find $A \cap C$:

$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{5, 6, 9, 10\}$

$A \cap C = \{5, 6\}$

$(A \cap B) \cup (A \cap C) = \{3, 4, 5, 6\} \cup \{5, 6\}$

$(A \cap B) \cup (A \cap C) = \{3, 4, 5, 6\}$



Hence

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved

Ex # 5.3

Q1: **If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$**
Draw Venn diagrams for the following.

(i) A'

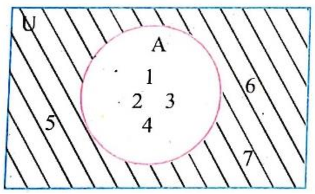
Solution:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4\}$

$= \{5, 6, 7\}$



(ii) B'

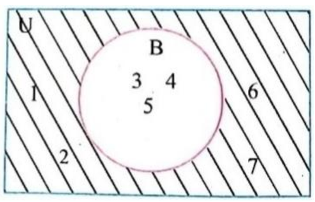
Solution:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{3, 4, 5\}$

$B' = U \setminus B$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 3, 5, 7, 9\}$

$= \{1, 2, 6, 7\}$



(iii) $A' \cup B'$

Solution:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4\}$

$= \{5, 6, 7\}$

Now find B'

$B' = U \setminus B$

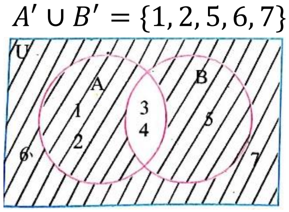
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 3, 5, 7, 9\}$

$= \{1, 2, 6, 7\}$

Now

$A' \cup B' = \{5, 6, 7\} \cup \{1, 2, 6, 7\}$

$A' \cup B' = \{1, 2, 5, 6, 7\}$



Ex # 5.3

(iv) $A' \cap B'$

Solution:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4\}$

$= \{5, 6, 7\}$

Now find B'

$B' = U \setminus B$

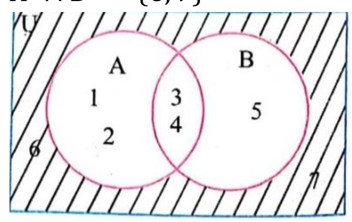
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 3, 5, 7, 9\}$

$= \{1, 2, 6, 7\}$

Now

$A' \cap B' = \{5, 6, 7\} \cap \{1, 2, 6, 7\}$

$A' \cap B' = \{6, 7\}$



(v) $(A \cup B)' = A' \cap B'$

Solution:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$

To Prove:

De-Morgan's Law:

$(A \cup B)' = A' \cap B'$

L.H.S: $(A \cup B)'$

First we find $A \cup B$:

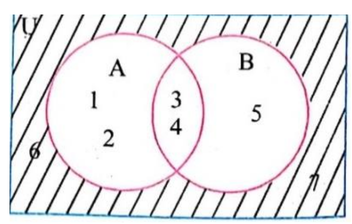
$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4, 5\}$

$(A \cup B)' = \{6, 7\}$



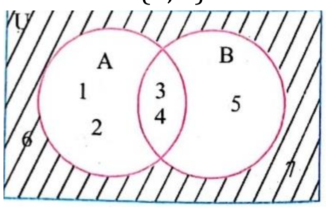
R.H.S: $A' \cap B'$

First we find A'

$A' = U \setminus A$

Ex # 5.3

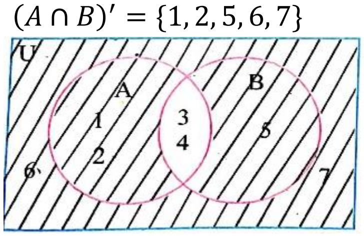
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4\}$
$= \ \{5, 6, 7\}$
Now find B'
$B' = U \setminus B$
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 3, 5, 7, 9\}$
$= \ \{1, 2, 6, 7\}$
Now
$A' \cap B' = \ \{5, 6, 7\} \cap \{1, 2, 6, 7\}$
$A' \cap B' = \{6, 7\}$



Hence
$(A \cup B)' = A' \cap B'$
Proved

(vi) $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$

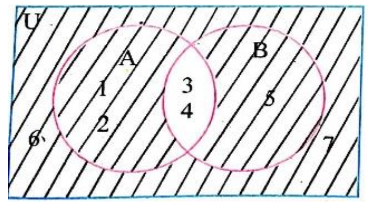
Solution:
U ={1, 2, 3, 4, 5, 6, 7} ,A ={1, 2, 3, 4} and
B ={3, 4, 5}
To Prove:
De-Morgan's Law:
$(A \cap B)' = A' \cup B'$
L.H.S: $(\boldsymbol{A} \cap \boldsymbol{B})'$
First we find $A \cap B$:
$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5\}$
$A \cap B = \{3, 4\}$
$(A \cap B)' = U \setminus \ (A \cap B)$
$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{3, 4\}$
$(A \cap B)' = \{1, 2, 5, 6, 7\}$



R.H.S: $\boldsymbol{A}' \cup \boldsymbol{B}'$
First we find A'
$A' = U \setminus A$
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 4\}$
$= \ \{5, 6, 7\}$
Now find B'
$B' = U \setminus B$
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 3, 5, 7, 9\}$
$= \ \{1, 2, 6, 7\}$

Ex # 5.3

Now
$A' \cup B' = \ \{5, 6, 7\} \cup \{1, 2, 6, 7\}$
$A' \cup B' = \{1, 2, 5, 6, 7\}$

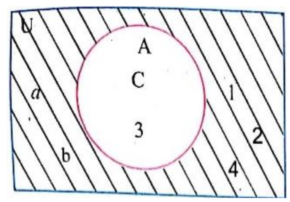


Hence
$(A \cap B)' = A' \cup B'$
Proved

Q4: **If $\boldsymbol{U}$ ={a, b, c, 1, 2, 3, 4} ,$\boldsymbol{A}$ ={c, 3} and $\boldsymbol{B}$ ={a, 3, 4} then draw Venn diagrams**

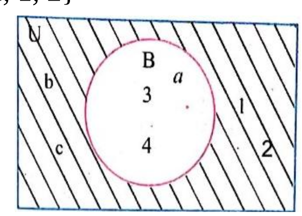
(i) $\boldsymbol{A}'$

Solution:
U ={a, b, c, 1, 2, 3, 4} ,A ={c, 3}
$A' = U \setminus A$
$= \{a, b, c, 1, 2, 3, 4\} \setminus \{c, 3\}$
$= \ \{a, b, 1, 2, 4\}$



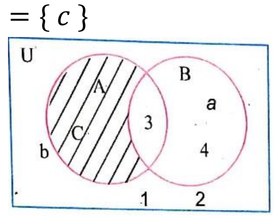
(ii) $\boldsymbol{B}'$

Solution:
U ={a, b, c, 1, 2, 3, 4} ,B ={a, 3, 4}
$B' = U \setminus B$
$= \{a, b, c, 1, 2, 3, 4\} \setminus \{a, 3, 4\}$
$= \ \{b, c, 1, 2\}$



(iii) $\boldsymbol{A} \setminus \boldsymbol{B}$

Solution:
A ={c, 3} ,B ={a, 3, 4}
$A \setminus B = \{c, 3\} \setminus \{a, 3, 4\}$
$= \{\ c\ \}$



Chapter # 5

Ex # 5.3

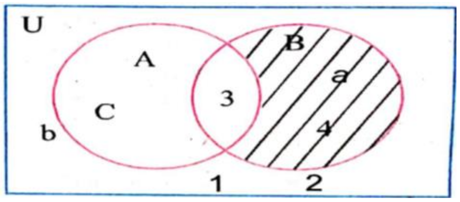
(iv) $\boldsymbol{B \setminus A}$

Solution:

$A = \{c, 3\}$, $B = \{a, 3, 4\}$

$B \setminus A = \{a, 3, 4\} \setminus \{c, 3\}$

$= \{a, 4\}$



Q5: If $\boldsymbol{U} = \{$a, b, c, d, e, f, g$\}$, $\boldsymbol{A} = \{$a, b, c$\}$ and $\boldsymbol{B} = \{$c, d, e$\}$ then verify De Morgan's laws with the help of Venn diagrams.

De-Morgan's Law:

Solution:

$U = \{a, b, c, d, e, f, g\}$, $A = \{a, b, c\}$ and

$B = \{c, d, e\}$

To Prove:

$(A \cup B)' = A' \cap B'$

L.H.S: $(\boldsymbol{A \cup B})'$

First we find $A \cup B$:

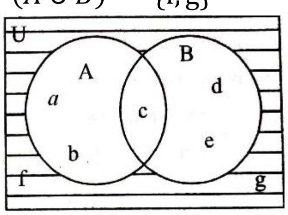
$A \cup B = \{a, b, c\} \cup \{c, d, e\}$

$A \cup B = \{a, b, c, d, e\}$

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{a, b, c, d, e, f, g\} \setminus \{a, b, c, d, e\}$

$(A \cup B)' = \{f, g\}$



R.H.S: $\boldsymbol{A' \cap B'}$

First we find A':

$A' = U \setminus A$

$= \{a, b, c, d, e, f, g\} \setminus \{a, b, c\}$

$= \{d, e, f, g\}$

And Also

$B' = U \setminus B$

$= \{a, b, c, d, e, f, g\} \setminus \{c, d, e\}$

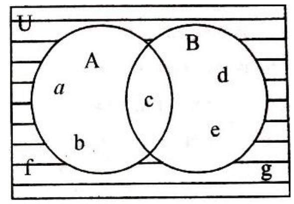
$= \{a, b, f, g\}$

Now

$A' \cap B' = \{d, e, f, g\} \cap \{a, b, f, g\}$

$A' \cap B' = \{f, g\}$

Ex # 5.3



Hence

$(A \cup B)' = A' \cap B'$

Proved

De-Morgan's Law:

Solution:

$U = \{a, b, c, d, e, f, g\}$, $A = \{a, b, c\}$ and

$B = \{c, d, e\}$

To Prove:

$(A \cap B)' = A' \cup B'$

L.H.S: $(\boldsymbol{A \cap B})'$

First we find $A \cap B$:

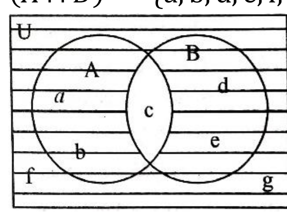
$A \cap B = \{a, b, c\} \cap \{c, d, e\}$

$A \cap B = \{c\}$

$(A \cap B)' = U \setminus (A \cap B)$

$(A \cap B)' = \{a, b, c, d, e, f, g\} \setminus \{c\}$

$(A \cap B)' = \{a, b, d, e, f, g\}$



R.H.S: $\boldsymbol{A' \cup B'}$

First we find A':

$A' = U \setminus A$

$= \{a, b, c, d, e, f, g\} \setminus \{a, b, c\}$

$= \{d, e, f, g\}$

And Also

$B' = U \setminus B$

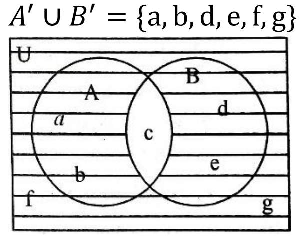
$= \{a, b, c, d, e, f, g\} \setminus \{c, d, e\}$

$= \{a, b, f, g\}$

Now

$A' \cup B' = \{d, e, f, g\} \cup \{a, b, f, g\}$

$A' \cup B' = \{a, b, d, e, f, g\}$



Hence $(A \cap B)' = A' \cup B'$ Proved

Chapter # 5

Ex # 5.4

Ordered Pairs and Cartesian Product

Ordered Pairs

Any two numbers x and y written in the form of (x, y) is called ordered pair. In (x, y), x is the first element and y is the second element.

Note

In (x, y), the order is of numbers is important.
(2, 3) is different from (3, 2)
$(x, y) \neq (y, x)$ unless $x = y$

The ordered pair of $(a, b) = (c, d)$, if and only if, $a = c$ and $b = d$

Example # 16

Find x and y given $(2x,\ x + y) = (6, 2)$

Solution:

$(2x, x + y) = (6,2)$

Two ordered pairs are equal, if and only if the corresponding elements are equal.

Hence

$2x = 6 \ldots\ldots equ$ (i)

$x + y = 2 \ldots\ldots equ$ (i)

Now

$2x = 6$

$x = \frac{6}{2}$

$x = 3$

Put $x = 3$ in equ (ii)

$3 + y = 2$

$y = 2 - 3$

$y = -1$

Cartesian Product

The Cartesian product of A and B is the set of all ordered pairs in which first element from A and second element from B.

It is denoted by $A \times B$ and read as A cross B

Symbolically

$$A \times B = \{(a, b) | a \in A \ and\ b \in B\}$$

Note

$A \times B \neq B \times A$ for non-empty and unequal sets A and B

$A \times \emptyset = \emptyset \times A = \emptyset$

Binary Relation

If A and B are any two non-empty sets, then a binary relation R from set A to set B is a subset of the Cartesian product $A \times B$. In other words$R \subseteq A \times B$

Ex # 5.4

When $(x$, y) ∈ R, we say x is related to y by R, written $x\ R\ y$

Otherwise, if (a, b) ∉ R, we write a R̸ b.

Example # 17

$A =$ {a, b} and $B =$ {1, 2} then find $A \times B$ and also write all possible binary relation

Solution:

$A =$ {a, b} and $B =$ {1, 2}

Now

$A \times B =$ {a, b} × {1,2}

= {(a, 1), (a, 2), (b, 1), (b, 2)}

As number of elements in $A \times B = 2 \times 2 = 4$

Thus number of all possible subset / binary relation of $A \times B = 2^4 = 16$

Now

$R_1 = \varphi$

$R_2 = \{(a, 1)\}$

$R_3 = \{(a, 2)\}$

$R_4 = \{(b, 1)\}$

$R_5 = \{(b, 2)\}$

$R_6 = \{(a, 1), (a, 2)\}$

$R_7 = \{(a, 1)(b, 1)\}$

$R_8 = \{(a, 1), (b, 2)\}$

$R_9 = \{(a, 2), (b, 1)\}$

$R_{10} = \{(a, 2), (b, 2)\}$

$R_{11} = \{(b, 1), (b, 2)\}$

$R_{12} = \{(a, 1), (a, 2), (b, 1)\}$

$R_{13} = \{(a, 1), (a, 2), (b, 2)\}$

$R_{14} = \{(a, 1), (b, 1), (b, 2)\}$

$R_{15} = \{(a, 2), (b, 1), (b, 2)\}$

$R_{16} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

Similarly, total number of binary relation in $B \times A = 2^4 = 16$

Example # 18

$A =$ {1, 2} and $B =$ {1, 2, 3} then find $A \times B$ and write any five relation from A to B.

Solution:

$A =$ {1, 2} and $B =$ {1, 2, 3}

$A \times B =$ {1, 2} × {1, 2, 3}

= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)}

Now

Five binary relation from A to B are

$R_1 = \varphi$

Ex # 5.4

$R_2 = \{(1,1),(1,2)\}$

$R_3 = \{(1,2),(2,1)\}$

$R_4 = \{(1,1)\}$

$R_5 = \{\ (2,1),(2,2),(2,3)\}$

Domain of a Binary Relation

The set of all first elements of the ordered pairs in binary relation is called domain of a binary relation. Domain of a relation is denoted by Dom(R)

Symbolically

$$\text{Dom (R)} = \{a \in A|(a\,,b) \in R\}$$

Range of a Binary Relation

The set of all second elements of the ordered pairs in binary relation is called range of a binary relation. Range of a relation is denoted by Ran(R)

Symbolically

$$\text{Range (R)} = \{b \in A|(a\,,b) \in R\}$$

Example # 19

$A = \{1, 2\}$ and $B = \{1, 2, 3\}$. Define a binary relation R from A to B as R = $\{(a\,,b) \in A \times B|a < b\}$

Find the ordered pairs in R

Find the Domain and Range of R.

Is 1R3, 2R2?

Solution:

$A = \{1, 2\}$ and $B = \{1, 2, 3\}$

First we find ordered pairs

$A \times B = \{1,2\} \times \{1, 2, 3\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

As $R = \{(a\,,b) \in A \times B|a < b\}$

In tabular form

$R = \{\ (1, 2), (1, 3), (2, 3)\ \}$

Now

Dom (R) $= \{1, 2\}$ and Range (R) $= \{2, 3\}$

As 1R3 means $(1, 3) \in R$ so it is true

And 2R2 means $(2, 2) \notin$ R so 2 is not related with 3

Ex # 5.4

Page # 109

Q1: If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ then

(i) Write three binary relations from A to B.

Solution:

$A = \{1, 2, 3\}$, $B = \{4, 5\}$

$A \times B = \{1, 2, 3\} \times \{4, 5\}$

$= \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

Now

Three binary relation from A to B are

$R_1 = \{(1, 4), (1, 5)\}$

$R_2 = \{\ (2, 4), (2, 5)\ \}$

$R_3 = \{(3, 4), (3, 5)\}$

(ii) Write four binary relations from B to A.

Solution:

$A = \{1, 2, 3\}$, $B = \{4, 5\}$

$B \times A = \{4, 5\} \times \{1, 2, 3\}$

$= \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$

Now

Four binary relation from B to A are

$R_1 = \{(4, 1)\}$

$R_2 = \{(4, 1), (4, 2)\}$

$R_3 = \{(4, 1), (4, 2), (4, 3)\}$

$R_4 = \{(4, 1), (4, 2), (4, 3), (5, 1)\}$

(iii) Write four binary relations on A.

Solution:

$A = \{1, 2, 3\}$

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Now

Four binary relation in A are

$R_1 = \{(1, 2), (1, 3)\}$

$R_2 = \{(1, 1), (1, 2), (1, 3)\}$

$R_3 = \{(2, 1), (2, 2)\}$

$R_4 = \{(1, 1)\ \}$

(iv) Write two binary relations on B.

Solution:

$B = \{4, 5\}$

$B \times B = \{4, 5\} \times \{4, 5\}$

$= \{(4, 4), (4, 5), (5, 4), (5, 5)\}$

Now

Two binary relation in B are

$R_1 = \{(4, 4)\}$

$R_2 = \{(4, 4), (4, 5)\ \}$

Chapter # 5

Ex # 5.4

Q2: If A ={1, 2, 3, 4} ,B ={1, 3, 5} and $R = \{(x, y) | y < x\}$ is a binary relation from A to B, then write it in tabular form.

<u>Solution</u>:

A ={1, 2, 3, 4} ,B ={1, 3, 5}

$R = \{(x, y) | y < x\}$

Now

$A \times B = \{1, 2, 3, 4\} \times \{1, 3, 5\}$

$A \times B$ = {(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)}

As the condition for binary relation is:

$y < x$

So Binary relation in Tabular form

R = {(2, 1), (3, 1), (4, 1), (4, 3)}

Q3: Domain of binary relation $R = \{(x, y) | y = 2x\}$ is the set {0, 4, 8}, find Range of R.

<u>Solution</u>:

Domain of R = {0, 4, 8}

Binary Relation $R = \{(x, y) | y = 2x\}$

To find:

Range of R = ?

As the condition is given:

$y = 2x$ … … equ(i)

As Dom (R) = x = {0, 4, 8}

Now

Put $x = 0$ in equ(i)

$y = 2(0)$

$y = 0$

Put $x = 4$ in equ(i)

$y = 2(4)$

$y = 8$

Put $x = 8$ in equ(i)

$y = 2(8)$

$y = 16$

Thus Ran(R) = {0, 8, 16}

Q4: Domain of binary relation $R = \{(x, y) | y + 1 = 2x^2\}$ is set N. Find its range.

<u>Solution</u>:

Domain of R = $N = \{1, 2, 3, 4 \ ...\}$

Binary Relation $R = \{(x, y) | y + 1 = 2x^2\}$

Ex # 5.4

To find:

Range of R = ?

As the condition is given:

$y + 1 = 2x^2$

$y = 2x^2 - 1$ … … equ(i)

As Dom (R) = $x = \{1, 2, 3, 4 \ ...\}$

Now

Put $x = 1$ in equ(i)

$y = 2(1)^2 - 1$

$y = 2(1) - 1$

$y = 2 - 1$

$y = 1$

Put $x = 2$ in equ(i)

$y = 2(2)^2 - 1$

$y = 2(4) - 1$

$y = 8 - 1$

$y = 7$

Put $x = 3$ in equ(i)

$y = 2(3)^2 - 1$

$y = 2(9) - 1$

$y = 18 - 1$

$y = 17$

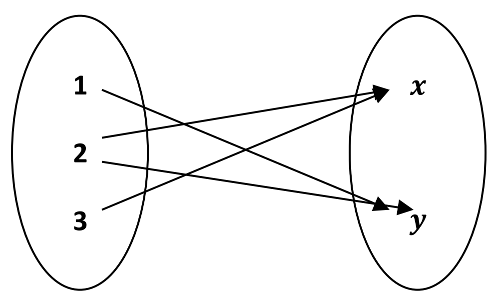
Thus Ran(R) = $\{1, 7, 17, … …\}$

Arrow Diagram of a Relation

Let A ={1, 2, 3} ,B ={x, y}

R = {(1, y), (2, x), (2, y), (3, x)} be a relation from A to B.

The arrow diagram of R is:



Ex # 5.5

Function

Let two non-empty sets, then a binary relation f is said to be a function if:

Dom $f = First\ Set$

There should be no repetition in domain in f

Explanation

Let A and B are two non-empty sets, then a binary relation f is said to be a function from A to B if:

Dom $f = Set\ A$

There should be no repetition in the first element of all ordered pairs in f

Symbolically, we write it as

$f: A \to B$ and say f is function from A to B.

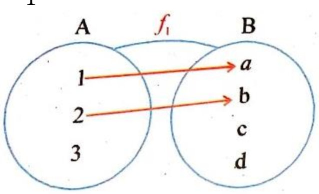
Example # 20

A ={1, 2, 3} and B ={a, b, c, d} then which of the following are functions?

(i) $\boldsymbol{f_1 = \{(1, a), (2, b)\}}$

Solution:

$f_1 = \{(1, a), (2, b)\}$



For function

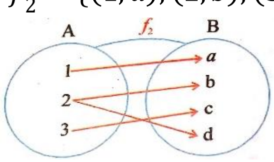
$Dom\ f_1 = \{1, 2\} \neq A$

Thus f_1 is not a function because it does not satisfy the first condition of function.

(ii) $\boldsymbol{f_2 = \{(1, a), (2, b), (3, c), (3, d)\}}$

Solution:

$f_2 = \{(1, a), (2, b), (3, c), (3, d)\}$



For function

$Dom\ f_2 = \{1, 2, 3\} = A$

As there is repetition in first element i.e. 3 is repeated.

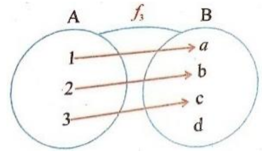
Thus f_2 is not a function because it does not satisfy the first condition of function.

Ex # 5.5

(iii) $\boldsymbol{f_3 = \{(1, a), (2, b), (3, c)\}}$

Solution:

$f_3 = \{(1, a), (2, b), (3, c)\}$



For function

$Dom\ f_3 = \{1, 2, 3\} = A$

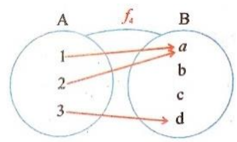
As there is no repetition in first element

Thus f_3 is a function because it satisfies both the conditions of function.

(iv) $\boldsymbol{f_4 = \{(1, a), (2, a), (3, d)\}}$

Solution:

$f_4 = \{(1, a), (2, b), (3, c)\}$



For function

$Dom\ f_4 = \{1, 2, 3\} = A$

As there is no repetition in first element

Thus f_4 is a function because it satisfies both the conditions of function.

Domain, Co-domain and Range of a function

Let f: $\longrightarrow B$ be a function, then the set A is called domain of "f"

The set B is co-domain of f and the set of second elements of all ordered pairs contained in f is called range of function.

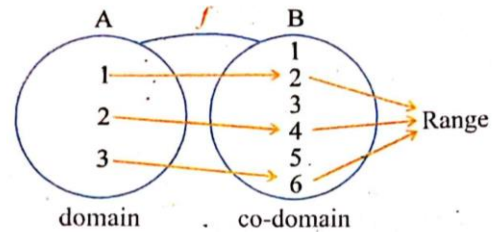
Note:

Range is always a subset of co-domain. i.e. Range $f \subseteq B$.

Example:

Let A ={1, 2, 3} and B ={1, 2, 3, 4, 5, 6}

$f : A \longrightarrow B$ as shown in the following figure.



Ex # 5.5

Kinds of a function

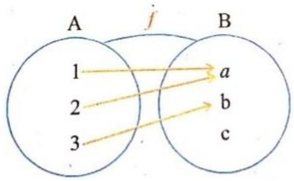
1. **Into function**

Let f be a function from A to B, then f is into function if $Range\ f \neq B$.

Example

Let A ={1, 2, 3} and B ={a, b, c} then a function f from A to B is defined by

$f = \{(1, a), (2, a), (3, b)\}$



As $Range\ f = \{a, b\} \neq B$

Thus f is into function

Written as: $f : A \xrightarrow{\text{into}} B$

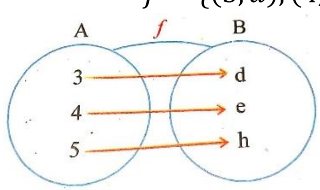
2. **Onto Function (Surjective Function)**

Let f be a function from A to B, then f is onto function if $Range\ f = B$.

Example

Let A ={3, 4, 5} and B ={d, e, h} then

$f = \{(3, d), (4, e), (5, h)\}$



As $Range\ f = \{d, e, h\} = B$

Thus f is onto function

Written as: $f : A \xrightarrow{\text{onto}} B$

3. **One-one Function**

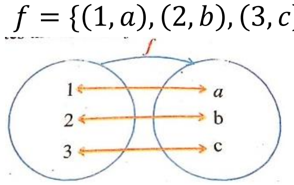
Let f be a function and if there is no repetition in the second elements (Range) then it is one-one function.

Written as: $f : A \xrightarrow{\text{one} - \text{one}} B$

Example

Let A ={1, 2, 3} and B ={a, b, c} then

$f = \{(1, a), (2, b), (3, c)\}$



As $Range\ f = \{a, b, c\}$

And also no repetition in range

Thus f is one-one function

Ex # 5.5

4. **Into and one-one function (Injective function)**

Let f be a function from A to B, then f is into and one-one function or injective function if .

$Range\ f \neq B$

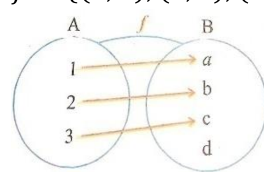
There is no repetition in the second element (Range)

Written as: $f : A \xrightarrow{\text{injective}} B$

Example

Let A ={1, 2, 3} and B ={a, b, c, d} then

$f = \{(1, a), (2, b), (3, c)\}$



As $Range\ f = \{a, b, c\} \neq B$

And also no repetition in range

Thus f is injective function

5. **One-one and onto function (Bijective Function)**

Let f be a function from A to B, then f is one-one and onto function or bijective function if

$Range\ f = B$.

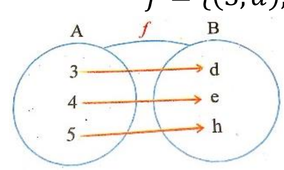
There is no repetition in the second element (Range)

Written as: $f : A \xrightarrow{\text{bijective}} B$

Example

Let A ={3, 4, 5} and B ={d, e, h} then

$f = \{(3, d), (4, e), (5, h)\}$



As $Range\ f = \{d, e, h\} = B$

And also no repetition in range

Thus f is bijective function

One-one correspondence

If A and B are two non-empty sets then each element of A is paired with one and only one element of B and each element of B is paired with one and only one element of A is called one-one correspondence.

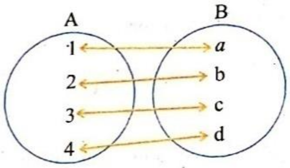
In other words, if both the sets have the same number of elements.

Ex # 5.5

Example

Let A ={1, 2, 3, 4} and B ={a, b, c, d} then one-one correspondence is given by

$f=\{(1,a),(2,b),(3,c),(4,d)\}$

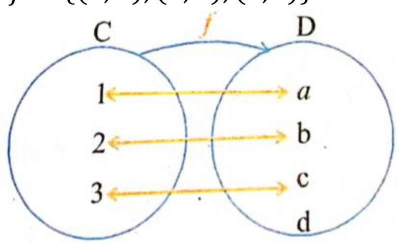


In one-one function every element of the set A is associated with one and only one element of set B. This means that Range f may not be equal to set B.

Example

Let C ={1, 2, 3} and D ={a, b, c, d} then one-one correspondence is given by

$f=\{(1,a),(2,b),(3,c)\}$



From figure, it is clear that there does not exist one – one correspondence between set C and D because $d\in D$ is unpaired.

Ex # 5.5

Page # 115

Q1: **A ={1, 2, 3, 4} ,B ={6, 7} and the following are the relations from A to B, then state whether these are functions are not?**
If these are functions then state which kind of functions are these?

(i) **R_1 = {(1, 6), (2, 7), (3, 6)}**

<u>Solution</u>:

R_1 = {(1, 6), (2, 7), (3, 6)}

For function, $Dom\ R_1=A$

$Dom\ R_1=\{1,2,3\}\neq A$

Thus R_1 is not a function because its $Dom\ R_1\neq A$.

Ex # 5.5

(ii) **R_2 = {(1, 6), (2, 6), (3, 7), (4, 7)}**

<u>Solution</u>:

R_2 = {(1, 6), (2, 6), (3, 7), (4, 7)}

For function, $Dom\ R_1=A$

$Dom\ R_1=\{1,2,3,4\}=A$

And there is no repetition in Domain.

Thus R_2 is a function from A to B

<u>Kind of function</u>

Now

$Range\ R_2=\{6,7\}=B$

Hence R_2 is Onto function.

(iii) **R_3 = {(1, 6), (2, 6), (3, 6), (4, 6)}**

<u>Solution</u>:

R_3 = {(1, 6), (2, 6), (3, 6), (4, 6)}

For function, $Dom\ R_1=A$

$Dom\ R_1=\{1,2,3,4\}=A$

And there is no repetition in Domain.

Thus R_2 is a function from A to B

<u>Kind of function</u>

Now

$Range\ R_3=\{6\}\neq B$

Hence R_3 is Into function.

Q2: **Which of the following relations on set {a, b, c, d} are functions? State the kind of functions as well.**

(i) **{(a, b), (c, d), (b, d), (d, b)}**

<u>Solution</u>:

{(a, b), (c, d), (b, d), (d, b)}

Let $R=\{(a,b),(c,d),(b,d),(d,b)\}$

For function, $Dom\ R=A$

$Dom\ R=\{a,b,c,d\}=A$

And there is no repetition in Domain.

Thus R is a function in A

<u>Kind of function</u>

Now

$Range\ R=\{b,d\}\neq B$

Hence R is Into function.

Chapter # 5

Ex # 5.5

(ii) {(b, a), (c, b), (a, b), (d, d)}

<u>Solution</u>:

{(b, a), (c, b), (a, b), (d, d)}

Let $R = \{(b, a), (c, b), (a, b), (d, d)\}$

For function, $Dom\ R = A$

$Dom\ R = \{a, b, c, d\} = A$

And there is no repetition in Domain.

Thus R is a function in A

<u>Kind of function</u>

Now

$Range\ R = \{a, b, d\} \neq B$

Hence R is Into function.

(iii) {(d, c), (c, b), (a, b), (d, d)}

<u>Solution</u>:

{(d, c), (c, b), (a, b), (d, d)}

Let $R = \{(\mathrm{d, c}), (\mathrm{c, b}), (\mathrm{a, b}), (\mathrm{d, d})\}$

For function, $Dom\ R = A$

$Dom\ R = \{a, c, d\} \neq A$

Thus R is not a function because its $Dom\ R \neq A$.

(iv) {(a, b), (b, c), (c, b), (d, a)}

<u>Solution</u>:

{(a, b), (b, c), (c, b), (d, a)}

Let $R = \{(\mathrm{a, b}), (\mathrm{b, c}), (\mathrm{c, b}), (\mathrm{d, a})\}$

For function, $Dom\ R = A$

$Dom\ R = \{a, b, c, d\} = A$

And there is no repetition in Domain.

Thus R is a function in A

<u>Kind of function</u>

Now

$Range\ R = \{a, b, c\} \neq B$

Hence R is Into function.

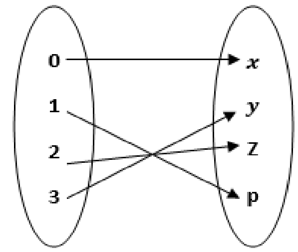
Q3: If A ={0, 1, 2, 3} ,B ={x, y, z, p} then state, whether the following relations shows that there exists one − onecorrespondence between the elements of sets A and B, if not, give reasons.

Ex # 5.5

(i) $\{(\mathbf{0}, \boldsymbol{x}), (\mathbf{2}, \boldsymbol{z}), (\mathbf{3}, \boldsymbol{y}), (\mathbf{1}, \boldsymbol{p})\}$

<u>Solution</u>:

$\{(0, x), (2, z), (3, y), (1, p)\}$

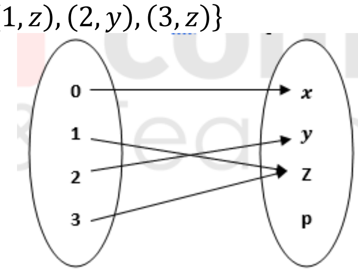


As each element of set A is paired with one and only element of set B. So, it is one – one correspondence.

(ii) $\{(\mathbf{0}, \boldsymbol{x}), (\mathbf{1}, \boldsymbol{z}), (\mathbf{2}, \boldsymbol{y}), (\mathbf{3}, \boldsymbol{z})\}$

<u>Solution</u>:

$\{(0, x), (1, z), (2, y), (3, z)\}$



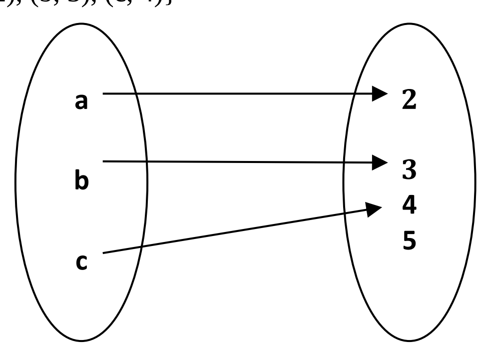
As each element of set A is not paired with each element of set B. Thus, it is not one – one correspondence.

Q4: If A ={a, b, c} ,B ={2, 3, 4, 5} then state, whether the following relations shows that there exists one − onecorrespondence between the elements of sets A and B, if not what kind of the relations they are?

(i) {(a, 2), (b, 3), (c, 4)}

<u>Solution</u>:

{(a, 2), (b, 3), (c, 4)}



As each element of set A is not paired with each element of set B. Thus, it is not one – one correspondence.

Ex # 5.5

Now

Let $f = \{(a, 2), (b, 3), (c, 4)\}$

For function, $Dom\ f = A$

$Dom\ f = \{a, b, c\} = A$

And there is no repetition in Domain.

Thus f is a function from A to B

Kind of function

Now

$Range\ R = \{2, 3, 4\} \neq B$

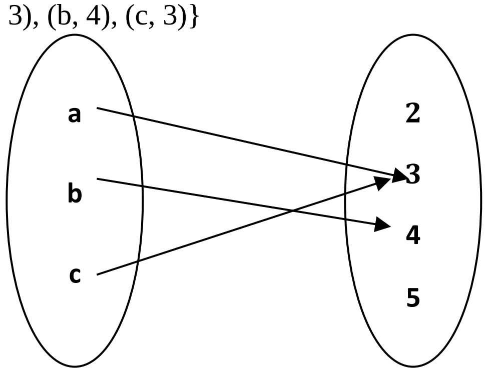
And also no repetition in range

Thus f is injective function

(ii) {(a, 3), (b, 4), (c, 3)}

Solution:

{(a, 3), (b, 4), (c, 3)}



As each element of set A is not paired with each element of set B. Thus, it is not one – one correspondence.

Now

Let $f = \{(a, 3), (b, 4), (c, 3)\}$

For function, $Dom\ f = A$

$Dom\ f = \{a, b, c\} = A$

And there is no repetition in Domain.

Thus f is a function from A to B

Kind of function

Now

$Range\ R = \{3, 4\} \neq B$

And also no repetition in range

Thus f is injective function

Hints for Q5

$X \times Y = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

$Y \times X = \{(5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (7, 1), (7, 2), (7, 3), (7, 4), (8, 1), (8, 2), (8, 3), (8, 4)\}$

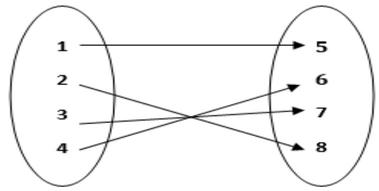
Ex # 5.5

Q5: If X ={1, 2, 3, 4} and Y ={5, 6, 7, 8} then write

(i) a function from X to Y.

Solution:

a function from X to Y.



$f = \{(1, 5), (2, 8), (3, 7), (4, 6)\}$

For function, $Dom\ f = X$

$Dom\ f = \{1, 2, 3, 4\} = X$

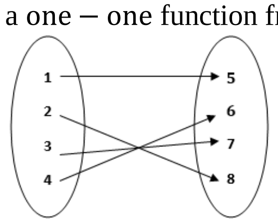
And there is no repetition in Domain.

Thus f is a function in $X \times Y$

(ii) a one – one function from X to Y.

Solution:

a one – one function from X to Y.



$f = \{(1, 5), (2, 8), (3, 7), (4, 6)\}$

For function, $Dom\ f = X$

$Dom\ f = \{1, 2, 3, 4\} = X$

And there is no repetition in Domain.

Thus f is a function in $X \times Y$

Now

$Range\ f = \{5, 6, 7, 8\} = B$

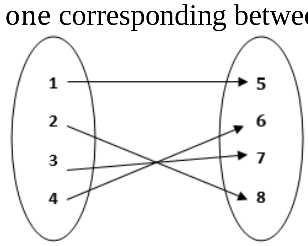
And also no repetition in range

Thus f is one – one function from X to Y

(iii) a relation which shows that there exist one – one corresponding between X and Y.

Solution:

a relation which shows that there exist one – one corresponding between X and Y.



Chapter # 5

Ex # 5.5

$Let\ f = \{(1, 5), (2, 8), (3, 7), (4, 6)\}$

$Dom\ f = \{1, 2, 3, 4\} = X$

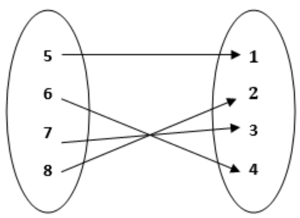
$Range\ f = \{5, 6, 7, 8\} = B$

As each element of set X is paired with one and only element of set Y. So, it is one – one correspondence.

(iv) a function which is onto from Y to X.

Solution:

a function which is onto from Y to X.



Let $f = \{(5, 1), (6, 4), (7, 3), (8, 2)\}$

For function, $Dom\ f = Y$

$Dom\ f = \{5, 6, 7, 8\} = Y$

And there is no repetition in Domain.

Thus f is a function in $Y \times X$

Now

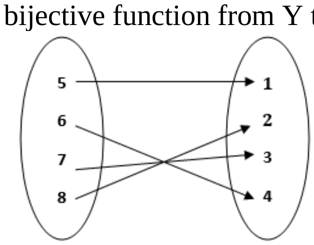
$Range\ f = \{1, 2, 3, 4\} = X$

Hence f is Onto function from Y to X.

(v) bijective function from Y to X.

Solution:

bijective function from Y to X



Let $f = \{(5, 1), (6, 4), (7, 3), (8, 2)\}$

For function, $Dom\ f = Y$

$Dom\ f = \{5, 6, 7, 8\} = Y$

And there is no repetition in Domain.

Thus f is a function in $Y \times X$

Now

$Range\ f = \{1, 2, 3, 4\} = X$

And also no repetition in range

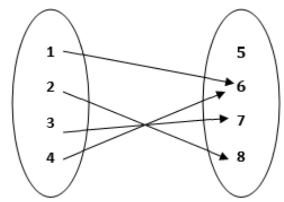
Hence f is bijective function from Y to X.

Ex # 5.5

(vi) a function from X to Y which is neither one – one nor onto.

Solution:

a function from X to Y which is neither one – one nor onto.



$f = \{(1, 6), (2, 8), (3, 7), (4, 6)\}$

For function, $Dom\ f = X$

$Dom\ f = \{1, 2, 3, 4\} = X$

And there is no repetition in Domain.

Thus f is a function in $X \times Y$

Now

$Range\ f = \{6, 7, 8\} \neq B$

And there is also repetition in range

Hence f is a function from X to Y which is neither one – one nor onto.

Q6: Let $A = \{1, 2, 3, 4, 5\}$. Check whether the following sets are functions on A. in case these are functions, indicate their ranges. Which function is onto.

(i) {(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)}

Solution:

{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)}

Let $R = \{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$

For function, $Dom\ R = A$

$Dom\ R = \{1, 2, 3, 4, 5\} = A$

And there is no repetition in Domain.

Thus R is a function in A

Kind of function

Now

$Range\ R = \{1, 2, 3, 5\} \neq A$

As $Range\ R \neq A$. Thus, it is not Onto function.

(ii) {(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)}

Solution:

{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)}

Let $R = \{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$

For function, $Dom\ R = A$

$Dom\ R = \{1, 2, 3, 4, 5\} = A$

Ex # 5.5

And there is no repetition in Domain.
Thus R is a function in A

Kind of function

Now

$$Range\ R = \{1, 2, 3, 4\} \neq A$$

As $Range\ R \neq A$. Thus, it is not Onto function.

(iii) **{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)}**

Solution:

{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)}
Let $R = \{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$
For function, $Dom\ R = A$
$Dom\ R = \{1, 2, 3, 4, 5\} = A$
And there is no repetition in Domain.
Thus R is a function in A

Kind of function

Now
$Range\ R = \{1, 2, 4, 5\} \neq A$
As $Range\ R \neq A$. Thus, it is not Onto function.

Review Ex # 5

Page # 116-117

Q2: **If U = set of natural numbers upto 100 *and* A = set of even numbers upto 100 B = set of odd numbers upto 100. Then find**

(i) $A' \cup B'$

Solution:

$U = \{1, 2, 3, 4 \ldots 100\}$, $A = \{2, 4, 6, \ldots 100\}$
$B = \{1, 3, 5, \ldots 99\}$
To Find:
$A' \cup B'$
First we find A':
A'
$A' = U \setminus A$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{2, 4, 6, \ldots 100\}$
$= \{1, 3, 5, \ldots 99\}$
Now find B':
B'
$B' = U \setminus B$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{1, 3, 5, \ldots 99\}$
$= \{2, 4, 6, \ldots 100\}$
$A' \cup B' = \{1, 3, 5, \ldots 99\} \cup \{2, 4, 6, \ldots 100\}$
$A' \cup B' = \{1, 2, 3, 4 \ldots 100\}$

Review Ex # 5

(ii) $A' \cap B'$

Solution:

$U = \{1, 2, 3, 4 \ldots 100\}$, $A = \{2, 4, 6, \ldots 100\}$
$B = \{1, 3, 5, \ldots 99\}$
To Find:
$A' \cup B'$
First we find A':
A'
$A' = U \setminus A$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{2, 4, 6, \ldots 100\}$
$= \{1, 3, 5, \ldots 99\}$
Now find B':
B'
$B' = U \setminus B$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{1, 3, 5, \ldots 99\}$
$= \{2, 4, 6, \ldots 100\}$
Now
$A' \cap B' = \{1, 3, 5, \ldots 99\} \cap \{2, 4, 6, \ldots 100\}$
$A' \cap B' = \{\ \}$

(iii) $A \cap B'$

Solution:

$U = \{1, 2, 3, 4 \ldots 100\}$, $A = \{2, 4, 6, \ldots 100\}$
$B = \{1, 3, 5, \ldots 99\}$
To Find:
$A \cup B'$
First we find B':
B'
$B' = U \setminus B$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{1, 3, 5, \ldots 99\}$
$= \{2, 4, 6, \ldots 100\}$
Now
$A \cap B' = \{2, 4, 6, \ldots 100\} \cap \{2, 4, 6, \ldots 100\}$
$A \cap B' = \{2, 4, 6, \ldots 100\}$

(iv) $A' \cap B$

Solution:

$U = \{1, 2, 3, 4 \ldots 100\}$, $A = \{2, 4, 6, \ldots 100\}$
$B = \{1, 3, 5, \ldots 99\}$
To Find:
$A' \cup B$
First we find A':
A'
$A' = U \setminus A$
$= \{1, 2, 3, 4 \ldots 100\} \setminus \{2, 4, 6, \ldots 100\}$
$= \{1, 3, 5, \ldots 99\}$
Now
$A' \cap B = \{1, 3, 5, \ldots 99\} \cap \{1, 3, 5, \ldots 99\}$
$A' \cap B = \{1, 3, 5, \ldots 99\}$

Chapter # 5

Review Ex # 5

Q3: $\boldsymbol{A}$ **={1, 2, 3, 5, 7} ,**$\boldsymbol{B}$ **={2, 4, 6} and** $\boldsymbol{C}$ **= {2, 5, 9}**

Verify the following.

(i) Associative property of Union

Solution:

A ={1, 2, 3, 5, 7},B ={2, 4, 6} and C = {2, 5, 9}

To Prove:

Associative Law of Union:

Now

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S: $\boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$

$B \cup C = \{2, 4, 6\} \cup \{2, 5, 9\}$

$B \cup C = \{2, 4, 5, 6, 9\}$

Now

$A \cup (B \cup C) = \{1, 2, 3, 5, 7\} \cup \{2, 4, 5, 6, 9\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 9\}$

R.H.S: $(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C}$

$A \cup B = \{1, 2, 3, 5, 7\} \cup \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

Now

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7\} \cup \{2, 5, 9\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$

Hence

$A \cup (B \cup C) = (A \cup B) \cup C$

Proved

(ii) Associative property of Intersection

Solution:

A ={1, 2, 3, 5, 7} ,B ={2, 4, 6} and

C = {2, 5, 9}

To Prove:

Associative property of Intersection

Now

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S: $\boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$

$B \cap C = \{2, 4, 6\} \cap \{2, 5, 9\}$

$B \cap C = \{ 2 \}$

Now

$A \cap (B \cap C) = \{1, 2, 3, 5, 7\} \cap \{ 2 \}$

$A \cap (B \cap C) = \{ 2 \}$

R.H.S: $(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}$

$A \cap B = \{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}$

$A \cap B = \{ 2 \}$

$(A \cap B) \cap C = \{ 2 \} \cap \{2, 5, 9\}$

$(A \cap B) \cap C = \{ 2 \}$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$ Proved

Review Ex # 5

(iii) Distributive property of union over intersection

Solution:

A ={1, 2, 3, 5, 7} ,B ={2, 4, 6} and

C = {2, 5, 9}

To prove:

Distributive property of union over intersection

Now

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

L.H.S: $\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C})$

$B \cap C = \{2, 4, 6\} \cap \{2, 5, 9\}$

$B \cap C = \{ 2 \}$

Now

$A \cup (B \cap C) = \{1, 2, 3, 5, 7\} \cup \{ 2 \}$

$A \cup (B \cap C) = \{1, 2, 3, 5, 7\}$

R.H.S: $(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

First we find $A \cup B$:

$A \cup B = \{1, 2, 3, 5, 7\} \cup \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

Now

$A \cup C = \{1, 2, 3, 5, 7\} \cup \{2, 5, 9\}$

$A \cup C = \{1, 2, 3, 4, 5, 7, 9\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 5, 7, 9\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 5, 7\}$

Hence

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

(iv) Distributive property of intersection over union

Solution:

A ={1, 2, 3, 5, 7} ,B ={2, 4, 6} and

C = {2, 5, 9}

To Prove:

Distributive property of intersection over union

Now

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C})$

$B \cup C = \{2, 4, 6\} \cup \{2, 5, 9\}$

$B \cup C = \{2, 4, 5, 6, 9\}$

Now

$A \cap (B \cup C) = \{1, 2, 3, 5, 7\} \cap \{2, 4, 5, 6, 9\}$

$A \cap (B \cup C) = \{2, 5\}$

R.H.S: $(\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

First we find $A \cap B$:

$A \cap B = \{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}$

$A \cap B = \{ 2 \}$

Now we find $A \cap C$:

$A \cap C = \{1, 2, 3, 5, 7\} \cap \{2, 5, 9\}$

$A \cap C = \{2, 5\}$

$(A \cap B) \cup (A \cap C) = \{ 2 \} \cup \{2, 5\}$

$(A \cap B) \cup (A \cap C) = \{2, 5\}$

Hence

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved

Q5: If $\boldsymbol{U} = \{\boldsymbol{x} \mid \boldsymbol{x} \in \boldsymbol{N} \wedge \mathbf{1} \leq \boldsymbol{x} \leq \mathbf{40}\}$, $\boldsymbol{A} = \{1, 6, 11, 16, 21, 26, 31\}$

$\boldsymbol{B} = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

Then Verify De Morgan's laws for the following sets.

<u>Solution</u>:

<u>De-Morgan's Law</u>:

$U = \{x \mid x \in N \wedge 1 \leq x \leq 40\}$,

$\mathrm{U} = \{1, 2, 3, 4 \ \dots 40\}$

$A = \{1, 6, 11, 16, 21, 26, 31\}$

$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

To Prove:

$(A \cup B)' = A' \cap B'$

L.H.S: $(\boldsymbol{A} \cup \boldsymbol{B})'$

First we find $A \cup B$:

$A \cup B = \{1, 6, 11, 16, 21, 26, 31\} \cup \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$A \cup B = \{1, 2, 5, 6, 8, 11, 14, 16, 17, 20, 21, 26, 29, 31, 32\}$

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{1, 2, 3, 4 \ \dots 40\} \setminus \{1, 2, 5, 6, 8, 11, 14, 16, 17, 20, 21, 26, 29, 31, 32\}$

$(A \cup B)' = \{3, 4, 7, 9, 10, 12, 13, 15, 18, 19, 22, 23, 24, 25, 27, 28, 30, 33, 34, \dots 40\}$

R.H.S: $\boldsymbol{A}' \cap \boldsymbol{B}'$

First we find A':

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \dots 40\} \setminus \{1, 6, 11, 16, 21, 26, 31\}$

$= \{2,3,4,5,7,8,9,10,12,13,14,15,17,18,19,20,22,23,24,25,27,28,29,30,32,33,34, \dots, 40\}$

And Also

$B' = B \setminus A$

$= \{1, 2, 3, 4 \ \dots 40\} \setminus \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$= \{1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,25,27,28,30,31,33,34,35, \dots, 40\}$

Now

$A' \cap B' = \{2,3,4,5,7,8,9,10,12,13,14,15,17,18,19,20,22,23,24,25,27,28,29,30,32,33,34, \dots, 40\} \cap$

$\{1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,25,27,28,30,31,33,34,35, \dots, 40\}$

$A' \cap B' = \{3,4,7,9,10,12,13,15,18,19,22,24,25, 27, 28,30,33,34,35, \dots, 40\}$

Hence $(A \cup B)' = A' \cap B'$ Proved

Chapter # 5

Review Ex # 5

<u>De-Morgan's Law</u>:

$U = \{x \mid x \in N \wedge 1 \le x \le 40\}$,

$U = \{1, 2, 3, 4 \ \dots 40\}$

$A = \{1, 6, 11, 16, 21, 26, 31\}$

$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

To Prove:

$(A \cap B)' = A' \cup B'$

L.H.S: $(\boldsymbol{A} \cap \boldsymbol{B})'$

First we find $A \cap B$:

$A \cap B = \{1, 6, 11, 16, 21, 26, 31\} \cap \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$A \cap B = \{11, 26\}$

$(A \cap B)' = U \setminus (A \cap B)$

$(A \cap B)' = \{1, 2, 3, 4 \ \dots 40\} \setminus \{11, 26\}$

$(A \cap B)' = \{1, 2, 3, \dots 10, 12, 13, 14 \ \dots 25, 27, 28 \dots 40\}$

R.H.S: $\boldsymbol{A}' \cup \boldsymbol{B}'$

First we find A':

$A' = U \setminus A$

$= \{1, 2, 3, 4 \ \dots 40\} \setminus \{1, 6, 11, 16, 21, 26, 31\}$

$= \{2,3,4,5,7,8,9,10,12,13,14,15,17,18,19,20,22,23,24,25,27,28,29,30,32,33,34,\dots,40\}$

And Also

$B' = B \setminus A$

$= \{1, 2, 3, 4 \ \dots 40\} \setminus \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$= \{1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,25,27,28,30,31,33,34,35,\dots,40\}$

Now

$A' \cup B' = \{2,3,4,5,7,8,9,10,12,13,14,15,17,18,19,20,22,23,24,25,27,28,29,30,32,33,34,\dots,40\}$
$\cup \{1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,25,27,28,30,31,33,34,35,\dots,40\}$

$A' \cup B' = \{1, 2, 3, \dots 10, 12, 13, 14 \ \dots 25, 27, 28 \dots 40\}$

Hence

$(A \cap B)' = A' \cup B'$

Proved

Q5: If $\boldsymbol{U}$ ={1, 2, 3, 4, 5, 6, 7} ,$\boldsymbol{A}$ ={2, 5, 6} and $\boldsymbol{B}$ = {1, 2, 3}
Then verify De – Morgan's laws with the help of Venn diagrams.

<u>De-Morgan's Law</u>:

$U = \{1, 2, 3, 4, 5, 6, 7\}$,$A = \{2, 5, 6\}$ and $B = \{1, 2, 3\}$

To Prove:

$(A \cup B)' = A' \cap B'$

L.H.S: $(\boldsymbol{A} \cup \boldsymbol{B})'$

First we find $A \cup B$:

$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$

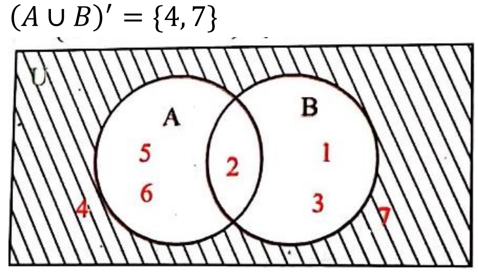
$A \cup B = \{1, 2, 3, 5, 6\}$

Now

$(A \cup B)' = U \setminus (A \cup B)$

$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 5, 6\}$

$(A \cup B)' = \{4, 7\}$



Chapter # 5

Review Ex # 5

R.H.S: $\boldsymbol{A' \cap B'}$

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$

$= \{1, 3, 4, 7\}$

Now find B'

$B' = U \setminus B$

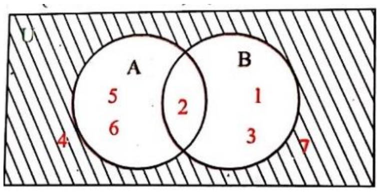
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

Now

$A' \cap B' = \{1, 3, 4, 7\} \cap \{4, 5, 6, 7\}$

$A' \cap B' = \{4, 7\}$



Hence

$(A \cup B)' = A' \cap B'$

Proved

De-Morgan's Law:

$U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 5, 6\}$ and $B = \{1, 2, 3\}$

To Prove:

$$(A \cap B)' = A' \cup B'$$

L.H.S: $\boldsymbol{(A \cap B)'}$

First we find $A \cap B$:

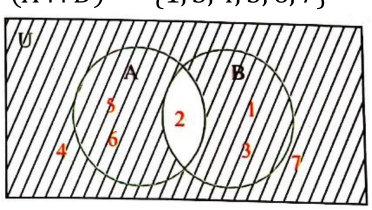
$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$

$A \cap B = \{2\}$

$(A \cap B)' = U \setminus (A \cap B)$

$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7\}$



R.H.S: $\boldsymbol{A' \cup B'}$

First we find A'

$A' = U \setminus A$

$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$

$= \{1, 3, 4, 7\}$

Review Ex # 5

Now find B'

$B' = U \setminus B$

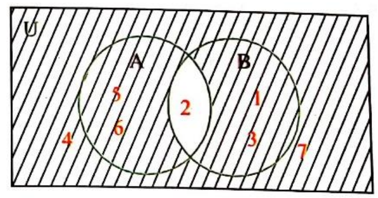
$= \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

Now

$A' \cup B' = \{1, 3, 4, 7\} \cup \{4, 5, 6, 7\}$

$A' \cup B' = \{1, 3, 4, 5, 6, 7\}$



Hence

$(A \cap B)' = A' \cup B'$

Proved

Q6: **If $\boldsymbol{U} = \{1,2,3,\ldots,10\}$, $\boldsymbol{A} = \{1,2,3,4\}$, $\boldsymbol{B} = \{3,4,5,6\}$, C $= \{3,4,7,8\}$ then verify distributive laws with help of Venn Diagram.**

Solution:

Distributive Property of Union over Intersection:

$A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$, $C = \{3,4,7,8\}$

To Prove:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S: $\boldsymbol{A \cup (B \cap C)}$

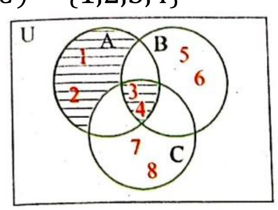
$B \cap C = \{3,4,5,6\} \cap \{3,4,7,8\}$

$B \cap C = \{3, 4\}$

Now

$A \cup (B \cap C) = \{1,2,3,4\} \cup \{3, 4\}$

$A \cup (B \cap C) = \{1,2,3,4\}$



R.H.S: $\boldsymbol{(A \cup B) \cap (A \cup C)}$

First we find $A \cup B$:

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$

$A \cup B = \{1,2,3,4,5,6\}$

Now

$A \cup C = \{1,2,3,4\} \cup \{3,4,7,8\}$

$A \cup C = \{1,2,3,4,7,8\}$

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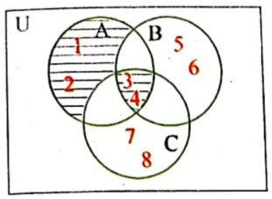
Now

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\} \cap \{1,2,3,4,7,8\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4\}$



Hence

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

Distributive Property of Intersection over Union:

$A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{3,4,7,8\}$

To Prove:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S: $\boldsymbol{A \cap (B \cup C)}$

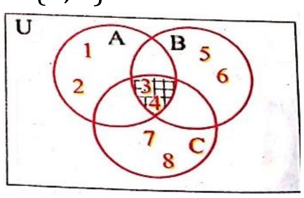
$B \cup C = \{3,4,5,6\} \cup \{3,4,7,8\}$

$B \cup C = \{3,4,5,6,7,8\}$

Now

$A \cap (B \cup C) = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$A \cap (B \cup C) = \{3,4\}$



R.H.S: $(\boldsymbol{A \cup B}) \cap (\boldsymbol{A \cup C})$

First we find $A \cap B$:

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6\}$

$A \cap B = \{3,4\}$

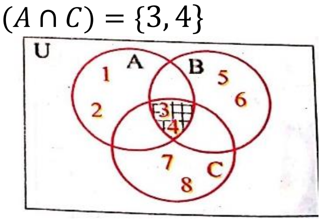
Now we find $A \cap B$:

$A \cap C = \{1,2,3,4\} \cap \{3,4,7,8\}$

$A \cap C = \{3,4\}$

$(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{3,4\}$

$(A \cap B) \cup (A \cap C) = \{3,4\}$



Hence

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved

Review Ex # 5

Q7: Let $A = \{-2, -1, 0, 1, 2\}$, $B = \{a, b, c, d, e\}$. Determine which sets of ordered pair represent a function. In case of a function, mention one – one function, onto function and bijective function.

(i) $\{(\boldsymbol{-2, a}), (\boldsymbol{-1, a}), (\boldsymbol{0, b}), (\boldsymbol{1, c}), (\boldsymbol{2, d})\}$

Solution:

$\{(-2, a), (-1, a), (0, b), (1, c), (2, d)\}$

Let $R = \{(-2, a), (-1, a), (0, b), (1, c), (2, d)\}$

For function, $Dom\ R = A$

$Dom\ R = \{-2, -1, 0, 1, 2\} = A$

And there is no repetition in Domain.

Thus R is a function from $A \times B$

Now

$Range\ R = \{a, b, c, d\} \neq B$

And also there is repetition in range

Hence R is Onto function from A to B

(ii) $\{(\boldsymbol{-1, a}), (\boldsymbol{1, e}), (\boldsymbol{-2, d}), (\boldsymbol{0, c}), (\boldsymbol{2, b})\}$

Solution:

$\{(-1, a), (1, e), (-2, d), (0, c), (2, b)\}$

Let $R = \{(-1, a), (1, e), (-2, d), (0, c), (2, b)\}$

For function, $Dom\ R = A$

$Dom\ R = \{-2, -1, 0, 1, 2\} = A$

And there is no repetition in Domain.

Thus R is a function from $A \times B$

Now

$Range\ R = \{a, b, c, d, e\} = B$

And there is no repetition in range

Hence R is bijective function from A to B

(iii) $\{(\boldsymbol{2, d}), (\boldsymbol{0, a}), (\boldsymbol{-2, b}), (\boldsymbol{-1, c}), (\boldsymbol{1, e})\}$

Solution:

$\{(2, d), (0, a), (-2, b), (-1, c), (1, e)\}$

Let $R = \{(2, d), (0, a), (-2, b), (-1, c), (1, e)\}$

For function, $Dom\ R = A$

$Dom\ R = \{-2, -1, 0, 1, 2\} = A$

And there is no repetition in Domain.

Thus R is a function from $A \times B$

Now

$Range\ R = \{a, b, c, d, e\} = B$

And there is no repetition in range

Hence R is bijective function from A to B

Review Ex # 5

(iv) $\{(-2,b),(-1,b),(0,a),(1,d),(-2,e)\}$

Solution:

$\{(-2,b),(-1,b),(0,a),(1,d),(-2,e)\}$

Let $R=\{(-2,b),(-1,b),(0,a),(1,d),(-2,e)\}$

For function, $Dom\ R=A$

$Dom\ R=\{-2,-1,0,1\}\neq A$

Thus R is not a function because its $Dom\ R\neq A$.

Q8 Let A ={1, 2, 3, 4, 5}, check whether the following sets are functions on A. in case these are functions, indicate their ranges. Which function is onto.

(i) {(1, 5), (2, 3), (3, 3), (4, 2), (5, 1) }

Solution:

{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1) }

Let R = {(1, 5), (2, 3), (3, 3), (4, 2), (5, 1) }

For function, $Dom\ R=A$

$Dom\ R=\{1,2,3,4,5\}=A$

And there is no repetition in Domain.

Thus R is a function from in A

Now

$Range\ R=\{1,2,3,5\}\neq A$

Thus, it is not onto function.

(ii) {(1, 1), (2, 4), (3, 2), (4, 1), (5, 3) }

Solution:

{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3) }

Let R = {(1, 1), (2, 4), (3, 2), (4, 1), (5, 3) }

For function, $Dom\ R=A$

$Dom\ R=\{1,2,3,4,5\}=A$

And there is no repetition in Domain.

Thus R is a function from in A

Now

$Range\ R=\{1,2,3,4\}\neq A$

Thus, it is not onto function.

(iii) {(1, 2), (2, 1), (3, 1), (4, 4), (5, 5) }

Solution:

{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5) }

Let R = {(1, 2), (2, 1), (3, 1), (4, 4), (5, 5) }

For function, $Dom\ R=A$

$Dom\ R=\{1,2,3,4,5\}=A$

Review Ex # 5

And there is no repetition in Domain.

Thus R is a function from in A

Now

$Range\ R=\{1,2,4,5\}\neq A$

Thus, it is not onto function.

(iv) {(1, 2), (2, 3), (1, 4), (3, 5) }

Solution:

{(1, 2), (2, 3), (1, 4), (3, 5) }

Let R = {(1, 2), (2, 3), (1, 4), (3, 5) }

For function, $Dom\ R=A$

$Dom\ R=\{1,2,3\}\neq A$

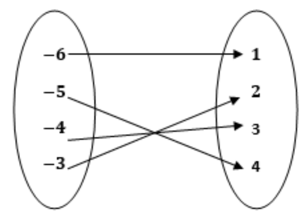
Thus R is not a function from in A

Q9: If $X=\{-6,-5,-4,-3\}$ and Y ={1, 2, 3, 4} then write

(i) a one – one function from X to Y.

Solution

a one – one function from X to Y.



$f=\{(-6,1),(-5,4),(-4,3),(-3,2)\}$

For function, $Dom\ f=X$

$Dom\ f=\{-6,-5,-4,-3\}=X$

And there is no repetition in Domain.

Thus f is a function in $X\times Y$

Now

$Range\ f=\{1,2,3,4\}=B$

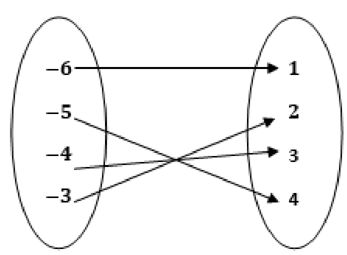
And also no repetition in range

Thus f is one – one function from X to Y

(ii) Onto function from X to Y.

Solution:

Onto function from X to Y.



$f=\{(-6,1),(-5,4),(-4,3),(-3,2)\}$

For function, $Dom\ f=X$

Review Ex # 5

$Dom\ f = \{-6, -5, -4, -3\} = X$

And there is no repetition in Domain.

Thus f is a function in $X \times Y$

Now

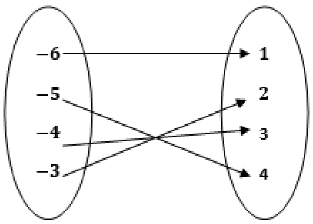
$Range\ f = \{1, 2, 3, 4\} = B$

Thus f is Onto function from X to Y.

(iii) a one – one and onto function from X to Y.

<u>Solution</u>:

a one – one and onto function from X to Y



$f = \{(-6, 1), (-5, 4), (-4, 3), (-3, 2)\}$

For function, $Dom\ f = X$

$Dom\ f = \{-6, -5, -4, -3\} = X$

And there is no repetition in Domain.

Thus f is a function in $X \times Y$

Now

$Range\ f = \{1, 2, 3, 4\} = B$

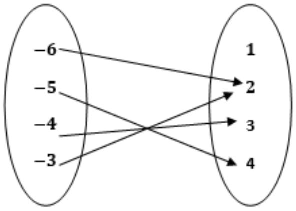
And also no repetition in range

Thus f is one – one and onto function from X to Y

a function from X to Y which is neither one − one nor onto.

<u>Solution</u>:

a function from X to Y which is neither one − one nor onto.



$f = \{(-6, 2), (-5, 4), (-4, 3), (-3, 2)\}$

For function, $Dom\ f = X$

$Dom\ f = \{-6, -5, -4, -3\} = X$

And there is no repetition in Domain.

Thus f is a function in $X \times Y$

Now

$Range\ f = \{2, 3, 4\} \neq B$

And there is also repetition in range

Hence f is a function from X to Y which is neither one − one nor onto.

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MATHEMATICS

Class 10th (KPK)

Unit # 6 Basic Statistics

NAME: ______________________________

F.NAME: _____________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: ____

____

SCHOOL: ____

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UNIT # 6

BASIC STATISTICS

Ex # 6.1

Frequency

The number of times a value appears on a set of data is called frequency.

Data

It can be defined as a systematic record of a particular quantity. Data is a collection of facts and figures to be used for a specific purpose.

Ungrouped data

The set of raw data is called ungrouped data.

Grouped data

The data represented in the form of frequency distribution is called grouped data.

Frequency Distribution

The frequency distribution table is a statistical method to organize and simplify a large set data into smaller groups.

The main purpose of the grouped frequency table is to find out how often each value occurred within each group of the entire data.

Construction of Frequency Table

There are wo types of grouped data.

Discrete frequency data

Continuous frequency data

Construction of Discrete Frequency Table

Steps

Find the minimum and maximum value in the data and write in values in the variable column from minimum to maximum.

Record the values by using tally marks (vertical bars "|")

Count the tally and write down in frequency column.

Example # 1

In a shoe store 40 customers bought shoes with the following shoe size.
6, 6, 7, 6, 8, 7, 7, 8, 6, 10, 6, 8, 8, 10, 7, 9, 7, 10, 6, 10, 10, 9, 7, 9, 6, 10, 10, 7, 11, 8, 8, 7, 6, 6, 8, 9, 7, 8, 7, 9. Construct a frequency table

Ex # 6.1

Solution:

$Let\ X = shoe\ size$

X	Tally Marks	Frequency (f)
6		9
7		10
8		8
9		4
10		8
11		1

Construction of Continuous Frequency Table

Find Range: Deduct lowest value from highest value ($X_{max} - X_{min}$

Determine the number of groups (k). The groups between 5 to 15 groups.

The groups depend upon the range. Larger the Range, more are the numbers of groups.

Determine the width (h) by dividing the Range by number of groups.

$$h = \frac{Range}{k}$$

Decide the upper and lower group data. All the groups should be formed accordingly.

Create the columns titled such as Groups, Tally Marks, Frequency etc.

Insert the data in the table.

Important Concepts

Class Limit

The selected number which shows the start and end of a class is called class limit. The start is lower limit and the end is called upper class limit.

Mid – point/ Class Mark

The midpoint of any class is known as mid – point.

Note:

For each class, the two limits may be fixed such that the midpoint of each class falls on an integer rather than a fraction.

The formula to find the mid – point is

$$Mid - point = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

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Unit # 6

Ex # 6.1

Class Width

The difference between two consecutive lower-class or upper-class limits is called class width. It is also found by dividing the range by the number of groups formed. It is denoted by h.

Formula

$$h = \frac{Range}{k}$$

Class Boundaries

Following is the formula to find the Class Boundaries (C.B)

$$\frac{Lower\ limit\ of\ 2nd\ Class - Upper\ limit\ of\ 1st\ Class}{2}$$

The value from the above formula, it must be subtracted from the lower limits and added to the upper limits of every class

Example

In this example, Class Boundaries are calculated like

$$\frac{Lower\ limit\ of\ 2nd\ Class - Upper\ limit\ of\ 1st\ Class}{2}$$

$$\frac{5-4}{2} = \frac{1}{2} = 0.5$$

Now 0.5 is subtracted from the lower limit and added to the upper limit of each class.

Class Limits	Class Boundaries
1 – 4	0.5 – 4 – 5
5 – 8	4.5 – 8.5
9 – 12	8.5 – 12.5
13 – 16	12.5 – 16.5
17 – 20	16.5 – 20.5

Example # 2

The heights of 30 students of 10th class in cm are as follows. Construct group frequency.
162, 165, 170, 170, 162, 159, 162, 163, 175, 166, 171, 174, 155, 160, 173, 140, 145, 140, 146, 150, 172, 158, 155, 163, 165, 171, 153, 158, 149, 153

Solution:

Minimum value = 140

Maximum value = 175

$Range = maximum\ value - minimum\ value$

$Range = 175 - 140$

$Range = 35$

Ex # 6.1

Let we take the 7 groups

Now

$$Class\ width = \frac{Range}{No.of\ groups}$$

$$Class\ width = \frac{35}{7}$$

$$Class\ width = 5$$

Groups	Class Boundaries	Heights (cm)	Frequency (f)
139 – 144	138.5 – 144.5	140, 140	2
145 – 150	144.5 – 150.5	146, 150, 149, 145	4
151 – 156	151.1 – 156.5	155, 155, 153, 153	4
157 – 162	156.5 – 162.5	158, 158, 159, 160, 162, 162, 162	7
163 – 168	162.5 – 168.5	163, 163, 165, 165, 166	5
169 – 174	168.5 – 174.5	170, 170, 171, 171, 172, 173, 174	7
175 – 180	174.5 – 180.5	175	1

Example # 3

Construct a frequency table of the weights (kg) of 30 students are the following data by using 5 as a class interval. Find the class boundaries and class marks also.
25, 30, 40, 21, 24, 25, 36, 30, 45, 50, 22, 25, 36, 46, 35, 38, 40, 28, 34, 45, 42, 46, 38, 48, 28, 29, 31, 33, 30, 26

Solution:

Groups	Tally Marks	Frequency (f)	Class Boundaries	Class marks
21 – 25	$\cancel{\vert\vert\vert\vert}\ \vert$	6	20.5 – 25.5	23
26 – 30	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	7	25.5 – 30.5	28
31 – 35	$\vert\vert\vert\vert$	4	30.5 – 35.5	33
36 – 40	$\cancel{\vert\vert\vert\vert}\ \vert$	6	35.5 – 40.5	38
41 – 45	$\vert\vert\vert$	3	40.5 – 45.5	43
46 – 50	$\vert\vert\vert\vert$	4	45.5 – 50.5	48

Histogram

A histogram is a vertical bar graph with no space between the bars. The area of each bar is proportional to the frequency it represents.

Advantage

A histogram has advantages over the other methods that it can used to represent data with both equal and unequal class intervals.

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Unit # 6

Ex # 6.1

Note

We have to make the class boundaries to avoid gaps between the bars.

Steps

Class Boundaries or values of variable should be taken along X – axis.

Frequencies should be taken along Y – axis.

The height/ area of the bar/ rectangle measures the frequency.

Example # 4

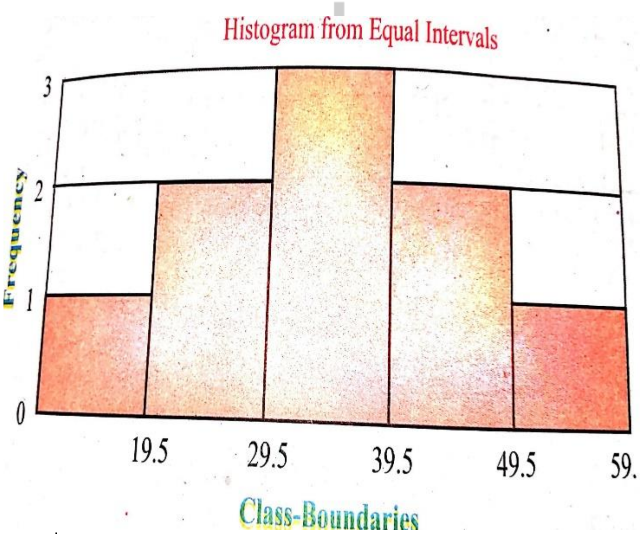
Construct a Histogram from the following table.

Class limits	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
F	1	2	3	2	1

Solution:

To draw a histogram class boundaries are marked along X – axis and frequencies of each class are marked along Y – axis.

Class limits	Frequency	Class Boundaries
20 – 29	1	19.5 – 29.5
30 – 39	2	29.5 – 39.5
40 – 49	3	39.5 – 49.5
50 – 59	2	59.5 – 59.5
60 – 69	1	59.5 – 69.5



Ex # 6.1

Example # 5

Draw a histogram for the following data.

Class limits	30 – 39	40 – 43	44 – 54	55 – 69	70 – 79	80 – 89	90 – 99
F	**10**	**12**	**44**	**75**	**40**	**30**	**10**

Solution:

Histogram with unequal class intervals.

The class intervals are not equal. In constructing the histogram, we must ensure that the area of rectangle are proportional to class frequencies, as the frequency in a histogram is represented b the area of each rectangle.

Class limits	Class Boundaries	Class intervals (h)	Frequency	Adjusted frequency $\frac{f}{h}$
30 – 39	29.5 – 39.5	10	10	1
40 – 43	39.5 – 43.5	4	12	3
44 – 54	43.5 – 54.5	11	44	4
55 – 69	54.5 – 69.5	15	75	5
70 – 79	69.5 – 79.5	10	40	4
80 – 89	79.5 – 89.5	10	30	3
90 – 99	89.5 – 99.5	10	10	1

Unit # 6

Ex # 6.1

Frequency Polygon

A frequency polygon is drawn by joining all the midpoints at the top of each rectangle. The midpoints at both ends are joined to the horizontal axis to accommodate the end points of the polygon.
We can draw the frequency polygon of a distribution without first drawing the histogram.
Frequency polygon are specially useful to compare two sets of data.

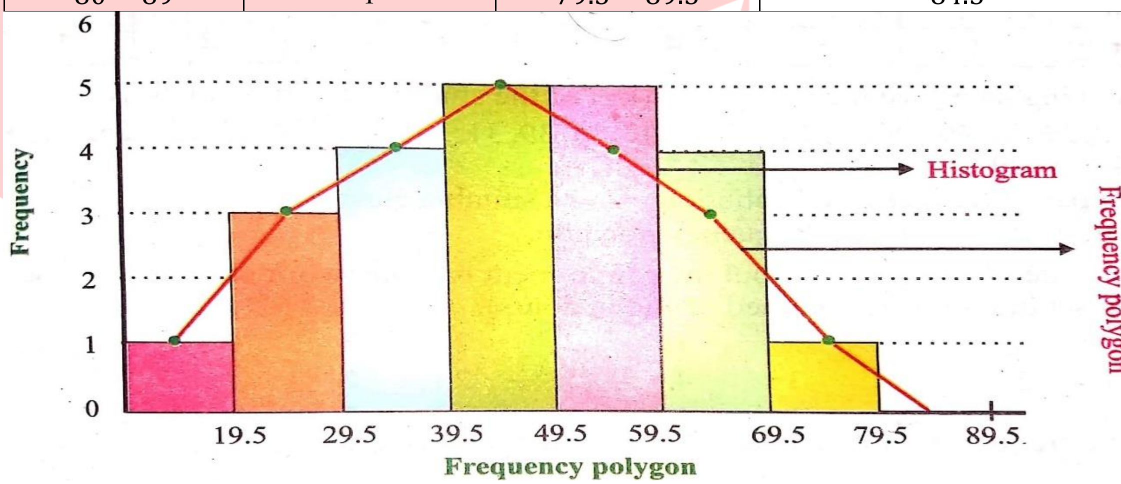
Example # 6

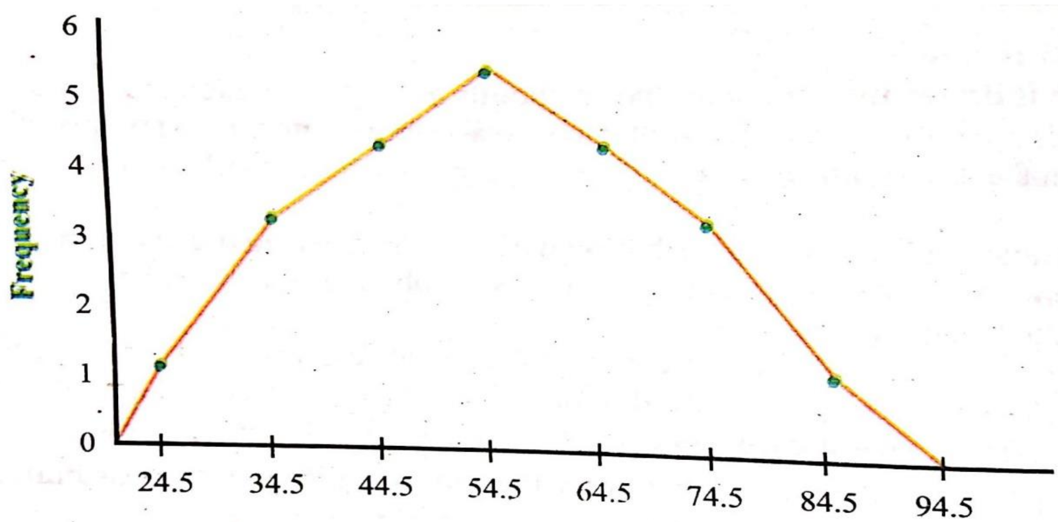
Draw a frequency polygon for the following frequency distribution.

Class limits	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	1	3	4	5	4	2	1

Solution:

Class limits	Frequency	Class Boundaries	Class Marks (Mid – Point)X
20 – 29	1	19.5 – 29.5	24.5
30 – 39	3	29.5 – 39.5	34.5
40 – 49	4	39.5 – 49.5	44.5
50 – 59	5	59.5 – 59.5	54.5
60 – 69	4	59.5 – 69.5	64.5
70 – 79	2	69.5 – 79.5	74.5
80 – 89	1	79.5 – 89.5	84.5





Ex # 6.1

Page # 175

Q1: Construct a frequency distribution of the marks of 30 students during a quiz with 100 points by taking 10 as the class interval. Indicate the class boundaries and class marks.
40, 60, 65, 70, 35, 50, 56, 74, 72, 49, 85, 76, 82, 83, 68, 90, 67, 66, 58, 46, 74, 88, 76, 69, 57, 63, 66, 47, 82, 90

Solution:

Minimum value = 35
Maximum value = 90
$Range = maximum\ value - minimum\ value$
$Range = 90 - 35$
$Range = 55$
As Class Interval is 10.
Now

$$Class\ width = \frac{Range}{No.of\ groups}$$

$$No.of\ groups = \frac{Range}{Class\ width}$$

$$No.of\ groups = \frac{55}{10}$$

$No.of\ groups = 5.5 \cong 6$

Class limits	Class Boundaries	Class marks	Tally Marks	Frequency
35 − 44	34.5 − 44.5	40, 35	||	2
45 − 54	44.5 − 54.5	50, 49, 46, 47	||||	4
55 − 64	54.5 − 64.5	60, 56, 58, 57, 63	𝍸	5
65 − 74	64.5 − 74.5	65, 70, 74, 72, 67, 66, 74, 66, 68, 69	𝍸 𝍸	10
75 − 84	74.5 − 84.5	76, 82, 83, 76, 82	𝍸	5
85 − 94	84.5 − 94.5	85, 90, 88, 90	||||	4
				$\sum f = 30$

Q2: Following are mistakes made by a group of students of class 10th in a test of easy writing. Using an appropriate size of class interval, make a frequency distribution and also indicate the number of class intervals.
4, 7, 12, 9, 21, 16, 3, 19, 17, 24, 14, 15, 8, 13, 11, 16, 15, 6, 5, 8, 11, 20, 18, 22, 6

Solution:

$Minimum\ Value = 3$
$Maximum\ value = 24$
$Range = 24 - 3$
$Range = 21$
$Class\ Size = 3$

$$Class\ interval = \frac{24-3}{3} = \frac{21}{3}$$

$Class\ interval = 7$

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Unit # 6

Ex # 6.1

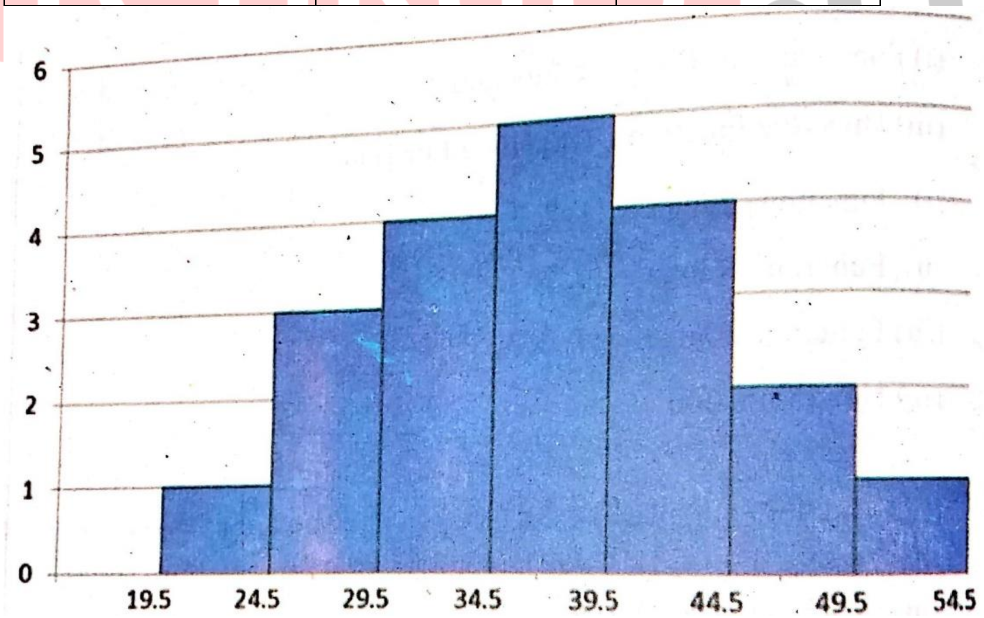
Class Interval	Mistakes	Frequency
3 − 5	4, 3, 5	3
6 − 8	7, 8, 6, 8, 6	5
9 − 11	9, 11, 11	3
12 − 14	12, 14, 13	3
15 − 17	16, 17, 15, 16, 15	5
18 − 20	19, 20, 18	3
22 − 24	21, 24, 22	3
		$\sum f = 25$

Q3: **Draw a Histogram for the following data.**

Class Limit	20 − 24	25 − 29	30 − 34	35 − 39	40 − 44	45 − 49	50 − 54
Frequency	1	3	4	5	4	2	1

Solution:

Class Interval	Frequency	Class Boundaries
20 − 24	1	19.5 − 24.5
25 − 29	3	24.5 − 29.5
30 − 34	4	29.5 − 34.5
35 − 39	5	34.5 − 39.5
40 − 44	4	39.5 − 44.5
45 − 49	2	44.5 − 49.5
50 − 54	1	49.5 − 54.5



Q4: **The following data give the weights in (kg) of the students in the 10th class.**
25, 30, 32, 29, 24, 40, 36, 37, 28, 27, 41, 42, 35, 39, 31, 32, 34, 42, 40, 43, 36, 26, 22, 23, 42, 39, 35, 41, 39, 29
Prepare a frequency distribution using a suitable class interval.
Draw histogram and frequency polygon.

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Unit # 6

Ex # 6.1

Solution:

$Minimum\ Value = 22$

$Maximum\ value = 43$

$Range = 43 - 22$

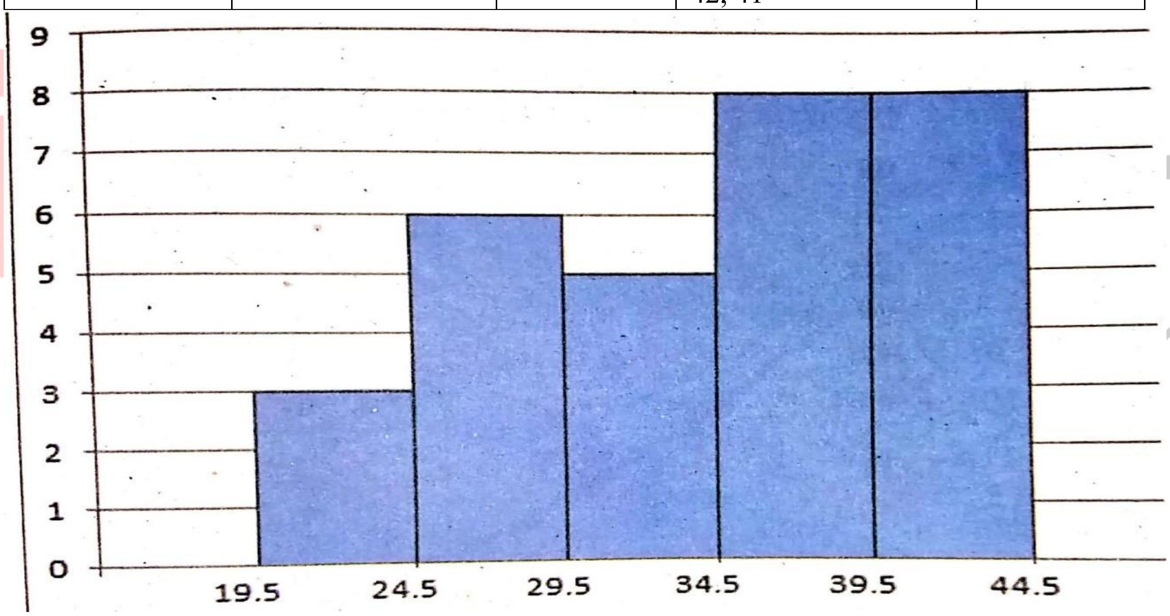
$Range = 21$

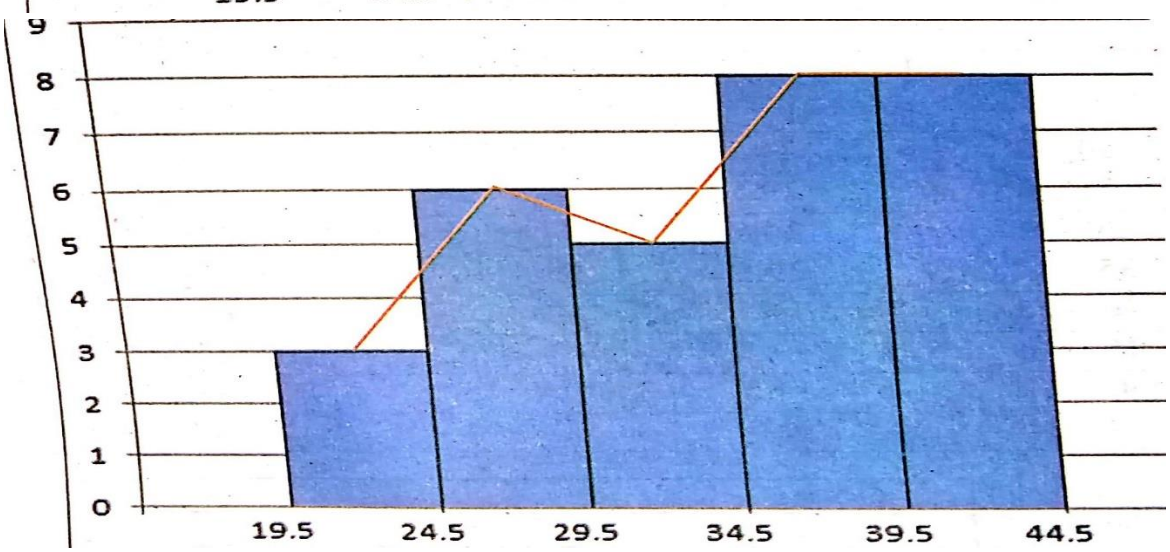
$Class\ Size = 5$

$Class\ interval = \frac{Range}{5} = \frac{21}{5}$

$Class\ interval = 4.2 \cong 4$

Class limits	Class Boundaries	Mid-point	Weights	Frequency
$20 - 24$	$19.5 - 24.5$	22	24, 22, 23	3
$25 - 29$	$24.5 - 29.5$	27	25, 29, 28, 27, 26, 29	6
$30 - 34$	$29.5 - 34.5$	32	30, 32, 31, 32, 34, 35	6
$35 - 39$	$34.5 - 39.5$	37	36, 37, 35, 39, 36, 39, 39	7
$40 - 44$	$39.5 - 44.5$	42	40, 41, 42, 42, 40, 43, 42, 41	8





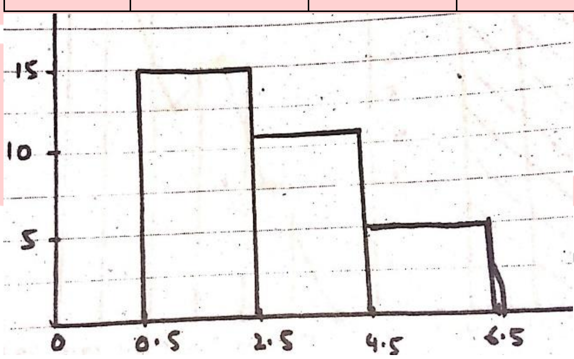
Unit # 6

Ex # 6.1

Q5: A teacher asked students about their time spent on homework completion. Following set of data as obtained.
4, 4, 6, 3, 1, 2, 2, 3, 1, 4, 1, 2, 5, 3, 4, 5, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 4, 2, 6, 2
Construct a frequency table and draw histogram showing the results.

Solution:

Class Limits	Class Boundaries	Tally	f
1 – 2	0.5 – 2.5	1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2	15
3 – 4	2.5 – 4.5	3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4	11
5 – 6	4.5 – 6.5	5, 5, 6, 6	4



Ex # 6.2

Cumulative Frequency

A cumulative frequency table provides information about the sum of a variable against the other values.

When the same data is presented on a graph paper the freehand curve formed is called an Ogive.

Example # 7

Find the cumulative frequency table.

X	3	4	5	6	7	8	9	10	11	12
F	1	2	3	4	5	6	7	4	3	8

Ex # 6.2

Solution:

Cumulative frequency table

X	F	Method of finding C.F	C.F
3	1	1	1
4	2	$1+2=3$	3
5	3	$3+3=6$	6
6	4	$4+6=10$	10
7	5	$5+10=15$	15
8	6	$6+15=21$	21
9	7	$7+21=28$	28
10	4	$4+28=32$	32
11	3	$3+32=35$	35
12	8	$8+35=43$	43

Example # 8

The consumption of petrol of 1000CC cars of a particular brand was surveyed. Construct a cumulative frequency distribution.

Distance km	10 – 12	13 – 15	16 – 18	19 – 21	22 – 24
F	16	20	36	21	7

Solution:

Cumulative frequency distribution

Mileage	Class Boundaries	Upper Class Boundaries	F	C.F
10-12	9.5-12.5	12.5	16	16
13-15	12.5-15.5	15.5	20	36
16-18	15.5-18.5	18.5	36	72
19-21	18.5-21.5	21.5	21	93
22-24	21.5-24.5	24.5	7	100

Cumulative Frequency Polygon

A polygon in which cumulative frequencies are used for ploting the curve is called cumulative frequency polygon. The curve is also called an Ogive.

Example # 9

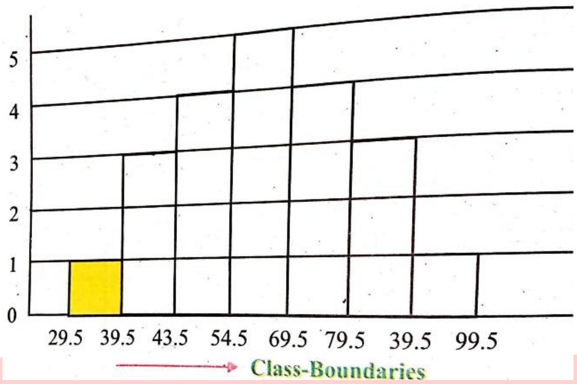
Marks of students are given during first pre – Board exam of mathematics
25, 30, 27, 28, 35, 36, 40, 41, 42, 45, 50, 44, 29, 26, 36, 31, 43, 46, 52, 53, 51, 42, 37, 27, 33, 46, 44, 34, 51, 54
By taking suitable class interval, prepare a frequency distribution, draw ogive.

Unit # 6

Ex # 6.2

Solution:

Class limits	Class Boundaries	F	C.F
25 – 29	24.5 – 29.5	6	6
30 – 34	29.5 – 34.5	4	10
35 – 39	34.5 – 39.5	4	14
40 – 44	39.5 – 44.5	7	21
45 – 49	44.5 – 49.5	3	24
50 – 54	49.5 – 54.5	6	30



Ex # 6.2

Pages # 132

Q1: The following data give the wages (in Rs.) of workers.
60,75,80,85,90,84,70,73,76,84,95,100,150,66, 58,90,98,120,77,90. By taking 10 as a class interval, prepare.
Cumulative frequency distribution.
Cumulative frequency polygon.

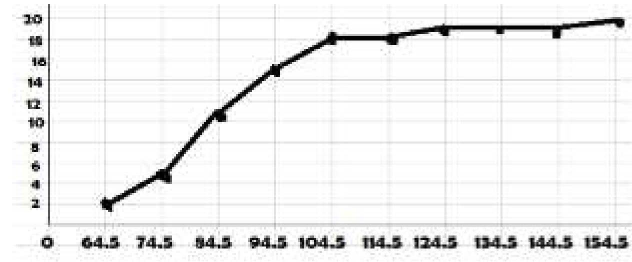
Solution:

Cumulative frequency polygon

Class Intervals	**Class Boundaries**	**Wages**	**f**	C.f
55-64	54.5-64.5	60, 58	2	2
65-74	64.5-74.5	66, 70, 73	3	5
75-84	74.5-84.5	76, 77, 75, 80, 84, 84	6	11
85-94	84.5-94.5	85, 90, 90, 90	4	15
95-104	94.5-104.5	95, 100, 98	3	18
105-114	104.5-114.5		0	18
115-124	114.5-124.5	120	1	19
125-134	124.5-134.5		0	19
135-144	134.5-144.5		0	19
145-154	144.5-154.5	150	1	20

Ex # 6.2

Cumulative frequency polygon



Q2: Make cumulative frequency table for the following data.

Age	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
No.	1	2	16	10	22	20	15	14

Solution:

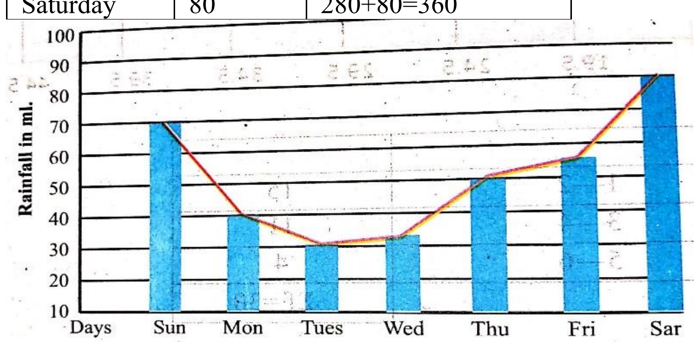
Age	**Class Boundaries**	**Upper Class Boundaries**	**f**	C.f
20–24	19.5–24.5	24.5	1	1
25–29	24.5–29.5	29.5	2	3
30–34	29.5–34.5	34.5	16	19
35–39	34.5–39.5	39.5	10	29
40–44	39.5–44.5	44.5	22	51
45–49	44.5–49.5	49.5	20	71
50–54	49.5–54.5	54.5	15	86
55–59	54.5–59.5	59.5	14	100

Q3: In a city during the first week of August rainfall recorded is as follows. Construct a cumulative frequency graph

Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat
Rainfall in ml	70	40	30	35	50	55	80

Solution:

Day	**Rainfall in ml**	**Cumulative frequency**
Sunday	70	70
Monday	40	70+40=110
Tuesday	30	110+30=140
Wednesday	35	140+35=175
Thursday	50	175+50=225
Friday	55	225+50=280
Saturday	80	280+80=360



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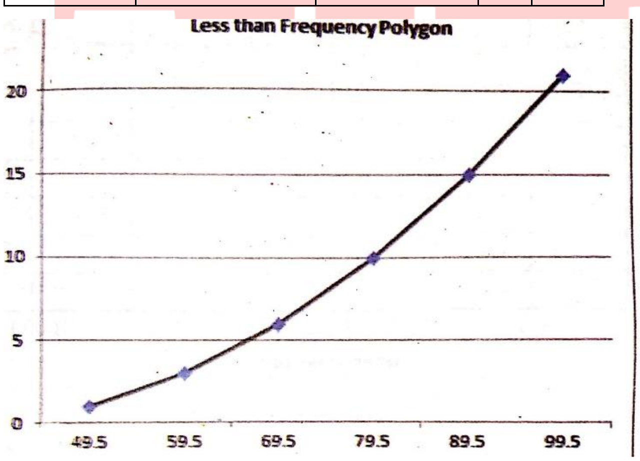
Ex # 6.2

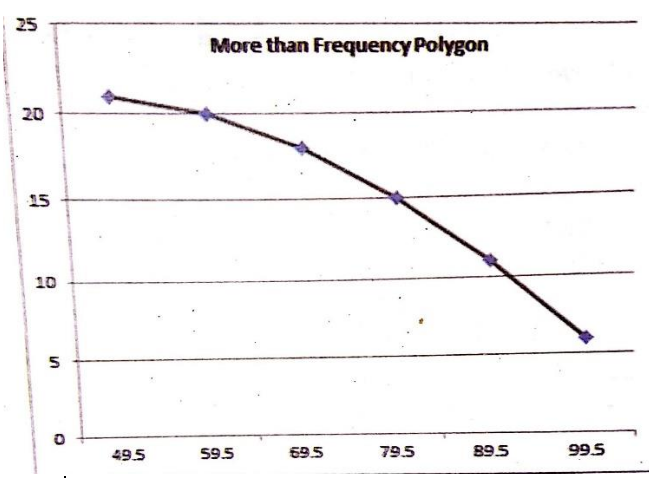
Q4: Draw less than and more than cumulative frequency polygon for the given data.

Marks	Number of Students
40 – 49	1
50 – 59	2
60 – 69	3
70 – 79	4
80 – 89	5
90 – 99	6

Solution:

Marks	Class Boundaries	Upper Class Boundaries	f	C.f
40 – 49	39.5 – 49.5	49.5	1	1
50 – 59	49.5 – 59.5	59.5	2	3
60 – 69	59.5 – 69.5	69.5	3	6
70 – 79	69.5 – 79.5	79.5	4	10
80 – 89	79.5 – 89.5	89.5	5	15
90 – 99	89.5 – 99.5	99.5	6	21





Ex # 6.2

Q5: Determine from the data of Q4, the following

Marks	Class Boundaries	Upper Class Boundaries	f	C.f
40 – 49	39.5 – 49.5	49.5	1	1
50 – 59	49.5 – 59.5	59.5	2	3
60 – 69	59.5 – 69.5	69.5	3	6
70 – 79	69.5 – 79.5	79.5	4	10
80 – 89	79.5 – 89.5	89.5	5	15
90 – 99	89.5 – 99.5	99.5	6	21

(i) Number of students who obtained more than 50 marks

The students who obtained more than 50 marks are 20.

Explanation

The students who obtained more than 50 marks are 2, 3, 4, 5, 6

Now add the number of students which are 20.

(ii) Number of students who obtained less than 70 marks

The students who obtained less than 70 marks are 6

Explanation

The students who obtained less than 70 marks are 1, 2, 3

Now add the number of students which are 6

(iii) Number of students who secured marks between 50 and 70

The students who secured marks between 50 and 70

Explanation

The students who secured marks between 50 and 70 are 2, 3

Now add the number of students which are 5

(iv) Class interval of all classes

Class interval of all classes is 10.

(v) Lower class boundary of 5th class

Lower class boundary of 5th class is 79.5

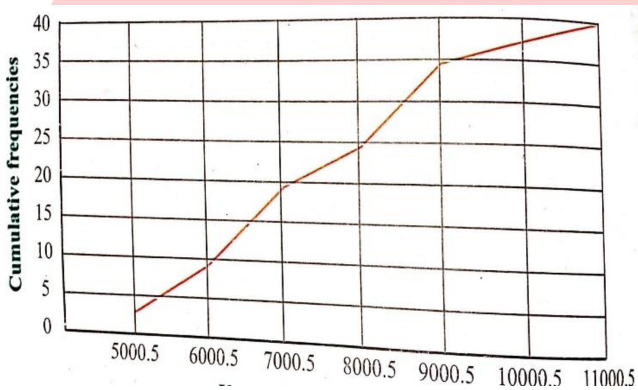
Ex # 6.2

Q6 Construct an Ogive for the following table.

Salary Groups	Workers
4000-5000	3
5001-6000	5
6001-7000	12
7001-8000	9
8001-9000	5
9001-10000	4
10001-11000	2

Solution:

Salary Groups	Class Boundaries	F	C. f
4000-5000	3999.5-5000.5	3	3
5001-6000	5000.5-6000.5	5	8
6001-7000	6000.5-7000.5	12	20
7001-8000	7000.5-8000.5	9	29
8001-9000	8000.5-9000.5	5	34
9001-10000	9000.5-10000.5	4	38
10001-11000	10000.5-11000.5	2	40



Ex # 6.3

Measure of central Tendency

Central Tendency of a data is the representation the whole data or the stage at which the largest number of item tends to concentrate and so it is called central tendency. Central Tendency or Averages are also sometimes called measures of location, because they locate the centre of a distribution.

Types of Centra Tendency Averages

(i) Arithmetic Mean (A.M)
(ii) Median
(iii Mode
(iv Geometric Mean (G.M)
(v) Harmonic Mean (H.M)
(iv Quartiles

Ex # 6.3

Arithmetic Mean for Ungroup Data

Arithmetic Mean is calculated by adding all values of the data divided by the number of items (values).

Denoted by Arithmetic Mean or A.M or Mean or $\overline{X}$

$$\text{Arithmetic Mean} = \frac{Sum\ of\ Quantities}{Number\ of\ Quantities}$$

$$\overline{X} = \frac{x_1 + x_2 + x_3 + x_4 + \cdots + x_n}{n}$$

OR

$$\text{Arithmetic Mean} = \frac{\sum X}{n}$$

$\sum = Sigma\ (Used\ for\ Summation)$

$n = Total\ number\ of\ items\ in\ the\ data.$

By Short – cut Method

$$\overline{X} = a + \frac{\sum D}{n}$$

Where

$a = Provisional\ Mean/\ Assume\ Mean$

$D = X - a$ (Deviation from provisional mean)

Exp # 10(i) Find A.M of 2, 3, 4, 5, 6, 7, 8, 9, 10

Solution:

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{2+3+4+5+6+7+8+9+10}{9}$$

$$\overline{X} = \frac{54}{9} = 6$$

Exp10(ii) Find A.M of 2, 3, 4, 5, 6, 7, 8, 9, 10 by shortcut method.

Solution:

Let assumed mean=2

X	$D = X - a$
2	$2 - 2 = 0$
3	$3 - 2 = 1$
4	$4 - 2 = 2$
5	$5 - 2 = 3$
6	$6 - 2 = 4$
7	$7 - 2 = 5$
8	$8 - 2 = 6$
9	$9 - 2 = 7$
10	$10 - 2 = 8$
$\sum X = 54$	$\sum D = 36$

$$\overline{X} = a + \frac{\sum D}{n}$$

Put the values

$$\overline{X} = 2 + \frac{36}{9}$$

$$\overline{X} = 2 + 4 = 6$$

Ex # 6.3

Arithmetic Mean for Group Data

$$\overline{X} = \frac{\sum fX}{\sum f}$$

Arithmetic Mean for Group Data by Short cut Method

$$\overline{X} = a + \frac{\sum fD}{\sum f}$$

Example # 11

In a coaching class of 13 students, a test conducted, and marks obtained are 10, 12, 12, 14, 9, 18, 9, 13, 16, 9, 17, 16, 14. Make frequency table and find arithmetic mean.

Solution:

X	f	fX
9	3	27
10	1	10
12	2	24
13	1	13
14	2	28
16	2	32
17	1	17
18	1	18
	$\sum f = 13$	$\sum fX = 169$

As we have

$$\overline{X} = \frac{\sum fX}{\sum f}$$

Put the values

$$\overline{X} = \frac{169}{13}$$

$$\overline{X} = 13$$

Example # 12(i)

The price of 2kw generators are given below along frequencies. Find mean by Direct method.

Price	90-94	95-99	100-104	105-109	110-114	115-119	120-124
f	4	11	15	24	18	9	3

Solution:

By Direct Method

Class interval	F	Mid point (X)	fX
$90-94$	4	92	368
$95-99$	11	97	1067
$100-104$	15	102	1530
$105-109$	24	107	2568
$110-114$	18	112	2016
$115-119$	9	117	1053
$120-124$	3	122	366
	$\sum f = 85$		$\sum fX = 8968$

Ex # 6.3

As we have

$$\overline{X} = \frac{\sum fX}{\sum f}$$

Put the values

$$\overline{X} = \frac{8968}{85}$$

$$\overline{X} = 105.5$$

Example # 12(ii)

The price of 2kw generators are given below along frequencies. Find mean by Direct method.

Price	90-94	95-99	100-104	105-109	110-114	115-119	120-124
f	4	11	15	24	18	9	3

Solution:

By Short cut Method

Let assumed mean=92

Class interval	F	Mid point (X)	$D = X - a$	FD
$90-94$	4	92	$92-92=0$	0
$95-99$	11	97	$97-92=5$	55
$100-104$	15	102	$102-92=10$	150
$105-109$	24	107	$107-92=15$	360
$110-114$	18	112	$112-92=20$	360
$115-119$	9	117	$117-92=25$	225
$120-124$	3	122	$122-92=30$	90
	$\sum f = 85$			$\sum fD = 1240$

$$\overline{X} = a + \frac{\sum fD}{\sum f}$$

$$\overline{X} = 92 + \frac{1240}{85}$$

$$\overline{X} = 92 + 14.58$$

$$\overline{X} = 106.58$$

Median

Median is a value which is in the center of observation when all the observations are arranged in ascending or descending order. i.e. Median divide the data in two equal parts.

Median for Ungroup Data

First the data should be ascending or descending order.

For Odd number of quantities

$$Median = \left(\frac{n+1}{2}\right) th\ value$$

Where n is number of quantities

For Even number of quantities

$$Median = \frac{1}{2}\left(\frac{n}{2} th + \frac{n+2}{2} th\right) value$$

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Ex # 6.3

Example # 13

Find the median for 2, 4, 5, 6, 3

Solution:

2, 4, 5, 6, 3

First arrange the data.

2, 3, 4, 5, 6

As number of quantities=5

So n=odd number

As we have

$$Median = \left(\frac{n+1}{2}\right) th\ value$$

$$Median = \left(\frac{5+1}{2}\right) th\ value$$

$$Median = \left(\frac{6}{2}\right) th\ value$$

$$Median = 3rd\ value$$

So

$$Median = 4$$

Example # 14

The following is the daily pocket money in rupees for children of a family 10, 20, 15, 30. Calculate Median.

Solution:

10, 20, 15, 30

First arrange the data.

10, 15, 20, 30

As number of quantities=4

So n=Even number

As we have

$$Median = \frac{1}{2}\left(\frac{n}{2}th + \frac{n+2}{2}th\right) value$$

$$Median = \frac{1}{2}\left(\frac{4}{2}th + \frac{4+2}{2}th\right) value$$

$$Median = \frac{1}{2}(2nd + 3rd) value$$

So

$$Median = \frac{1}{2}(15 + 20)$$

$$Median = \frac{1}{2}(35)$$

$$Median = 17.5$$

Ex # 6.3

Median for Group data (Discrete Data)

Make the cumulative frequency column.

Find out the median value in cumulative frequency column by $\left(\frac{n}{2}\right) th\ value$

Where n is cumulative frequency

Example # 15

The following are the marks obtained by 35 students in a test..

X	10	12	15	20	25	30
F	1	10	5	13	2	4

Solution:

Here n=35

So n=odd number

Now

$$Median = \left(\frac{n+1}{2}\right) th\ value$$

$$Median = \left(\frac{35+1}{2}\right) th\ value$$

$$Median = \left(\frac{36}{2}\right) th\ value$$

$$Median = 18th\ value$$

$See\ 18\ in\ Cumulative\ frequency\ column$

So

$$Median = 20$$

Example # 16: Find median marks from

Marks	10	20	22	25
No. of students	0	2	4	6

Solution:

X	f	C.f
10	0	0
20	2	2
22	4	6
25	6	12

Here n=12

So n=Even number

Now

$$Median = \left(\frac{n}{2}\right) th\ value$$

$$Median = \left(\frac{12}{2}\right) th\ value$$

$$Median = 6\ th\ value$$

$See\ 6\ in\ Cumulative\ frequency\ column$

So

$$Median = 22$$

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Ex # 6.3

Example # 17

Find Median of the following distribution

Wages	60-69	70-79	80-89	90-99	100-109
Labour	4	6	8	10	15

Solution:

Wages	Class Boundaries	f	C.f
$60-69$	$59.5-69.5$	4	4
$70-79$	$69.5-79.5$	6	10
$80-89$	$79.5-89.5$	8	18
$90-99$	$89.5-99.5$	10	28
$100-109$	$99.5-109.5$	5	33

First we find median class

Here n=33

Now

$Median = \left(\frac{n}{2}\right) th\ value$

$Median = \left(\frac{33}{2}\right) th\ value$

$Median = 16.5\ th\ value$

$See\ 16.5\ in\ Cumulative\ frequency\ column$

Now

$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$

Here

$L = 79.5$

$h = 10$

$f = 8$

$C = 10$

$n = 33$

Put the values

$$\text{Median} = 79.5 + \frac{10}{8}\left(\frac{33}{2} - 10\right)$$

$$\text{Median} = 79.5 + 1.25(16.5 - 10)$$

$$\text{Median} = 79.5 + 1.25(6.5)$$

$$\text{Median} = 79.5 + 8.125$$

$$\text{Median} = 87.625$$

Mode for ungroup data

The value that appears more times in a data is called mode

Or

The most repeated or frequent value in a data.

Ex # 6.3

Example # 18

From the following sizes of kids trousers, find the model size 25, 30, 31, 25, 35, 25

Solution:

As the most repeated value is 25

So

Mode=25

Example # 19

The following data shows the weights of the students. Find the model weight.

Weight	40	42	50	51	55
Students	10	8	3	2	1

Solution:

Weight	40	42	50	51	55
Students	10	8	3	2	1

As the highest frequency is 10

So the weight of highest frequency is 40

Thus

Mode=40

Mode of Group Data (Continuous Data)

$$Mode = l + \frac{f_m - f_0}{(f_m - f_0) + (f_m - f_1)} \times h$$

Or

$$Mode = l + \frac{f_m - f_0}{2f_m - f_0 - f_1} \times h$$

L=Lower Class boundary of model class

h=width of class interval

$f_m = Heighest\ Frequency$

$f_0 = Frequency\ before\ Model\ Class$

$f_1 = Frequency\ after\ Model\ Class$

Calculate Mode from the following.

Marks	0-4	4-8	8-12	12-16	16-20
Student	3	5	4	6	2

Solution:

Marks	No. of Students
$0-4$	3
$4-8$	5
$8-12$	4
$12-16$	6
$16-20$	2

As the highest frequency is 6

Thus $12-16$ Model Class

As we have

$$Mode = l + \frac{f_m - f_0}{(f_m - f_0) + (f_m - f_1)} \times h$$

Here

$$l = 12$$

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Ex # 6.3

$f_m = 6$
$f_0 = 4$
$f_1 = 2$
$h = 4$

Put the values

$$Mode = 12 + \frac{6-4}{(6-4)+(6-2)} \times 4$$

$$Mode = 12 + \frac{2}{2+4} \times 4$$

$$Mode = 12 + \frac{2}{6} \times 4$$

$$Mode = 12 + 0.33 \times 4$$

$$Mode = 12 + 1.32$$

$$Mode = 13.32$$

Geometric Mean (G.M)

Geometric Mean is the n^{th} positive root of n values.

Geometric Mean of Ungrouped Data

$$\text{Geometric Mean} = Anti - \log\left(\frac{1}{n}\sum \log X\right)$$

Example # 21

Find Geometric Mean of the marks 60, 65, 70, 80, 85, 90, 75

Solution:

X	LogX
60	1.7781
65	1.8129
70	1.8450
75	1.8750
80	1.9030
85	1.9294
90	1.9542
	$\sum D$ =13.0976

$$\text{Geometric Mean} = Anti - \log\left(\frac{1}{n}\sum \log X\right)$$

$$\text{Geometric Mean} = Anti - \log\left(\frac{1}{7}(13.0976)\right)$$

$$\text{Geometric Mean} = Anti - \log(1.8711)$$

$$\text{Geometric Mean} = 74.32$$

Geometric Mean of Group data

$$G.M = anti - log\left(\frac{\sum f \log X}{\sum f}\right)$$

Ex # 6.3

Example # 22

Calculate the Geometric Mean for

Marks	0 – 20	20 – 40	40 – 60	60 – 80
Students	3	4	10	11

Solution:

Marks	f	X	$\log X$	$f \log X$
0 – 20	3	10	1	3
20 – 40	4	30	1.4771	5.9084
40 – 60	10	50	1.6989	16.989
60 – 80	11	70	1.8450	20.295
	$\sum f = 28$			$\sum f \log X$ = 46.1924

$$G.M = anti - log\left(\frac{\sum f \log X}{\sum f}\right)$$

Put the values

$$G.M = anti - log\left(\frac{46.1924}{28}\right)$$

$$G.M = anti - log(1.697)$$

$$G.M = 44.64$$

Harmonic Mean of ungroup data

Harmonic Mean is the reciprocal of the Arithmetic Mean of the reciprocal values.

$$H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}$$

$$H.M = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

Example # 23

Find Harmonic mean of 5, 6, 8, 9, 10.

Solution:

5, 6, 8, 9, 10

As we have

$$H.M = \frac{5}{\frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}}$$

$$H.M = \frac{5}{0.2 + 0.16 + 0.125 + 0.11 + 0.1}$$

$$H.M = \frac{5}{0.695}$$

$$H.M = 7.194$$

Harmonic Mean of Group Data

$$H.M = \frac{\sum f}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + \cdots + \frac{f_n}{x_n}}$$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

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Ex # 6.3

Example # 24

Find Harmonic mean for

Classes	0-6	6-12	12-18	18-24	24-30
f	1	2	5	4	6

Solution:

Classes	f	X	f/X
$0-6$	1	3	0.33
$6-12$	2	9	0.22
$12-18$	5	15	0.33
$18-24$	4	21	0.19
$24-30$	6	27	0.22
	$\sum f = 18$		$\sum \frac{f}{X} = 1.29$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{X}\right)}$$

Put the values

$$H.M = \frac{18}{1.29}$$

$$H.M = 13.95$$

Weight mean for ungroup data

The numerical values which show the relative importance of different items are called weights and the average of different items having different weights is called weighted mean.

Let $x_1, x_2, x_3 \ldots x_n$ are different values of items having weights $w_1, w_2, w_3 \ldots w_n$ then **Weighted Mean** is:

$$\overline{X}_w = \frac{x_1w_1 + x_2w_2 + x_3w_3 \ldots x_nw_n}{w_1 + w_2 + w_3 + \cdots + w_n}$$

$$\overline{X}_w = \frac{\sum x_iw_i}{\sum w_i} = \frac{\sum xw}{\sum w}$$

Example # 25

The marks obtained by a student in Maths, English, Urdu and Statistics were 70, 60, 80, 65 respectively. Find the average if weights of 2, 1, 3, 1 are assigned to the marks.

Solution:

x	w	xw
70	2	140
60	1	60
80	3	240
65	1	65
	$\sum w = 7$	$\sum w = 505$

Ex # 6.3

As we have

$$\overline{X}_w = \frac{\sum xw}{\sum w}$$

$$\overline{X}_w = \frac{505}{7}$$

$$\overline{X}_w = 72.14$$

Moving Average

It is succession of averages derived from the successive segments of series of values. It continuously recomputed as new data becomes available. If progresses by dropping the earliest value and adding the latest value.

Example # 26

During first week of May, daily temperatures were recorded as given in the table. Calculate 3 – day moving average temperature.

Days	Temperature
Saturday	40
Sunday	37
Monday	36
Tuesday	38
Wednesday	37
Thursday	41
Friday	39

Solution:

Days	**Temperature**	**3 – day Moving Average**
Saturday	40	...
Sunday	37	$\frac{40+37+36}{3} = 37.67$
Monday	36	$\frac{37+36+38}{3} = 37$
Tuesday	38	$\frac{36+38+37}{3} = 37$
Wednesday	37	$\frac{38+37+41}{3} = 38.67$
Thursday	41	$\frac{37+41+39}{3} = 39$
Friday	39	...

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Ex # 6.3

Example # 27
Find median graphically from the following frequency distribution.

Classes	10-14	15-19	20-24	25-29	30-34	35-39
F	1	5	7	2	6	4

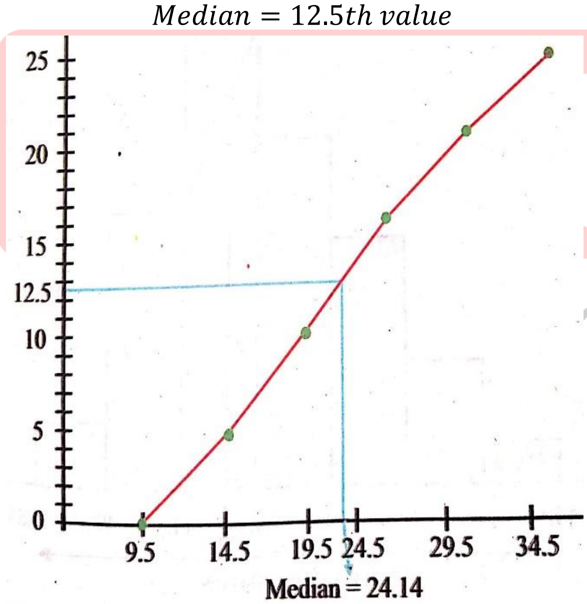
Solution:

Classes	Class Boundaries	f	C.f
10-14	9.5-14.5	1	1
15-19	14.5-19.5	5	6
20-24	19.5-24.5	7	13
25-29	24.5-29.5	2	15
30-34	29.5-34.5	6	21
35-39	34.5-39.5	4	25

$$Median = \frac{n}{2} th\ value$$

$$Median = \frac{25}{2} th\ value$$

$$Median = 12.5th\ value$$



So

$$Median = 24.5$$

Quartiles

$$1st\ Quartile = \frac{n}{4} th\ value$$

$$2nd\ Quartile = \frac{2n}{4} th\ value$$

$$2nd\ Quartile = \frac{n}{2} th\ value = Median$$

$$3rd\ Quartile = \frac{3n}{4} th\ value$$

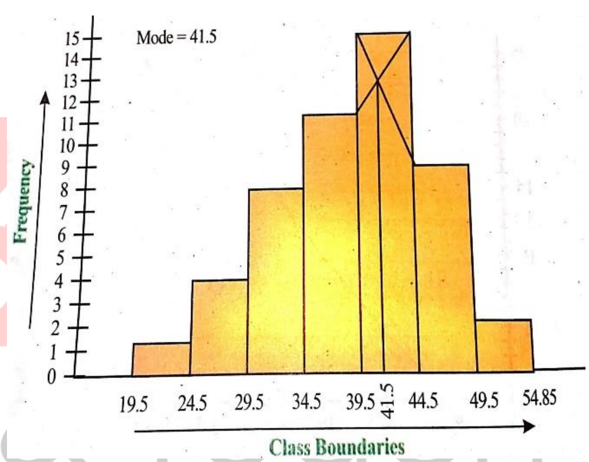
Ex # 6.3

Example # 28
Find mode graphically from the following frequency distribution.

Classes	20-24	25-29	30-34	35-39	40-44	45-49	50-54
F	1	4	8	11	15	9	2

Solution:

Classes	Class Boundaries	f
20-24	19.5-24.5	1
25-29	24.5-29.5	4
30-34	29.5-34.5	8
35-39	34.5-39.5	11
40-44	39.5-44.5	15
45-49	44.5-49.5	9
50-54	49.5-54.5	2



Find Q_1 and Q_2 from the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50
f	3	5	9	3	2

Solution:

Marks	f	C.f
0-10	3	3
10-20	5	8
20-30	9	17
30-40	3	20
40-50	2	22

$$Location\ of\ Q_1 = \frac{n}{4} th\ value$$

$$Location\ of\ Q_1 = \frac{22}{4} th\ value$$

$$Location\ of\ Q_1 = 5.5\ th\ value$$

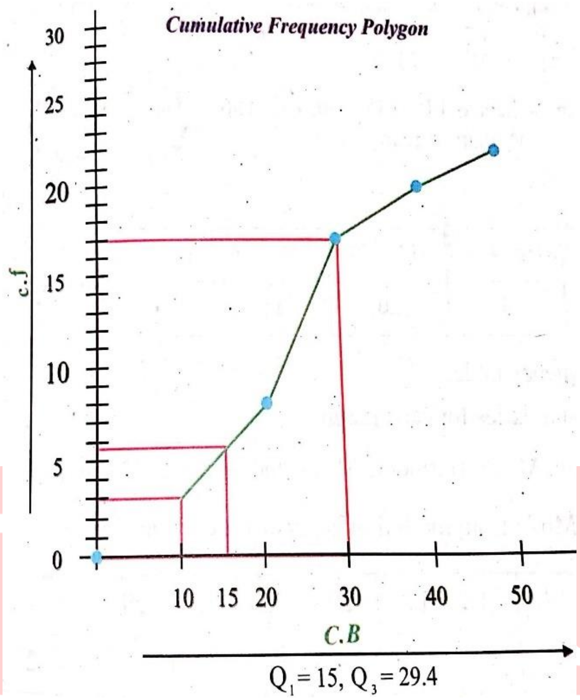
$$Location\ of\ Q_3 = \frac{3n}{4} th\ value$$

Ex # 6.3

$Location\ of\ Q_3 = \frac{3(22)}{4} th\ value$

$Location\ of\ Q_3 = \frac{66}{4} th\ value$

$Location\ of\ Q_3 = 16.5\ th\ value$



Ex # 6.3

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Q1: The following are weights (in kg) of students of 10th grade are 45, 30, 25, 36, 42, 27, 31, 43, 49, 50. Calculate mean of the weights.

Solution:

Let

X=45, 30, 25, 36, 42, 27, 31, 43, 49, 50

As we have

$\text{Mean} = \frac{\sum X}{n}$

$\text{Mean} = \frac{45 + 30 + 25 + 36 + 42 + 27 + 31 + 43 + 49 + 50}{10}$

$\text{Mean} = \frac{378}{10}$

$\text{Mean} = \frac{37.8}{10}$

Ex # 6.3

Q2: Weights of students of 10th grade are 45, 30, 25, 36, 42, 27, 31, 43, 49, 50. Calculate mean by Short cut Method

Solution:

Let assumed mean=25

X	$D = X - a$
45	$45 - 25 = 20$
30	$30 - 25 = 5$
25	$25 - 25 = 0$
36	$36 - 25 = 11$
42	$42 - 25 = 17$
27	$27 - 25 = 2$
31	$31 - 25 = 6$
43	$43 - 25 = 18$
49	$49 - 25 = 24$
50	$50 - 25 = 25$
$\sum X = 378$	$\sum D = 128$

$\bar{X} = a + \frac{\sum D}{n}$

Put the values

$\bar{X} = 25 + \frac{128}{10}$

$\bar{X} = 25 + 12.8$

$\bar{X} = 37.8$

Q3: Using an assumed mean, find the mean of following numbers 1242, 1248, 1252, 1244, 1249

Solution:

Let assumed mean=1242

X	D=X-a
1242	1242-1242=0
1248	1248-1242=6
1252	1252-1242=10
1244	1244-1242=2
1249	1249-1242=7
	$\sum D = 25$

$\bar{X} = a + \frac{\sum D}{n}$

Put the values

$\bar{X} = 1242 + \frac{25}{5}$

$\bar{X} = 1242 + 5$

$\bar{X} = 1247$

Ex # 6.3

Q4: **Find the mean marks obtained by students of 9th class in maths.**

Score	0 – 15	16 – 31	32 – 47	48 – 63	64 – 75
F	0	10	40	70	45

Solution:

Score	f	$X = \frac{L+U}{2}$	fX
0 – 15	0	7.5	0
16 – 31	10	23.5	235
32 – 47	40	39.5	1580
48 – 63	70	55.5	3885
64 – 75	45	69.5	3127.5
	$\sum f = 165$		$\sum fX = 8827.5$

As we have

$$\overline{X} = \frac{\sum fX}{\sum f}$$

Put the values

$$\overline{X} = \frac{8827.5}{165}$$

$$\overline{X} = 53.5$$

Q5: (i) **Find the median of Heights of boys in inches 64, 65, 65, 66, 66, 67**

Solution:

64, 65, 65, 66, 66, 67

As n=6

So

$$Median = \frac{65+66}{2}$$

$$Median = \frac{131}{2}$$

$$Median = 65.5$$

Q5: (ii) **Find the median of Salaries of 8 workers of a factory 7000, 6600, 8000, 4500, 7500, 11000, 9000, 7500**

Solution:

7000, 6600, 4500, 7500, 11000, 9000, 7500

First arrange the data in ascending order

4500, 6600, 7000, 7500, 7500, 8000, 9000, 11000

As n=8

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So

$$Median = \frac{7500+7500}{2}$$

$$Median = \frac{1500}{2}$$

$$Median = 7500$$

Q6: **Find the Arithmetic mean, Geometric mean, Median and Mode of the following data 58, 59, 60,62, 64, 64, 65, 67, 67, 68, 70, 71, 71, 7, 73**

Solution:

X	Log x
58	1.7634
59	1.7709
60	1.7782
62	1.7924
64	1.8062
64	1.8062
65	1.8129
67	1.8261
67	1.8261
68	1.8325
70	1.8451
71	1.8513
71	1.8513
71	1.8513
73	1.8633
$\sum X = 990$	$\sum \log X = 27.2770$

$$\text{Arithmetic Mean} = \frac{\sum X}{n}$$

$$\text{Arithmetic Mean} = \frac{990}{15}$$

$$\text{Arithmetic Mean} = 66$$

Geometric Mean

$$\text{Geometric Mean} = Anti - \log\left(\frac{1}{n}\sum \log X\right)$$

$$\text{Geometric Mean} = Anti - \log\left(\frac{1}{15}(27.2770)\right)$$

$$\text{Geometric Mean} = Anti - \log(1.8185)$$

$$\text{Geometric Mean} = 65.83$$

Median

As n=15

Hence central exact value is median.

So

$$Median = 67$$

Mode

As mode is the most repeated value in a data

So

$$Mode = 71$$

Ex # 6.3

Q7: A set of data contains the values of 148, 145, 160, 157, 156, 160. Show that Mode>Median>Mean

Solution:

148, 145, 160, 157, 156, 160

To Show

Mode>Median>Mean

Now

$$Mean = \frac{\sum X}{n}$$

$$Mean = \frac{148 + 145 + 160 + 157 + 156 + 160}{6}$$

$$Mean = \frac{926}{6}$$

$$Mean = 154.33$$

Median

First arrange the data in ascending order

145, 148, 156, 157, 160, 160

As n=6

So

$$Median = \frac{156 + 157}{2}$$

$$Median = \frac{313}{2}$$

$$Median = 156.5$$

Mode

As mode is the most repeated value in a data

So

$$Mode = 160$$

Thus

Mode>Median>Mean

Q8: From the following distribution

Wages	112 – 116	117 – 121	122 – 126	127 – 131	132 – 136
Workers	3	20	11	4	5

(i) Construct a frequency table

(ii) Find class boundaries for each group

(iii) Calculate Median, Mode, Harmonic Mean and Geometric Mean

Solution:

Wages	f	Class Boundaries
112 – 116	3	111.5 – 116.5
117 – 121	20	116.5– 121.5
122 – 126	11	121.5 – 126.5
127 – 131	4	126.5 – 131.5
132 – 136	5	131.5 – 136.5

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Median

Wages	f	Class Boundaries	C.f
112 – 116	3	111.5 – 116.5	3 C
117 – 121	20 f	116.5– 121.5	23
122 – 126	11	121.5 – 126.5	34
127 – 131	4	126.5 – 131.5	38
132 – 136	5	131.5 – 136.5	43 n

$$Median = \frac{n}{2} th\ term$$

$$Median = \frac{43}{2} th\ term$$

$$Median = 21.5\ th\ term$$

Now

$$Median = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$l = 116.5$

$h = 5$

$C = 3$

Put the values

$$Median = 116.5 + \frac{5}{20}\left(\frac{43}{2} - 3\right)$$

$$Median = 116.5 + 0.25(21.5 - 3)$$

$$Median = 116.5 + 0.25(18.5)$$

$$Median = 116.5 + 4.625$$

$$Median = 121.125$$

Mode

Wages	f	Class Boundaries
112 – 116	3 f_0	111.5 – 116.5
117 – 121	20 f_m	116.5– 121.5
122 – 126	11 f_1	121.5 – 126.5
127 – 131	4	126.5 – 131.5
132 – 136	5	131.5 – 136.5

$$Mode = l + \frac{f_m - f_0}{(f_m - f_0) + (f_m - f_1)} \times h$$

$l = 116.5$

$h = 5$

$f_m = 20$

$f_0 = 3$

$f_1 = 11$

Put the values

$$Mode = 116.5 + \frac{20 - 3}{(20 - 3) + (20 - 11)} \times 5$$

$$Mode = 116.5 + \frac{17}{17 + 9} \times 5$$

Ex # 6.3

$$Mode = 116.5 + \frac{85}{17+9}$$

$$Mode = 116.5 + \frac{85}{26}$$

$$Mode = 116.5 + 3.27$$

$$Mode = 119.77$$

Harmonic Mean

Wages	f	X	f/X
112 – 116	3	114	0.026
117 – 121	20	119	0.168
122 – 126	11	124	0.089
127 – 131	4	129	0.031
132 – 136	5	134	0.037
	$\sum f = 43$		$\sum \frac{f}{X} = 0.351$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{X}\right)}$$

Put the values

$$H.M = \frac{43}{0.351}$$

$$H.M = 122.5$$

Geometric Mean

Wages	f	X	$\log X$	$f\log X$
112 – 116	3	114	2.0569	6.1707
117 – 121	20	119	2.0775	41.5500
122 – 126	11	124	2.0934	23.0274
127 – 131	4	129	2.1106	8.424
132 – 136	5	134	2.1271	10.6355
	$\sum f = 43$			$\sum f\log X = 89.8260$

$$G.M = anti-log\left(\frac{1}{\sum f} \times \sum f\log X\right)$$

Put the values

$$G.M = anti-log\left(\frac{1}{43} \times 89.8260\right)$$

$$G.M = anti-log(2.0890)$$

$$G.M = 122.7374$$

Ex # 6.3

Q9: Find Median, Q1, Q3 and mode Graphically

Classes	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34
F	1	3	7	12	2

Solution:

Classes	Boundaries	F	C.f
10 – 14	9.5 – 14.5	1	1
15 – 19	14.5 – 19.5	3	4
20 – 24	19.5 – 24.5	7	11
25 – 29	24.5 – 29.5	12	23
30 – 34	29.5 – 34.5	2	25

Median

$$Median = \frac{n}{2} th\ value$$

$$Median = \frac{25}{2} th\ value$$

$$Median = 12.5\ th\ value$$

Quartile

$$Q_1 = \frac{n}{4} th\ value$$

$$Q_1 = \frac{25}{4} th\ value$$

$$Q_1 = 8.25\ th\ value$$

And Also

$$Q_3 = \frac{3n}{4} th\ value$$

$$Q_3 = \frac{3(25)}{4} th\ value$$

$$Q_3 = \frac{75}{4} th\ value$$

$$Q_3 = 18.75\ th\ value$$

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Ex # 6.4

Measure of Dispresion

Disprssion is the scatterdness of values from its central value (Average)

Types of Measure of Dispresion are:

Range

Standard Deviation

Variance

Range

The range is the difference between the smallest observation and the largest observation.

Formula

$$Range = Largest\ value - Smallest\ value$$

Note:

Range is very rarely used as it does not tell us about the observation in between the largest and smallest values.

Example # 30

What is the range of the data 209, 260, 270, 311, 311

Solution:

$Largest\ value = 311$

$Smallest\ value = 209$

$Range = ?$

$As\ we\ have$

$Range = largest\ value - smallest\ value$

$Range = 311 - 209$

$Range = 102$

Example # 31: Following are the names and heights of mountains in Karakoram. Find the range of heights.

K – 2	8611 m
Gasherbrum I	8068 m
Broad	8047 m
Gasherbrum II	8035 m
Gasherbrum III	7952 m
Gasherbrum IV	7925 m
Rakaposhi	7788 m

Solution:

$Largest\ height = 8611\ m$

$Smallest\ height = 7788\ m$

$Range = ?$

$As\ we\ have$

$Range = largest\ height - smallest\ height$

$Range = 8611 - 7788$

$Range = 823\ m$

Ex # 6.4

Exp32

Calculate the range from the given data.

Classes	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29
F	10	15	12	21	3

Solution:

Classes	Boundaries	Frequency
5 – 9	4.5 – 9.5	10
10 – 14	9.5 – 14.5	15
15 – 19	14.5 – 19.5	12
20 – 24	19.5 – 24.5	21
25 – 29	24.5 – 29.5	3

$Lower\ Limit\ of\ first\ group = 4.5$

$Upper\ Limit\ of\ last\ group = 29.5$

$Range = ?$

$As\ we\ have$

$Range = 29.5 - 4.5$

$Range = 25$

Example # 33

The number of grams in various candy bars are listed below.

Find the mean, median, mode, and range. Round to the nearest tenth if necessary. Then selected the appropriate measure of central tendency or range to describe the data. Justify your answer.

9, 8, 9, 8, 9, 13, 24

Solution:

9, 8, 9, 8, 9, 13, 24

Mean

$$Mean = \frac{9+8+9+8+9+13+24}{7}$$

$$Mean = \frac{80}{7}$$

$Mean = 11.43$

Median

First arrange the data

8, 8, 9, 9, 9, 13, 24

As number of quantities=7

So n=odd number

As we have

$$Median = \left(\frac{n+1}{2}\right) th\ value$$

$$Median = \left(\frac{7+1}{2}\right) th\ value$$

$$Median = \left(\frac{8}{2}\right) th\ value$$

$Median = 4th\ value$

So

$Median = 9$

Ex # 6.4

Range

$Range = Max.value - Min.value$

$Range = 24 - 8$

$Range = 16$

The appropriate measure of central tendency or range to describe the data is median or mode. The mean is affected by the highest value 24 gram

Standard Deviation

It is the positive square root of the average of squared deviations measured from Arithmetic Mean (A.M).

Standard Deviation for Ungrouped Data

$$\text{Standard Deviation} = \sqrt{\frac{\sum(X-\overline{X})^2}{n}}$$

OR

$$\text{Standard Deviation} = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

Standard Deviation for Grouped Data (Discrete and Continuous data)

$$\text{Standard Deviation} = \sqrt{\frac{\sum f(X-\overline{X})^2}{\sum f}}$$

OR

$$\text{Standard Deviation} = \sqrt{\frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2}$$

Variance

Variance is the square of standard deviation. Variance is usually denoted by the symbol "S".

Variance for Ungrouped Data

$$\text{Variance} = S^2 = \frac{\sum(X-\overline{X})^2}{n}$$

OR

$$\text{Variance} = S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

Variance for Grouped Data (Discrete and Continuous data)

$$\text{Variance} = S^2 = \frac{\sum f(X-\overline{X})^2}{\sum f}$$

OR

$$\text{Variance} = S^2 = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

Example # 34

Find variance and standard deviation of 6, 8, 10, 12, 14

Ex # 6.4

Solution:

X	X^2
6	36
8	64
10	100
12	144
14	196
$\sum X = 50$	$\sum X^2 = 540$

$$\text{Variance} = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

Put the values

$$\text{Variance} = \frac{540}{5} - \left(\frac{50}{5}\right)^2$$

$\text{Variance} = 108 - (10)^2$

$\text{Variance} = 108 - 100$

$\text{Variance} = 8$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

Put the values

$$\text{Standard Deviation} = \sqrt{\frac{540}{5} - \left(\frac{50}{5}\right)^2}$$

$\text{Standard Deviation} = \sqrt{108 - (10)^2}$

$\text{Standard Deviation} = \sqrt{108 - 100}$

$\text{Standard Deviation} = \sqrt{8}$

$\text{Standard Deviation} = 2.83$

Example # 35

Find Standard Deviation and Variance

Rotten Eggs	0-4	4-8	8-12	12-16	16-20	20-24
Crates	5	10	15	20	6	4

Solution:

Defective	f	X	fX	fX^2
0-4	5	2	10	20
4-8	10	6	60	360
8-12	15	10	150	1500
12-16	20	14	280	3920
16-20	6	18	108	1944
20-24	4	22	88	1936
	$\sum f = 60$		$\sum fX = 696$	$\sum fX^2 = 9680$

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Ex # 6.4

Variance

$$Variance = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

$$Variance = \frac{9680}{60} - \left(\frac{696}{60}\right)^2$$

$$Variance = 161.33 - (11.6)^2$$

$$Variance = 161.33 - 134.56$$

$$Variance = 26.77$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\text{variance}}$$

$$\text{Standard Deviation} = \sqrt{26.77}$$

$$\text{Standard Deviation} = 5.17$$

Ex # 6.4

Q1: Find the range of 11, 13, 15, 21, 19, 23

Solution:

11, 13, 15, 21, 19, 23

$$\text{Maximum} = 23$$

$$\text{Minimum} = 11$$

$$\text{Range} = \text{Maximum} - \text{Minimum}$$

$$\text{Range} = 23 - 11$$

$$\text{Range} = 12$$

Q2: A bank branch manager interested in waiting times of customers carried out a survey. A random sample of 12 customers is selected and yielded following 5.90, 9.66, 5.79, 8.02, 8.73, 8.01, 10.49, 8.35, 6.68, 5.64, 5.47, 9.91

Solution:

5.90, 9.66, 5.79, 8.02, 8.73, 8.01, 10.49, 8.35, 6.68, 5.64, 5.47, 9.91

First arrange the data

5.47, 5.64, 5.79, 5.90, 6.68, 8.01, 8.02, 8.35, 8.73, 9.66, 9.91, 10.49

X	X^2
5.47	29.9209
5.64	31.8096
5.79	33.5241
5.90	34.81
6.68	44.6224
8.01	64.1601
8.02	64.3204
8.35	69.7225
8.73	76.2129
9.66	93.3156
9.91	98.2081
10.49	110.0401
$\sum X = 92.65$	$\sum X^2 = 750.6667$

Ex # 6.4

Average

$$\text{Average} = \frac{\sum X}{n}$$

$$\text{Average} = \frac{92.65}{12}$$

$$\text{Average} = 7.72$$

Median

As n=12

So

$$Median = \frac{8.01 + 8.02}{2}$$

$$Median = \frac{16.03}{2}$$

$$Median = 8.015$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum X^2}{\text{n}} - \left(\frac{\sum X}{\text{n}}\right)^2}$$

Put the values

$$\text{Standard Deviation} = \sqrt{\frac{750.6667}{12} - \left(\frac{92.65}{12}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{62.5556 - (7.7208)^2}$$

$$\text{Standard Deviation} = \sqrt{62.5556 - 59.6107}$$

$$\text{Standard Deviation} = \sqrt{2.9449}$$

$$\text{Standard Deviation} = 1.7160$$

Q3: Calculate the Range, Variance, and Standard Deviation for discrete data

X	5	10	11	13	15
f	2	3	4	1	5

Solution:

X	f	fX	fX^2
5	2	10	50
10	3	30	300
11	4	44	484
13	1	13	169
15	5	75	1125
	$\sum f = 15$	$\sum fX = 172$	$\sum fX^2 = 2128$

Range

$$Minimum\ Value = 5$$

$$Maximum\ Value = 15$$

$$Range = ?$$

As we have

$$Range = 15 - 5$$

$$Range = 10$$

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Ex # 6.4

Variance

$$Variance = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

$$Variance = \frac{2128}{15} - \left(\frac{172}{15}\right)^2$$

$$Variance = 141.87 - (11.47)^2$$

$$Variance = 141.87 - 131.56$$

$$Variance = 10.31$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{\frac{2128}{15} - \left(\frac{172}{15}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{141.87 - (11.47)^2}$$

$$\text{Standard Deviation} = \sqrt{141.87 - 131.56}$$

$$\text{Standard Deviation} = \sqrt{10.31}$$

$$\text{Standard Deviation} = 3.21$$

Q4: The following table shows the marks obtained by 10 students of two sections of 10th class.

Sec A	7	9	6	9	4	7	5	8	8	7
Sec B	6	10	6	4	2	8	10	6	9	9

Solution:

Section A

X	X^2
7	49
9	81
6	36
9	81
4	16
7	49
5	25
8	64
8	64
7	49
$\sum X = 70$	$\sum X^2 = 514$

Arithmetic Mean

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{70}{10}$$

$$\overline{X} = 7$$

Ex # 6.4

Variance

$$\text{Variance} = \frac{\sum X^2}{\text{n}} - \left(\frac{\sum X}{\text{n}}\right)^2$$

Put the values

$$\text{Variance} = \frac{514}{10} - \left(\frac{70}{10}\right)^2$$

$$\text{Variance} = 51.4 - (7)^2$$

$$\text{Variance} = 51.4 - 49$$

$$\text{Variance} = 2.4$$

Section B

X	X^2
6	36
10	100
6	36
4	16
2	4
8	64
10	100
6	36
9	81
9	81
$\sum X = 70$	$\sum X^2 = 554$

Arithmetic Mean

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{70}{10}$$

$$\overline{X} = 7$$

Variance

$$\text{Variance} = \frac{\sum X^2}{\text{n}} - \left(\frac{\sum X}{\text{n}}\right)^2$$

Put the values

$$\text{Variance} = \frac{554}{10} - \left(\frac{70}{10}\right)^2$$

$$\text{Variance} = 55.4 - (7)^2$$

$$\text{Variance} = 55.4 - 49$$

$$\text{Variance} = 6.4$$

Q5: Following are the marks (out of 75) of eight students in two subjects.

Student	A	B	C	D	E	F	G	H
Maths	54	63	59	45	52	35	61	68
Physics	52	55	57	51	56	58	50	59

Compare the standard deviation of the marks and tell that in which subject students are more consistent.

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Ex # 6.4

Solution:

Maths

X	X^2
54	2916
63	3969
59	3481
45	2025
52	2704
35	1225
61	3721
68	4624
$\sum X = 437$	$\sum X^2 = 24665$

Arithmetic Mean

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{437}{8}$$

$$\overline{X} = 54.625$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

Put the values

$$\text{Standard Deviation} = \sqrt{\frac{24665}{8} - \left(\frac{437}{8}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{3083.125 - (54.625)^2}$$

$$\text{Standard Deviation} = \sqrt{3083.125 - 2983.891}$$

$$\text{Standard Deviation} = \sqrt{99.234}$$

$$\text{Standard Deviation} = 9.962$$

Physics

X	X^2
52	2704
55	3025
57	3249
51	2601
56	3136
58	3364
50	2500
59	3481
$\sum X = 438$	$\sum X^2 = 24060$

Arithmetic Mean

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{438}{8}$$

Ex # 6.4

$$\overline{X} = 54.75$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

Put the values

$$\text{Standard Deviation} = \sqrt{\frac{24060}{8} - \left(\frac{438}{8}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{3007.5 - (54.75)^2}$$

$$\text{Standard Deviation} = \sqrt{3007.5 - 2997.6}$$

$$\text{Standard Deviation} = \sqrt{9.94}$$

$$\text{Standard Deviation} = 3.15$$

Q6: The following is the distribution for the number of defective bulbs in 30 cartons (Packs). Find variance and standard deviation of defective bulbs.

Defective	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10
Packs	1	3	15	10	2

Solution:

Defective	f	X	fX	fX^2
0 – 2	1	1	1	1
2 – 4	3	3	9	27
4 – 6	15	5	75	375
6 – 8	10	7	70	490
8 – 10	2	9	18	162
	$\sum f = 31$		$\sum fX = 173$	$\sum fX^2 = 1055$

Variance

$$Variance = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

$$Variance = \frac{1055}{31} - \left(\frac{173}{31}\right)^2$$

$$Variance = 34.03 - (5.58)^2$$

$$Variance = 34.03 - 31.14$$

$$Variance = 2.89$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{\text{variance}}$$

$$\text{Standard Deviation} = \sqrt{2.89}$$

$$\text{Standard Deviation} = 1.7$$

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MATHEMATICS

Class 10th

Unit # 7

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UNIT # 7

INTRODUCTION TO TRIGNOMETRY

Ex # 7.1

1^o = One Degree
$1'$ = One minute
$1''$ = One second

Note

$1^o = 60 \text{ minutes} = 60'$
$1' = 60 \text{ seconds} = 60''$
$1^o = 3600 \text{ seconds} = 3600''$

Also

$1' = \left(\frac{1}{60}\right)^o$
$1'' = \left(\frac{1}{3600}\right)^o$
$1'' = \left(\frac{1}{60}\right)'$

Example # 1: Convert $15^o30'25''$ to decimal form.

Solution:

$15^o30'25''$

As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

So

$15^o30'25''$

$= 15^o + \left(30 \times \frac{1}{60}\right)^o + \left(25 \times \frac{1}{3600}\right)^o$
$= 15^o + 0.5^o + 0.0069^o$
$= 15.5069^o$

Thus

$15^o30'25'' = 15.5069^o$

Example # 2: Convert 38.39^o to $D^oM'S''$ form.

Solution:

42.25^o

As $1^o = 60'$ and $1' = 60''$

$38.39^o = 38^o + 0.39^o$
$38.39^o = 38^o + (0.39 \times 60)'$
$38.39^o = 38^o + 23.4'$
$38.39^o = 38^o + 23' + 0.4'$
$38.39^o = 38^o + 23' + (0.4 \times 60)''$
$38.39^o = 38^o + 23' + 24''$
$38.39^o = 38^o23'24''$

Relation between radians and degrees

As circumference of a circle $= 2\pi r$

Ex # 7.1

So $2\pi\, radians = 360^O$
$\pi\, radians = 180^O$
$\frac{\pi}{2}\, radians = 90^O$
$\frac{\pi}{3}\, radians = 60^O$
$\frac{\pi}{5}\, radians = 30^O$ and so on ...

As

$\pi\, radians = 180^O$
$1\, radian = \frac{180^O}{\pi}$
$1\, radian = \frac{180^O}{3.14159}$
$1\, radian = 57.296^O$
$1\, radian = 57.3^O$

Similarly

$180^O = \pi\, radians$
$1^O = \frac{\pi}{180}\, radians$
$1^O = \frac{3.14159}{180}\, radians$
$1^O = 0.0175\, radians$

Example # 3: Convert $\frac{4\pi}{7}$ radians to degrees.

Solution:

$\frac{4\pi}{7}$ radians

As π radian $= 180^o$

Now

$\frac{4\pi}{7}$ radians $= \frac{4}{7} \times 180^o$
$\frac{4\pi}{7}$ radians $= 4 \times 25.71^o$
$\frac{4\pi}{7}$ radians $= 102.84^o$

Example # 4 Convert $31^o45'$ to radians.

Solution:

$31^o45'$

First, we convert it into Decimal form

As $1' = \left(\frac{1}{60}\right)^o$

So

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$31^o45' = 31^o + \left(45 \times \frac{1}{60}\right)^o$
$31^o45' = 31^o + 0.75^o$
$31^o45' = 31.75^o$
Now we have 31.75^o
As $1^o = \frac{\pi}{180}$ radians
Now
$31.75^o = 31.75 \times \frac{\pi}{180}$ radians
$31.75^o = 31.75 \times \frac{3.14159}{180}$ radians
$31.75^o = 31.75 \times 0.0175$ radians
$31.75^o = 0.5556$ radians

Q1: Convert the following angles from $D^oM'S''$ forms to decimal forms.

(i) $80^o15'35''$

Solution:
$80^o15'35''$
As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$
So
$80^o15'35'' = 80^o + \left(15 \times \frac{1}{60}\right)^o + \left(35 \times \frac{1}{3600}\right)^o$
$80^o15'35'' = 80^o + 0.25^o + 0.0097^o$
$80^o15'35'' = 80.2597^o$

(ii) $39^o48'55''$

Solution:
$39^o48'55''$
As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$
So
$39^o48'55'' = 39^o + \left(48 \times \frac{1}{60}\right)^o + \left(55 \times \frac{1}{3600}\right)^o$
$39^o48'55'' = 39^o + 0.8^o + 0.0153^o$
$39^o48'55'' = 39.8153^o$

(iii) $84^o19'10''$

Solution:
$84^o19'10''$
As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$
So
$84^o19'10'' = 84^o + \left(19 \times \frac{1}{60}\right)^o + \left(10 \times \frac{1}{3600}\right)^o$
$84^o19'10'' = 84^o + 0.32^o + 0.0028^o$
$84^o19'10'' = 84.3228^o$

(iv) $18^o6'21''$

Solution:
$18^o6'21''$
As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$
So
$18^o6'21'' = 18^o + \left(6 \times \frac{1}{60}\right)^o + \left(21 \times \frac{1}{3600}\right)^o$
$18^o6'21'' = 18^o + 0.1^o + 0.0058^o$
$18^o6'21'' = 18.1058^o$

Q2: Convert the following angles from decimal forms to $D^oM'S''$

(i) 42.25^o

Solution:
42.25^o
As $1^o = 60'$ and $1' = 60''$
$42.25^o = 42^o + 0.25^o$
$42.25^o = 42^o + (0.25 \times 60)'$
$42.25^o = 42^o + 15'$

(ii) 57.325^o

Solution:
57.325^o
As $1^o = 60'$ and $1' = 60''$
$57.325^o = 57^o + 0.325^o$
$57.325^o = 57^o + (0.325 \times 60)'$
$57.325^o = 57^o + 19.5'$
$57.325^o = 57^o + 19' + 0.5'$
$57.325^o = 57^o + 19' + (0.5 \times 60)''$
$57.325^o = 57^o + 19' + 30''$
$57.325^o = 57^o19'30''$

(iii) 12.9956^o

Solution:
12.9956^o
As $1^o = 60'$ and $1' = 60''$
$12.9956^o = 12^o + 0.9956^o$
$12.9956^o = 12^o + (0.9956 \times 60)'$
$12.9956^o = 12^o + 59.736'$
$12.9956^o = 12^o + 59' + 0.736'$
$12.9956^o = 12^o + 59' + (0.736 \times 60)''$
$12.9956^o = 12^o + 59' + 44.16''$
$12.9956^o = 12^o + 59' + 44''$
$12.9956^o = 12^o59'44''$

(iv) 32.625^o

Solution:
32.625^o

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As $1^o = 60'$ and $1' = 60''$

$32.625^o = 32^o + 0.625^o$

$32.625^o = 32^o + (0.625 \times 60)'$

$32.625^o = 32^o + 37.5'$

$32.625^o = 32^o + 37' + 0.5'$

$32.625^o = 32^o + 37' + (0.5 \times 60)''$

$32.625^o = 32^o + 37' + 30''$

$32.625^o = 32^o 37' 30''$

Q3: Convert the following radian measures of angles into the measures of degrees.

(i) 2 radians

Solution:

2 radians

As 1 radian $= \frac{180^o}{\pi}$

Now

$2 \text{ radians} = 2 \times \frac{180^o}{\pi}$

$2 \text{ radians} = 2 \times \frac{180^o}{3.14159}$

$2 \text{ radians} = 2 \times 57.3^o$

$2 \text{ radians} = 114.6^o$

(ii) $\frac{5\pi}{3}$ radians

Solution:

$\frac{5\pi}{3}$ radians

As π radian $= 180^o$

Now

$\frac{5\pi}{3} \text{ radians} = \frac{5}{3} \times 180^o$

$\frac{5\pi}{3} \text{ radians} = 5 \times 60^o$

$\frac{5\pi}{3} \text{ radians} = 300^o$

(iii) $\frac{\pi}{6}$ radians

Solution:

$\frac{\pi}{6}$ radians

As π radian $= 180^o$

Now

$\frac{\pi}{6} \text{ radians} = \frac{180^o}{6}$

$\frac{\pi}{6} \text{ radians} = 30^o$

(iv) $\frac{-3\pi}{4}$ radians

Solution:

$\frac{-3\pi}{4}$ radians

As π radian $= 180^o$

Now

$\frac{-3\pi}{4} \text{ radians} = \frac{-3}{4} \times 180^o$

$\frac{-3\pi}{4} \text{ radians} = -3 \times 45^o$

$\frac{-3\pi}{4} \text{ radians} = -135^o$

Q4: Convert the following angles in terms of radians.

(i) 45^o

Solution:

45^o

As $1^o = \frac{\pi}{180}$ radians

Now

$45^o = 45 \times \frac{\pi}{180}$ radians

$45^o = \frac{\pi}{4}$ radians

(ii) 120^o

Solution:

120^o

As $1^o = \frac{\pi}{180}$ radians

Now

$120^o = 120 \times \frac{\pi}{180}$ radians

$120^o = 12 \times \frac{\pi}{18}$ radians

$120^o = 2 \times \frac{\pi}{3}$ radians

$120^o = \frac{2\pi}{3}$ radians

(iii) -210^o

Solution:

-210^o

As $1^o = \frac{\pi}{180}$ radians

Now

$-210^o = -210 \times \frac{\pi}{180}$ radians

$-210^o = -21 \times \frac{\pi}{18}$ radians

$-210^o = -7 \times \frac{\pi}{6}$ radians

$-210^o = \frac{-7\pi}{6}$ radians

(iv) $60^o 35' 48''$

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Solution:

$60^o35'48''$

First, we convert it into Decimal form

As $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

So

$60^o35'48''$

$= 60^o + \left(35 \times \frac{1}{60}\right)^o + \left(48 \times \frac{1}{3600}\right)^o$

$= 60^o + 0.58^o + 0.0133^o$

$= 60.5933^o$

Thus

$60^o35'48'' = 60.5933^o$

Now we have 60.5933^o

As $1^o = \frac{\pi}{180}$ radians

Now

$60.5933^o = 60.5993 \times \frac{\pi}{180}$ radians

$60.5933^o = 60.5993 \times \frac{3.14159}{180}$ radians

$60.5933^o = 60.5993 \times 0.0175$ radians

$60.5933^o = 1.06$ radians

Exercise 7.2

Sector of a circle

Length of an arc of circle

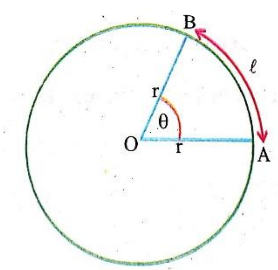
Consider a circle with "O" and radius r, which subtends an angle θ radians at the center O. Let $\widehat{AB}$ is the minor arc of the circle whose length is equal to l as shown in figure.

Now

Radian $= \frac{length\ of\ an\ arc\ \widehat{AB}}{radius\ of\ circle}$

$\theta = \frac{l}{r}$

$Or\quad l = r\,\theta$



Example # 5:

Find the length of an arc of a circle of radius 5 cm which subtends an angle of $\frac{3\pi}{4}$ radians at the centre.

Solution:

$\theta = \frac{3\pi}{4}$ radians, r = 5 cm

To Find:

$l = ?$

As we have

$l = r\,\theta$

Put the values

$l = 5\left(\frac{3\pi}{4}\right)$

$l = \frac{15\pi}{4}$

$l = \frac{15(3.14159)}{4}$

$l = \frac{47.12}{4}$

$l = 11.78\ cm$

Example # 6:

Find the distance travelled by a cyclist moving on a circle of radius 15 m, if he makes 3.5 revolutions.

Solution:

$r = 15\ m$

As 1 revolution $= 2\pi\ radins$

3.5 revolutions $= 3.5 \times 2\pi\ radins$

3.5 revolutions $= 7\pi\ radins$

To Find:

Distance travelled $= l = ?$

As we have

$l = r\,\theta$

Put the values

$l = 15 \times 7\pi$

$l = 105\ \pi$ m

Example # 7:

An arc of length 2.5 cm of a circle subtends an angle θ at the centre O of diameter 6 cm. find the value of θ

Solution:

$l = 2.5$ cm,

As Diameter = 6 cm

Then $r = \frac{d}{2} = \frac{6}{2} = 3$ cm

To Find:

θ = ?

As we have

$l = r\,\theta$

Put the values

$2.5 = 3\,\theta$

Divide B.S by 3

$\frac{2.5}{3} = \frac{3\,\theta}{3}$

$0.833 = \theta$

$\theta = 0.833\ radians$

Example # 8:

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If length of an arc of a circle is 5 cm which subtends an angle of measure 60^o, find the radius of the circle.

Solution:

$\theta = 60^o, \quad r = 5$ cm

To Find:

$r = ?$

First, we convert degree to radians

As $1^o = \frac{\pi}{180}$ radians

Now

$60^o = 60 \times \frac{\pi}{180}$ radians

$60^o = 6 \times \frac{\pi}{18}$ radians

$60^o = \frac{\pi}{3}$ radians

Thus $\theta = \frac{\pi}{3}$ radians

Now we have

$l = r\,\theta$

Put the values

$5 = r \times \frac{\pi}{3}$

Multiply B.S by $\frac{3}{\pi}$

$5 \times \frac{3}{\pi} = r \times \frac{\pi}{3} \times \frac{3}{\pi}$

$\frac{15}{\pi} = r$

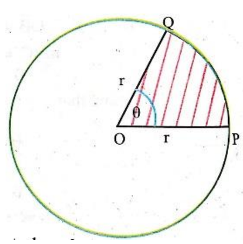
$\frac{15}{3.14159} = r$

$4.77 = r$

$r = 4.77$ cm

Area of sector

Consider a circle of radius r with centre O, PQ as an arc which subtends an angle θ radians at the centre as shown in figure.



By Proportion

$$\frac{\text{Area of sector POQ}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of sector POQ} = \frac{\theta}{2\pi} \times \pi r^2$$

$$\text{Area of sector POQ} = \frac{1}{2} r^2 \theta$$

Thus

$$\text{Area } = \frac{1}{2} r^2 \theta$$

Example # 9: Find the area of sector with central angle of 60^o in a circular region whose radius is 5 cm.

Solution:

Given that:

$r = 5$ m, $\quad \theta = 60^o$

To Find:

$Area\ of\ sector = A = ?$

First, we convert degree to radians

As $1^o = \frac{\pi}{180}$ radians

Now

$60^o = 60 \times \frac{\pi}{180}$ radians

$60^o = 6 \times \frac{\pi}{18}$ radians

$60^o = \frac{\pi}{3}$ radians

Thus $\theta = \frac{\pi}{3}$ radians

Now we have

$\text{Area of sector} = \frac{1}{2} r^2 \theta$

Put the values

$\text{Area of sector} = \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right)$

$\text{Area of sector} = \frac{1}{2}(25)\left(\frac{3.14159}{3}\right)$

$\text{Area of sector} = \frac{1}{2}(25)(1.047)$

$\text{Area of sector} = \frac{1}{2}(26.175)$

$\text{Area of sector} = 13.09\ \text{cm}^2$

Exercise # 7.2

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Q1: Find l when

(i) $\boldsymbol{\theta = \frac{\pi}{6}}$ **radians,** $\quad$ **r = 2 cm**

Solution:

$\theta = \frac{\pi}{6}$ radians, $\quad r = 2$ cm

To Find:

$l = ?$

As we have

$l = r\,\theta$

Put the values

$l = 2\left(\frac{\pi}{6}\right)$

$l = \frac{3.14159}{3}$

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$l = 1.047\ cm$

(ii) $\theta = 30^o$, $r = 6$ cm

Solution:

$\theta = 30^o$, $r = 6$ cm

To Find:

$l = ?$

First, we convert degree to radians

As $1^o = \frac{\pi}{180}$ radians

Now

$30^o = 30 \times \frac{\pi}{180}$ radians

$30^o = 3 \times \frac{\pi}{18}$ radians

$30^o = \frac{\pi}{6}$ radians

Thus $\theta = \frac{\pi}{6}$ radians

Now we have

$l = r\,\theta$

Put the values

$l = 6\left(\frac{\pi}{6}\right)$

$l = \pi$cm

$l = 3.14159$ cm

(iii) $\theta = \frac{4\pi}{6}$ radians, $r = 6$ cm

Solution:

$\theta = \frac{4\pi}{6}$radians, $r = 6$ cm

To Find:

$l = ?$

As we have

$l = r\,\theta$

Put the values

$l = 6\left(\frac{4\pi}{6}\right)$

$l = 4\pi$

$l = 4(3.14159)$

$l = 12.57$ cm

Q2: Find θ when

(i) $l = 5$cm, $r = 2$ cm

Solution:

$l = 5$cm, $r = 2$ cm

To Find:

$\theta = ?$

As we have

$l = r\,\theta$

Put the values

$5 = 2\,\theta$

Divide B.S by 2

$\frac{5}{2} = \frac{2\,\theta}{2}$

$2.5 = \theta$

$\theta = 2.5\ radians$

(ii) $l = 30$ cm, $r = 6$ cm

Solution:

$l = 30$ cm, $r = 6$ cm

To Find:

$\theta = ?$

As we have

$l = r\,\theta$

Put the values

$30 = 6\,\theta$

Divide B.S by 6

$\frac{30}{6} = \frac{6\,\theta}{6}$

$5 = \theta$

$\theta = 5\ radians$

(iii) $l = 6$ cm, $r = 2.87$ cm

Solution:

$l = 6$ cm, $r = 2.87$ cm

To Find:

$\theta = ?$

As we have

$l = r\,\theta$

Put the values

$6 = 2.87\,\theta$

Divide B.S by 2.87

$\frac{6}{2.87} = \frac{2.87\,\theta}{2.87}$

$2.091 = \theta$

$\theta = 2.091\ radians$

Q3: Find r when

(i) $\theta = \frac{\pi}{6}$radians, $l = 2$ cm

Solution:

$\theta = \frac{\pi}{6}$radians, $l = 2$ cm

To Find:

$r = ?$

As we have

$l = r\,\theta$

Put the values

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$2 = r \times \frac{\pi}{6}$

Multiply B.S by $\frac{6}{\pi}$

$2 \times \frac{6}{\pi} = r \times \frac{\pi}{6} \times \frac{6}{\pi}$

$\frac{12}{\pi} = r$

$\frac{12}{3.14159} = r$

$3.82 = r$

$r = 3.82 \text{ cm}$

(ii) $\boldsymbol{\theta = 3\frac{1}{2}}$ **radians,** $\boldsymbol{l = \frac{4}{7}}$ **m**

Solution:

$\theta = 3\frac{1}{2}$ radians, $l = \frac{4}{7}$ m

$\theta = \frac{7}{2}$ radians, $l = \frac{4}{7}$ m

To Find:

$r = ?$

As we have

$l = r\,\theta$

Put the values

$\frac{4}{7} = r \times \frac{7}{2}$

Multiply B.S by $\frac{2}{7}$

$\frac{4}{7} \times \frac{2}{7} = r \times \frac{7}{2} \times \frac{2}{7}$

$\frac{8}{49} = r$

$0.16 = r$

$r = 0.16 \text{ cm}$

(iii) $\boldsymbol{\theta = \frac{3\pi}{4}}$ **radians,** $\boldsymbol{l = 15}$ **cm**

Solution:

$\theta = \frac{3\pi}{4}$ radians, $l = 15$ cm

To Find:

$r = ?$

As we have

$l = r\,\theta$

Put the values

$15 = r \times \frac{3\pi}{4}$

Multiply B.S by $\frac{4}{3\pi}$

$15 \times \frac{4}{3\pi} = r \times \frac{3\pi}{4} \times \frac{4}{3\pi}$

$5 \times \frac{4}{\pi} = r$

$\frac{20}{\pi} = r$

$\frac{20}{3.14159} = r$

$6.366 = r$

$r = 6.366 \text{ cm}$

Q4: Find the area of sector whose radius is 4 m, with central angle 12 radian.

Solution:

Given that:

r = 4 m, $\theta = 12$ radians

To Find:

$Area\ of\ sector = A = ?$

As we have

Area of sector $= \frac{1}{2}r^2\theta$

Put the values

Area of sector $= \frac{1}{2}(4)^2(12)$

Area of sector $= \frac{1}{2}(16)(12)$

Area of sector $= (8)(12)$

Area of sector $= 96\ \text{m}^2$

Q5: The arc of a circle subtends an angle of 30^0 at the centre. The radius of a circle is 5 cm. find;

(i) Length of the arc

(ii) Area of sector formed.

Solution:

Length of the arc

r = 5 m, $\theta = 30^o$

To Find:

$l = ?$

First, we convert degree to radians

As $1^o = \frac{\pi}{180}$ radians

Now

$30^o = 30 \times \frac{\pi}{180}$ radians

$30^o = \frac{\pi}{6}$ radians

Thus $\theta = \frac{\pi}{6}$ radians

Now we have

$l = r\,\theta$

Put the values

$l = 5\left(\frac{\pi}{6}\right)$

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$l = 5\left(\frac{3.14159}{6}\right)$

$l = 5(0.524)$

$l = 2.62$ cm

Area of sector formed

Given that:

$r = 5$ m, $\quad \theta = 30^o = \frac{\pi}{6}$ radians

To Find:

Area of sector $= A = ?$

As we have

Area of sector $= \frac{1}{2}r^2\theta$

Put the values

Area of sector $= \frac{1}{2}(5)^2\left(\frac{\pi}{6}\right)$

Area of sector $= \frac{1}{2}(25)\left(\frac{3.14159}{6}\right)$

Area of sector $= \frac{1}{2}(25)(0.524)$

Area of sector $= \frac{1}{2}(13.1)$

Area of sector $= 6.55$ cm^2

Q6: An arc of a circle subtends an angle of 2 radian at the centre. If area of sector formed is 64 cm^2. Find the radius of circle.

Solution:

Given that:

$\theta = 12$ radian

Area of sector $= 64$ cm^2

To Find:

$r = ?$

As we have

Area of sector $= \frac{1}{2}r^2\theta$

Put the values

$64 = \frac{1}{2}r^2(2)$

$64 = r^2$

$r^2 = 64$

Taking Square root on B.S

$\sqrt{r^2} = \sqrt{64}$

$r = 8\ cm$

Q7: In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolutions. ($\mathbf{3.5\ revolutions = 7\pi}$)

Solution:

$r = 10\ m$

$\theta = 3.5$ revolutions $= 7\pi$

To Find:

Distance travelled $= l = ?$

As we have

$l = r\,\theta$

Put the values

$l = 10 \times 7\pi$

$l = 70\,\pi$ m

Q8: What is the circle measure of the angle between the hands of the watch at 3 o'clock?

Solution:

Since one complete rotaion $= 360^o$

As 3 o'clock shows $= \frac{1}{4}$ of complete rotaion

So 3 o'clock $= \frac{1}{4} \times 360^o$

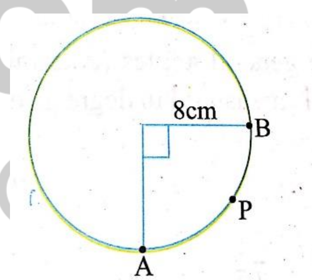
So 3 o'clock $= 90^o$

Hence 3 o'clock $= 90^o = \frac{\pi}{2}$ *radian*

Q9: What is the length of the arc *APB* ?

Solution:

$r = 8\ cm$



To Find:

$l = ?$

From the figure

$\theta = 90^o = \frac{\pi}{2}$ *radian*

As we have

$l = r\,\theta$

Put the values

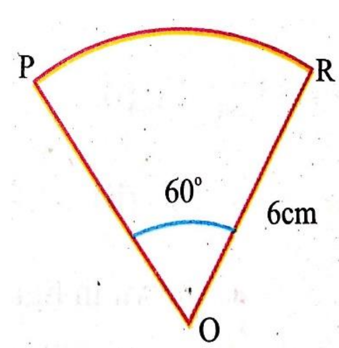
$l = 8 \times \frac{\pi}{2}$

$l = 4\,\pi\ cm$

Q10: Find the area of sector *OPR.*

Solution:

From the figure



$r = 6$ m, $\quad \theta = 60^o$

To Find:

Area of sector $= A = ?$

First, we convert degree to radians

As $1^o = \frac{\pi}{180}$ radians

Now

$60^o = 60 \times \frac{\pi}{180}$ radians

$60^o = \frac{\pi}{3}$ radians

Thus $\theta = \frac{\pi}{3}$ radians

Now we have

Area of sector $= \frac{1}{2} r^2 \theta$

Put the values

Area of sector $= \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

Area of sector $= \frac{1}{2}(36)\left(\frac{\pi}{3}\right)$

Area of sector $= (18)\left(\frac{\pi}{3}\right)$

Area of sector $= 6\pi\ \text{cm}^2$

Ex # 7.3

The General Angle (Coterminal Angles)

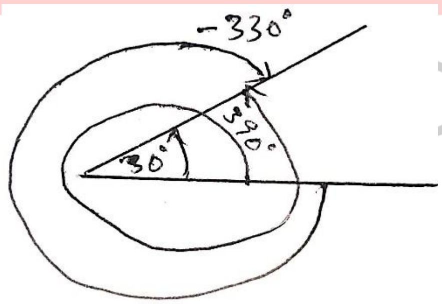
Angles having the same initial and terminal sides are called coterminal angles, and they differ by a multiple of 2π radians or 360^o. They are also called general angles.

Let $\theta = 30^o$

Then $30^o + 360^o = 390^o$

And $30^o - 360^o = -330^o$

The coterminal angles of 30^o are 390^o and -330^o



Angle in standard position

In XY – plane, an angle is in standard position if:

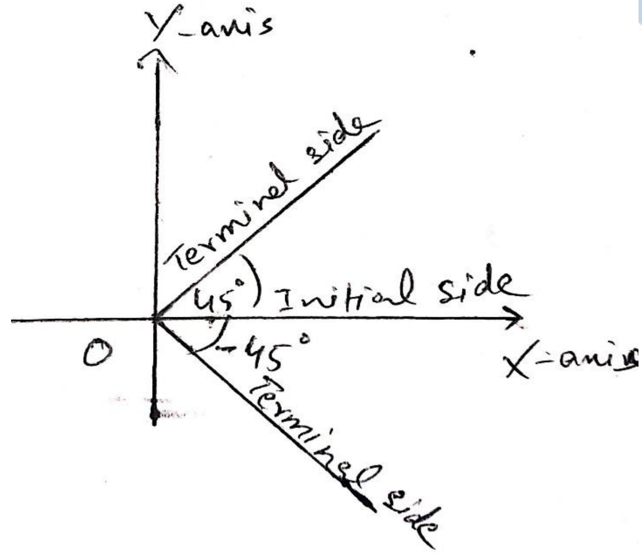
- Its vertex is at origin of XY – plane
- Its initial side lies along the positive x – axis.

Note:

- When an angle is positive then it shows anti – clock wise direction.
- When an angle is negative then it shows clock wise direction.

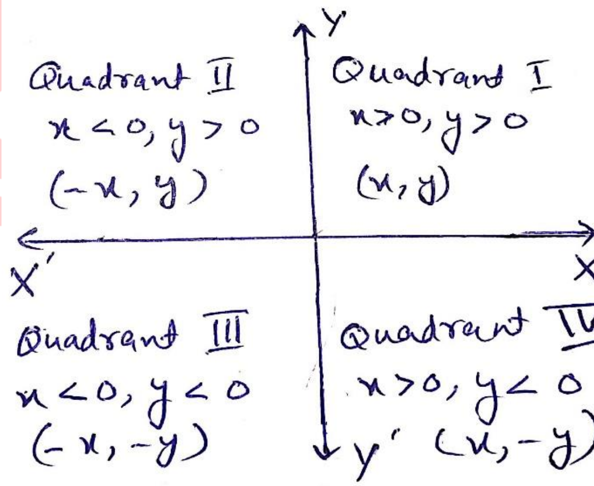
In the given figure

An angle of measure 45^o shows anti – clock wise direction while -45^o shows clock wise direction.



Quadrants

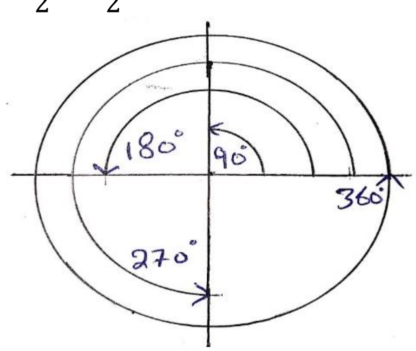
The Cartesian plane is divided into four quadrants and the angle θ is said to be in that quadrant where OP lies.



Quadrantal Angles

Quadrantal angles are those angles whose terminal sides coincide with co – ordinate axis i.e., x – axis or y – axis. The Quadrantal measure of angles are 0^o, 90^o, 180^o, 270^o and 360^o or

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ *and* 2π



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Exercise # 7.3

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Q1: Find coterminal angles of the following angles.

(i) 55^o

Solution:

55^o

As $55^o + 360^o = 415^o$

And $55^o - 360^o = -305^o$

The coterminal angles of 55^o are 415^o and -305^o

(ii) -45^o

Solution:

-45^o

As $-45^o + 360^o = 315^o$

And $-45^o - 360^o = -405^o$

The coterminal angles of -45^o are 315^o and -405^o

(iii) $\frac{\pi}{6}$

Solution:

$\frac{\pi}{6}$

As $\frac{\pi}{6} + 2\pi = \frac{\pi + 12\pi}{6} = \frac{13\pi}{6}$

And $\frac{\pi}{6} - 2\pi = \frac{\pi - 12\pi}{6} = \frac{-11\pi}{6}$

The coterminal angles of $\frac{\pi}{6}$ are $\frac{13\pi}{6}$ and $\frac{-11\pi}{6}$

(iv) $\frac{-3\pi}{4}$

Solution:

$\frac{-3\pi}{4}$

As $\frac{-3\pi}{4} + 2\pi = \frac{-3\pi + 8\pi}{4} = \frac{5\pi}{5}$

And $\frac{-3\pi}{4} - 2\pi = \frac{-3\pi - 8\pi}{6} = \frac{-11\pi}{6}$

The coterminal angles of $\frac{-3\pi}{4}$ are $\frac{5\pi}{6}$ and $\frac{-11\pi}{6}$

Q2: State the quadrant in which the following angles lie?

(i) $\frac{8\pi}{5}$

Solution:

$\frac{8\pi}{5}$

As π radian $= 180^o$

$\frac{8\pi}{5} = \frac{8 \times 180^o}{5}$

$\frac{8\pi}{5} = 8 \times 36^o$

$\frac{8\pi}{5} = 288^o$

Thus $\frac{8\pi}{5}$ *lies in 2nd Quadrant*

(ii) 75^o

Solution:

75^o

Thus 75^o *lies in 1st Quadrant*

(iii) -818^o

Solution:

As $-818^o = -2(360^o) - 98^o$

As -818^o is negative so in anti – clock direction

Thus -98^o *lies in 3rd Quadrant*

(iv) $\frac{-5\pi}{4}$

Solution:

$\frac{-5\pi}{4}$

As π radian $= 180^o$

$\frac{-5\pi}{4} = \frac{-5 \times 180^o}{4}$

$\frac{-5\pi}{4} = -5 \times 45^o$

$\frac{-5\pi}{4} = -225^o$

As -225^o is negative so in anti – clock direction

Thus -225^o *lies in 2nd Quadrant*

103^o

Solution:

103^o

Thus 103^o *lies in 2nd Quadrant*

Ex # 7.4

Trigonometric Ratios

Hypotenuse

The side opposite to 90^o is called hypotenuse.

Perpendicular

The side opposite to θ or angle in consideration is called perpendicular or opposite side.

Base

The side adjacent to θ or angle in consideration is called base or adjacent side.

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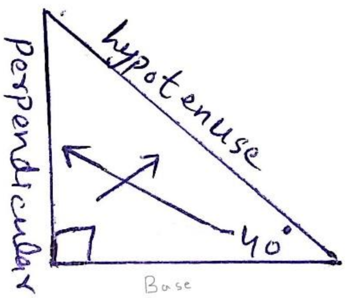
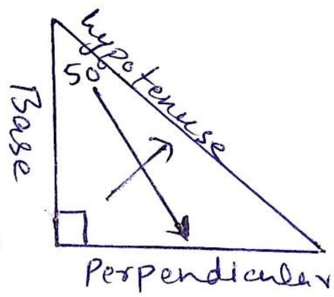
Unit # 7





Reciprocal Identities

$\sin\theta = \dfrac{1}{cosec\,\theta}$ *or* $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{sec\,\theta}$ *or* $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{cot\,\theta}$ *or* $\cot\theta = \dfrac{1}{\tan\theta}$

Trigonometric Ratios

A trigonometric ratio is a ratio of lengths of two sides in a right triangle. Trigonometric ratios are used to find the measure of a side or an acute angle in a right-angled triangle.

The trigonometric ratios for any acute angle of a right-angled triangle are given:

$\sin\theta = \dfrac{perp}{hyp}$

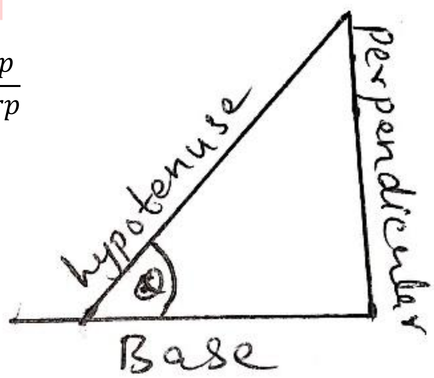
$cosec\ \theta = \dfrac{hyp}{perp}$

$\cos\theta = \dfrac{base}{hyp}$

$\sec\theta = \dfrac{hyp}{base}$

$\tan\theta = \dfrac{perp}{base}$

$\cot\theta = \dfrac{base}{perp}$



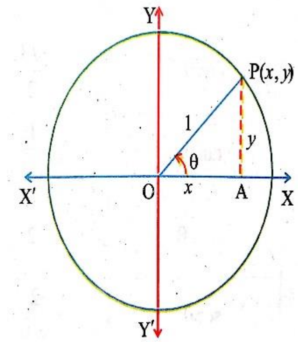
Trick to remember Trigonometric Formulas

Some **P**eople **H**ave	سلیم پڑھتا ہے۔
Curly **B**rown **H**air	کتا بھونکتا ہے۔
Till **P**ainted **B**lack	ٹرین پر بیٹھ۔

Trigonometric Ratios with the help of a unit circle

Consider a circle with centre O and radius 1 (unit circle). $P(x,\ y)$ is any point on the circle. Radius $\overline{OP}$ makes an angle θ. Draw $\overline{PA}$ on x – axis to form OAP a right-angled triangle with $\angle A = 90^{o}$

In the figure



Perpendicular $\overline{AP} = y$

Base $\overline{OA} = x$

Hypotenuse $\overline{OP} = 1$

$\sin\theta = \dfrac{perp}{hyp} = \dfrac{\overline{AP}}{\overline{OP}} = \dfrac{y}{1} = y$

$\cos\theta = \dfrac{base}{hyp} = \dfrac{\overline{OA}}{\overline{OP}} = \dfrac{x}{1} = x$

$\tan\theta = \dfrac{perp}{base} = \dfrac{\overline{AP}}{\overline{OA}} = \dfrac{y}{x}$

Thus, trigonometric ratios of unit circle are

$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \dfrac{y}{x}$

Reciprocal trigonometric ratios

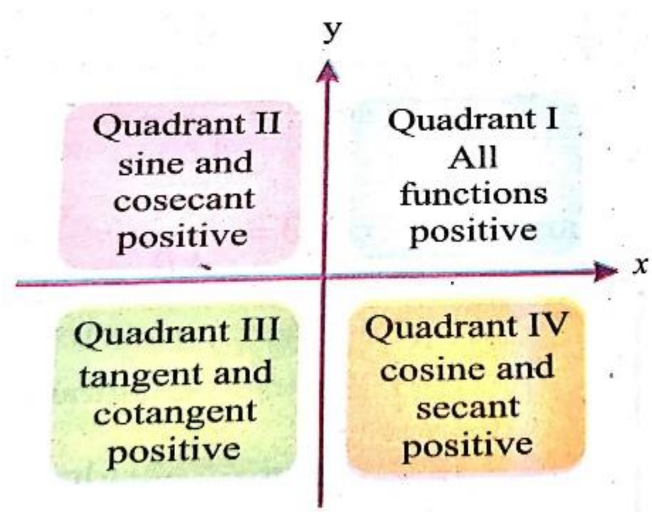
$cosec\ \theta = \dfrac{1}{y}$

$\sec\theta = \dfrac{1}{x}$

$\cot\theta = \dfrac{x}{y}$

θ	**30º**	**45º**	**60º**
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Signs of Trigonometric Ratios in Different Quadrants.



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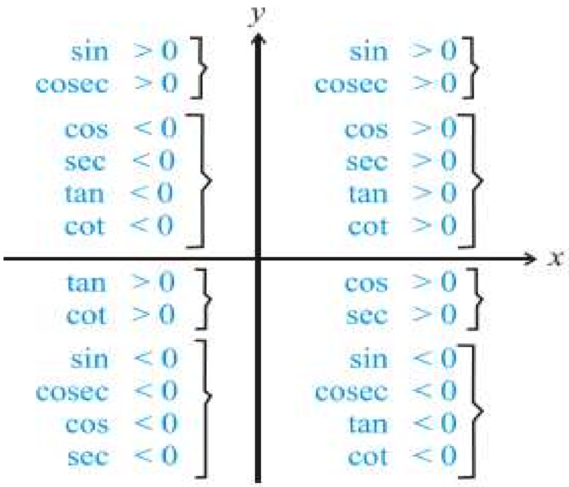
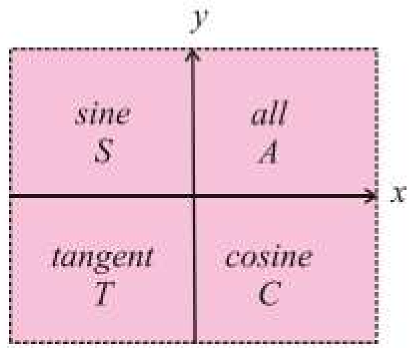


Trick to remember Signs of Trigonometric Ratios



ASTC Add Sugar To Coffee

Example # 11: Find the signs of the following trigonometric ratios and tell in which quadrant the lie?

(i) $\sin 105^o$

Solution:

$\sin 105^o$

Since 105^o lies in 2^{nd} Quadrant.

As $\sin\theta$ is positive in 2^{nd} Quadrant.

So $\sin 105^o$ is positive in 2^{nd} Quadrant.

(ii) $\tan\frac{-5\pi}{6}$

Solution:

$\tan\frac{-5\pi}{6}$

First convert into degree

As π radian $= 180^o$

Now

$\frac{-5\pi}{6}$ radians $= \frac{-5}{6} \times 180^o$

$\frac{-3\pi}{4}$ radians $= -5 \times 30^o$

$\frac{-5\pi}{6}$ radians $= -150^o$

As θ is negative which shows clock wise direction.

Since -150^o lies in 2^{nd} Quadrant.

As $\tan\theta$ is positive in 3^{rd} Quadrant.

So $\tan\frac{-5\pi}{6}$ is positive in 3rd Quadrant.

(iii) $\sec 1030^o$

Solution:

$\sec 1030^o$

As $1030^o = 2(360^o) + 310^o$

Since 310^o lies in 4^{th} Quadrant.

As $\cos\theta$ and $\sec\theta$ are positive in 4^{th} Quadrant.

So $\sec 1030^o$ is positive in 4^{th} Quadrant.

(iv) $\cot 710^o$

Solution:

$\cot 710^o$

As $710^o = 360^o + 350^o$

Since 350^o lies in 4^{th} Quadrant.

As $\tan\theta$ and $\cot\theta$ are negative in 4^{th} Quadrant.

So $\sec 1030^o$ is negative in 4^{th} Quadrant.

Example # 13: If $\tan\theta = 1$, find the other trigonometric ratios, when θ lies in first quadrant.

Solution:

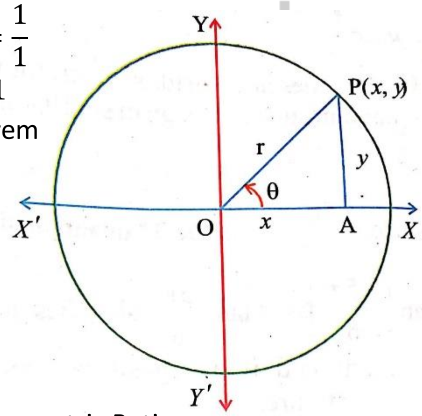
As $\tan\theta = 1$ and θ lies in 1^{st} quadrant.

Here $x = Base, r = hyp, y = perp$

$\tan\theta = \frac{perp}{base} = \frac{y}{x} = \frac{1}{1}$

$So\ \ y = 1\ and\ x = 1$

By Pythagoras Theorem



$(r)^2 = (x)^2 + (y)^2$

$(r)^2 = (1)^2 + (1)^2$

$(r)^2 = 1 + 1$

$(r)^2 = 2$

$\sqrt{(r)^2} = \sqrt{2}$

$r = \sqrt{2}$

Now the other Trigonometric Ratios are

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}}$

$\text{cosec}\,\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{1} = 1$

$\cot\theta = \frac{x}{y} = \frac{1}{1} = 1$

Example # 14: If $\tan\theta = -\frac{2}{3}$ and θ lies in 2nd quadrant. Find the other trigonometric ratios

Solution:

As $\tan\theta = \frac{-2}{3}$ and θ lies in 2nd quadrant.

Here $x = Base, r = hyp, y = perp$

As θ lies in 2nd quadrant then $x = -3$

$\tan\theta = \frac{perp}{base} = \frac{y}{x} = \frac{2}{-3}$

$So\ \ y = 2\ and\ x = -3$

Now by Pythagoras Theorem

$(r)^2 = (x)^2 + (y)^2$

$(r)^2 = (-2)^2 + (3)^2$

$(r)^2 = 4 + 9$

$(r)^2 = 13$

$\sqrt{(r)^2} = \sqrt{13}$

$r = \sqrt{13}$

Now the other Trigonometric Ratios are

$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}}$

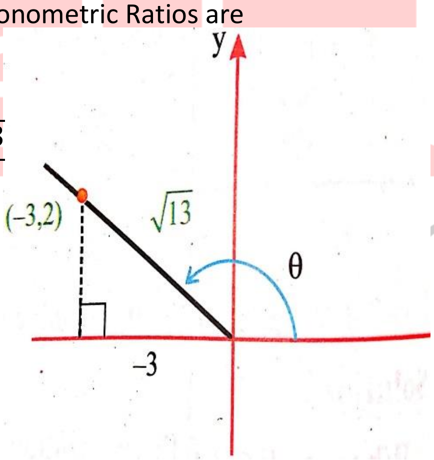
$\operatorname{cosec}\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3}$

$\tan\theta = \frac{y}{x} = \frac{2}{-3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{2}$



Example # 15: If $\cos\theta = \frac{4}{5}$ and θ lies in 4th quadrant. Find the other trigonometric ratios

Solution:

As $\cos\theta = \frac{4}{5}$ and θ lies in 4th quadrant.

Here $x = Base, r = hyp, y = perp$

$\cos\theta = \frac{base}{hyp} = \frac{x}{r} = \frac{4}{5}$

$So\ \ x = 4\ and\ r = 5$

Now by Pythagoras Theorem

$(x)^2 + (y)^2 = (r)^2$

$(4)^2 + (y)^2 = (5)^2$

$16 + (y)^2 = 25$

$(y)^2 = 25 - 16$

$(y)^2 = 9$

$\sqrt{(y)^2} = \pm\sqrt{9}$

$y = \pm 3$

As θ lies in 4th quadrant then

$y = -3$

Now the other Trigonometric Ratios are

$\sin\theta = \frac{y}{r} = \frac{-3}{5}$

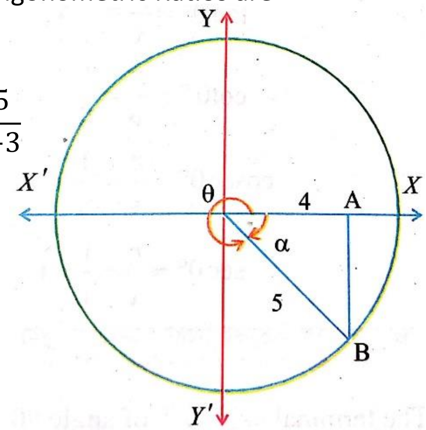
$\operatorname{cosec}\theta = \frac{r}{y} = \frac{5}{-3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$

$\sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{-3}{4}$

$\cot\theta = \frac{x}{y} = \frac{4}{-3}$



Trigonometric Ratios of Quadrantal Angles

Trigonometric Ratios of 0^{o}

Solution:

To find:

Trigonometric ratios of 0^{o}.

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OX}$.

Thus $\overline{OP} = r = x$

or $\quad x = r$ and $y = 0$

Now

$\sin(0^{o}) = \frac{y}{r} = \frac{0}{r} = 0$

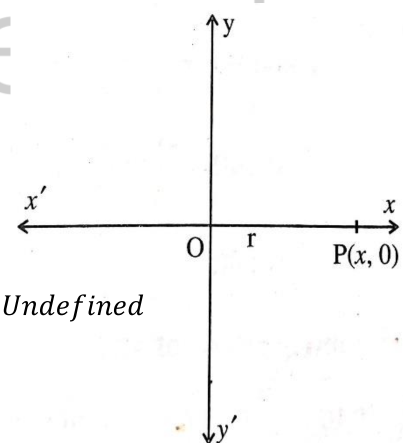
$\operatorname{cosec}(0^{o}) = \frac{r}{y} = \frac{r}{0} = Undefined$

$\cos(0^{o}) = \frac{x}{r} = \frac{r}{r} = 1$

$\sec(0^{o}) = \frac{r}{x} = \frac{r}{r} = 1$

$\tan(0^{o}) = \frac{y}{x} = \frac{0}{r} = 0$

$\cot(0^{o}) = \frac{x}{y} = \frac{r}{0} = Undefined$



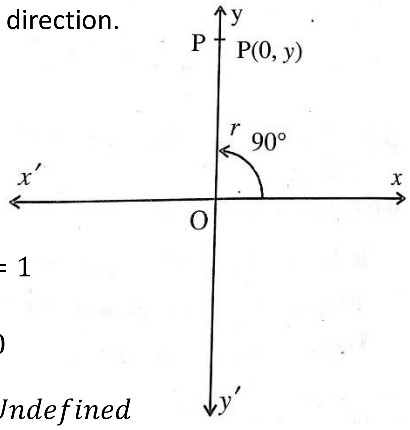
Trigonometric Ratios of 90^{o}

Solution:

To find:

Trigonometric ratios of 90^{o}

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OY}$ in anti – clock wise direction.



Thus $\overline{OP} = r = y$

or $y = r$ and $x = 0$

Now

$\sin(90^o) = \frac{y}{r} = \frac{r}{r} = 1$

$\text{cosec}(90^o) = \frac{r}{y} = \frac{r}{r} = 1$

$\cos(90^o) = \frac{x}{r} = \frac{0}{r} = 0$

$\sec(90^o) = \frac{r}{x} = \frac{r}{0} = Undefined$

$\tan(90^o) = \frac{y}{x} = \frac{r}{0} = Undefined$

$\cot(90^o) = \frac{x}{y} = \frac{0}{r} = 0$

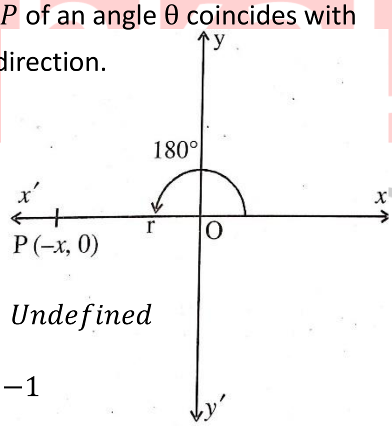
Trigonometric Ratios of 180^0

Solution:

To find:

Trigonometric ratios of 180^o.

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OX'}$ in anti – clock wise direction.



Thus $\overline{OP} = r = -x$

or $x = -r$ and $y = 0$

Now

$\sin(180^o) = \frac{y}{r} = \frac{0}{r} = 0$

$\text{cosec}(180^o) = \frac{r}{y} = \frac{r}{0} = Undefined$

$\cos(180^o) = \frac{x}{r} = \frac{-r}{r} = -1$

$\sec(180^o) = \frac{r}{x} = \frac{r}{-r} = -1$

$\tan(180^o) = \frac{y}{x} = \frac{0}{-r} = 0$

$\cot(180^o) = \frac{x}{y} = \frac{-r}{0} = Undefined$

Trigonometric Ratios of 270^0

Solution:

To find:

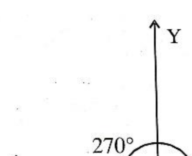
Trigonometric ratios of 270^o

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OY'}$ in anti – clock wise direction.

Thus $\overline{OP} = r = -y$

or $y = -r$ and $x = 0$

Now



$\sin(270^o) = \frac{y}{r} = \frac{-r}{r} = -1$

$\text{cosec}(270^o) = \frac{r}{y} = \frac{r}{-r} = -1$

$\cos(270^o) = \frac{x}{r} = \frac{0}{r} = 0$

$\sec(270^o) = \frac{r}{x} = \frac{r}{0} = Undefined$

$\tan(270^o) = \frac{y}{x} = \frac{-r}{0} = Undefined$

$\cot(270^o) = \frac{x}{y} = \frac{0}{-r} = 0$

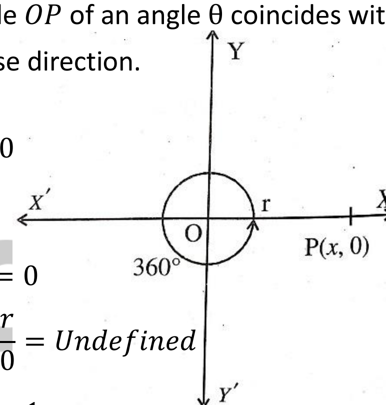
Trigonometric Ratios of 360^0

Solution:

To find:

Trigonometric ratios of 3600^o.

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OX}$ in anti – clock wise direction.



Thus $\overline{OP} = r = x$

or $x = r$ and $y = 0$

Now

$\sin(360^o) = \frac{y}{r} = \frac{0}{r} = 0$

$\text{cosec}(360^o) = \frac{r}{y} = \frac{r}{0} = Undefined$

$\cos(360^o) = \frac{x}{r} = \frac{r}{r} = 1$

$\sec(360^o) = \frac{r}{x} = \frac{r}{r} = 1$

$\tan(360^o) = \frac{y}{x} = \frac{0}{r} = 0$

$\cot(360^o) = \frac{x}{y} = \frac{r}{0} = Undefined$

Exercise # 7.4

Page # 183

Q1: Find the signs of the following trigonometric ratios and tell in which quadrant the lie?

(i) $\sin 98^o$

Solution:

$\sin 98^o$

Since 98^o lies in 2^{nd} Quadrant.

As $\sin\theta$ is positive in 2^{nd} Quadrant.

So $\sin 98^o$ is positive in 2^{nd} Quadrant.

(ii) $\sin 160^o$

Solution:

$\sin 160^o$

Since 160^o lies in 2^{nd} Quadrant.

As $\sin\theta$ is positive in 2nd Quadrant.
So $\sin 160^o$ is positive in 2nd Quadrant.

(iii) $\tan 200^o$

Solution:

$\tan 200^o$

Since 200^o lies in 3rd Quadrant.
As $\tan\theta$ is positive in 3rd Quadrant.
So $\tan 200^o$ is positive in 3rd Quadrant.

(iv) $\sec 120^o$

Solution:

$\sec 120^o$

Since 120^o lies in 2nd Quadrant.
As $\cos\theta$ and $\sec\theta$ are negative in 2nd Quadrant.
So $\sec 120^o$ is negative in 2nd Quadrant.

(v) $\text{cosec}198^o$

Solution:

$\text{cosec}198^o$

Since 198^o lies in 3rdQuadrant.
Assin θ and cosec $\theta$are negative in 3rd Quadrant.
So $\text{cosec}198^o$ is negative in 3rd Quadrant.

(vi) $\sin 460^o$

Solution:

$\sin 460^o$

As $460^o = 360^o + 100^o$
Since 100^o lies in 2nd Quadrant.
As $\sin\theta$ is positive in 2nd Quadrant.
So $\sin 460^o$ is positive in 2nd Quadrant.

Q2: Find the trigonometric ratios of the following angles.

(i) -180^o

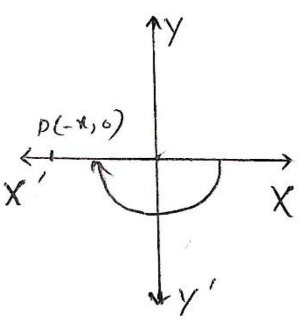
Solution:

-180^o

To find:

Trigonometric ratios of -180^o.

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OX'}$ in clock wise direction.



Thus $\overline{OP} = r = -x$
or $x = -r$ and $y = 0$
Now

$\sin(-180^o) = \frac{y}{r} = \frac{0}{r} = 0$

$\text{cosec}(-180^o) = \frac{r}{y} = \frac{r}{0} = Undefined$

$\cos(-180^o) = \frac{x}{r} = \frac{-r}{r} = -1$

$\sec(-180^o) = \frac{r}{x} = \frac{r}{-r} = -1$

$\tan(-180^o) = \frac{y}{x} = \frac{0}{-r} = 0$

$\cot(-180^o) = \frac{x}{y} = \frac{-r}{0} = Undefined$

(ii) -270^o

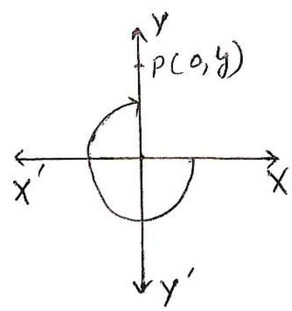
Solution:

-270^o

To find:

Trigonometric ratios of-270^o

Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OY}$ in clock wise direction.



Thus $\overline{OP} = r = y$
or $y = r$ and $x = 0$
Now

$\sin(-270^o) = \frac{y}{r} = \frac{r}{r} = 1$

$\text{cosec}(-270^o) = \frac{r}{y} = \frac{r}{r} = 1$

$\cos(-270^o) = \frac{x}{r} = \frac{0}{r} = 0$

$\sec(-270^o) = \frac{r}{x} = \frac{r}{0} = Undefined$

$\tan(-270^o) = \frac{y}{x} = \frac{r}{0} = Undefined$

$\cot(-270^o) = \frac{x}{y} = \frac{0}{r} = 0$

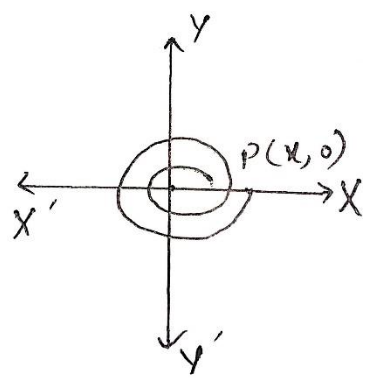
(iii) 720^o

Solution:

720^o

To find:

Trigonometric ratios of 720^o
As 720^o and 0^o have same terminal side.
Here the terminal side $\overline{OP}$ of an angle θ coincides with $\overrightarrow{OX}$ in anti – clock wise direction.



Thus $\overline{OP} = r = x$
or $x = r$ and $y = 0$
Now

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$\sin(720^o) = \frac{y}{r} = \frac{0}{r} = 0$

$\text{cosec}(720^o) = \frac{r}{y} = \frac{r}{0} = Undefined$

$\cos(720^o) = \frac{x}{r} = \frac{r}{r} = 1$

$\sec(720^o) = \frac{r}{x} = \frac{r}{r} = 1$

$\tan(720^o) = \frac{y}{x} = \frac{0}{r} = 0$

$\cot(720^o) = \frac{x}{y} = \frac{r}{0} = Undefined$

(iv) 1470^o

<u>Solution</u>:

1470^o

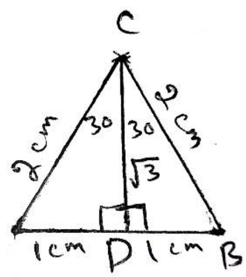
To find:

Trigonometric ratios of 1470^o

As $1470^o = 4(360^o) + 30^o$

So, we take 30^o

As 1470^o and 30^o have same terminal side.



Let an equilateral triangle ABC of sides 2 cm.

Now draw an angle bisector of $\angle C$ which cuts $\overline{AB}$ at point D. Thus, a right-angled triangle ACD is formed.

Hence $m\angle ADC = 90^o$ and $m\angle ACD = 30^o$.

Also $\overline{AC} = 2cm$ and $\overline{AD} = 1cm$.

From the figure

$\overline{AC}$ is hypotenuse

$\overline{AD}$ is perpendicular

$\overline{AC}$ is Base

By Pythagoras Theorem

$(Perp)^2 + (Base)^2 = (Hyp)^2$

$(CD)^2 + (AD)^2 = (AC)^2$

$(CD)^2 + (1)^2 = (2)^2$

$(CD)^2 + 1 = 4$

$(CD)^2 = 4 - 1$

$(CD)^2 = 3$

$\sqrt{(CD)^2} = \sqrt{3}$

$CD = \sqrt{3}$

Now

$\sin(1470^o) = \sin 30^o = \frac{perp}{hyp} = \frac{1}{2}$

$\text{cosec}(1470^o) = cosec\ 30^o = \frac{hyp}{perp} = \frac{2}{1} = 2$

$\cos(1470^o) = \cos 30^o = \frac{base}{hyp} = \frac{\sqrt{3}}{2}$

$\sec(1470^o) = \sec 30^o = \frac{hyp}{base} = \frac{2}{\sqrt{3}}$

$\tan(1470^o) = \tan 30^o = \frac{perp}{base} = \frac{1}{\sqrt{3}}$

$\cot(1470^o) = \cot 30^o = \frac{base}{perp} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Q2: If $\sec\theta = 2$ where θ lies in 4th quadrant, find the other values of trigonometric ratios

<u>Solution</u>:

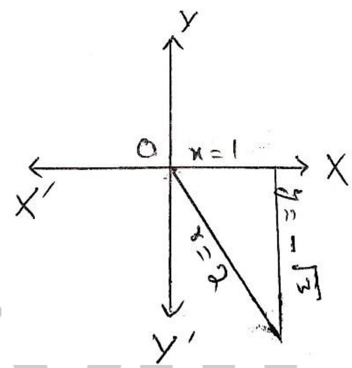
As $\sec\theta = 2$ and θ lies in 4th quadrant.

Here $x = Base, r = hyp, y = perp$

$\sec\theta = \frac{hyp}{base} = \frac{r}{x} = \frac{2}{1}$

$So\ \ r = 2\ and\ x = 1$

Now by Pythagoras Theorem



$(x)^2 + (y)^2 = (r)^2$

$(1)^2 + (y)^2 = (2)^2$

$1 + (y)^2 = 4$

$(y)^2 = 4 - 1$

$(y)^2 = 3$

$\sqrt{(y)^2} = \pm\sqrt{3}$

$y = \pm\sqrt{3}$

As θ lies in 4th quadrant then

$y = -\sqrt{3}$

Now the other Trigonometric Ratios are

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2}$

$\text{cosec}\,\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}}$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$

$\sec\theta = \frac{r}{x} = \frac{2}{1} = 2$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

$\cot\theta = \frac{x}{y} = \frac{1}{-\sqrt{3}}$

Q3: If $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$ then find other trigonometric ratios.

Solution:

$As\ \ \sin\theta = \frac{4}{5}$

$Also\ \ \frac{\pi}{2} < \theta < \pi = 90^o < \theta < 180^o$

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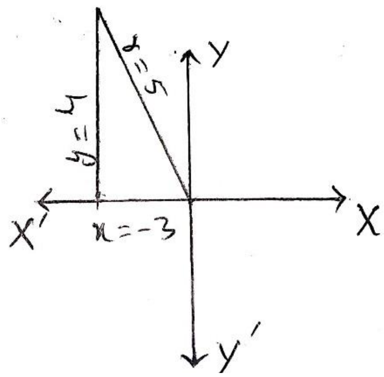
Thus θ lies in 2nd quadrant.

Here $x = Base, r = hyp, y = perp$

$\sin\theta = \frac{perp}{hyp} = \frac{y}{r} = \frac{4}{5}$

$So\ \ y = 4\ and\ r = 5$

Now by Pythagoras Theorem



$(x)^2 + (y)^2 = (r)^2$

$(x)^2 + (4)^2 = (5)^2$

$(x)^2 + 16 = 25$

$(x)^2 = 25 - 16$

$(x)^2 = 9$

$\sqrt{(x)^2} = \pm\sqrt{9}$

$x = \pm 3$

As θ lies in 4th quadrant then

$x = -3$

Now the other Trigonometric Ratios are

$\sin\theta = \frac{y}{r} = \frac{4}{5}$

$\operatorname{cosec}\theta = \frac{r}{y} = \frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5}$

$\sec\theta = \frac{r}{x} = \frac{5}{-3}$

$\tan\theta = \frac{y}{x} = \frac{4}{-3} = -\sqrt{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{4}$

Q5: Find the values of

(i) $2\sin 45^o \cos 45^o$

<u>Solution</u>:

$2\sin 45^o \cos 45^o$

As $\sin 45^o = \frac{1}{\sqrt{2}}$ and $\cos 45^o = \frac{1}{\sqrt{2}}$

Now

$$2\sin 45^o \cos 45^o = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$2\sin 45^o \cos 45^o = 2\left(\frac{1}{\sqrt{2\times 2}}\right)$$

$$2\sin 45^o \cos 45^o = 2\left(\frac{1}{2}\right)$$

$$2\sin 45^o \cos 45^o = 1$$

(ii) $\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o}$

<u>Solution</u>:

$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o}$

As $\tan 30^o = \frac{1}{\sqrt{3}}$ and $\tan 60^o = \sqrt{3}$

Now

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}$$

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{\frac{\sqrt{3}\sqrt{3} - 1}{\sqrt{3}}}{1 + 1}$$

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{\frac{(\sqrt{3})^2 - 1}{\sqrt{3}}}{2}$$

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{3 - 1}{\sqrt{3}} \div 2$$

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{2}{\sqrt{3}} \times \frac{1}{2}$$

$$\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o} = \frac{1}{\sqrt{3}}$$

(iii) $\frac{\cos 45^o}{\sin 45^o + \tan 45^o}$

<u>Solution</u>:

$\frac{\cos 45^o}{\sin 45^o + \tan 45^o}$

As $\sin 45^o = \frac{1}{\sqrt{2}}, \cos 45^o = \frac{1}{\sqrt{2}}$ and $\tan 45^o = 1$

Now

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + 1}$$

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{\frac{1}{\sqrt{2}}}{\frac{1 + \sqrt{2}}{\sqrt{2}}}$$

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{1}{\sqrt{2}} \div \frac{1 + \sqrt{2}}{\sqrt{2}}$$

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1 + \sqrt{2}}$$

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{1}{1 + \sqrt{2}}$$

(iv) $\tan 30^o \tan 60^o + \tan 45^o$

<u>Solution</u>:

$\tan 30^o \tan 60^o + \tan 45^o$

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As $\tan 30^o = \dfrac{1}{\sqrt{3}}, \tan 45^o = 1 \; and \;\; \tan 60^o = \sqrt{3}$

$\tan 30^o \tan 60^o + \tan 45^o = \left(\dfrac{1}{\sqrt{3}}\right)(\sqrt{3}) + 1$

$\tan 30^o \tan 60^o + \tan 45^o = 1 + 1$

$\tan 30^o \tan 60^o + \tan 45^o = 2$

(v) $\boldsymbol{\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6}}$

Solution:

$\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6} - \sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6}$

As $\cos\dfrac{\pi}{3} = \cos 60^o = \dfrac{1}{2}$

$\cos\dfrac{\pi}{6} = \cos 30^o = \dfrac{\sqrt{3}}{2}$

$\sin\dfrac{\pi}{3} = \sin 60^o = \dfrac{\sqrt{3}}{2}$

$\sin\dfrac{\pi}{6} = \sin 30^o = \dfrac{1}{2}$

Now

$\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6} - \sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6} = \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right)$

$\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6} - \sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4}$

$\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6} - \sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6} = 0$

Q6: In which quadrant θ lies?

(i) $\boldsymbol{\sin\theta > 0 \,, \quad \tan\theta > 0}$

Solution:

$\sin\theta > 0 \,, \quad \tan\theta > 0$

As $\sin\theta$ *and* $\tan\theta$ are positive in first quadrant.

Thus θ lies in first quadrant.

(ii) $\boldsymbol{\sin\theta < 0 \,, \quad \cot\theta > 0}$

Solution:

$\sin\theta < 0 \,, \quad \cot\theta > 0$

As $\cot\theta$ is positive and $\sin\theta$ is negative in 3rd quadrant.

Thus θ lies in 3rd quadrant.

(iii) $\boldsymbol{\sin\theta > 0 \,, \quad \cos\theta < 0}$

Solution:

$\sin\theta > 0 \,, \quad \cos\theta < 0$

As $\sin\theta$ is positive and $\cos\theta$ is negative in 2nd quadrant.

Thus θ lies in 3rd quadrant.

(iv) $\boldsymbol{\cos\theta > 0 \,, \quad \mathrm{cosec}\,\theta < 0}$

Solution:

$\cos\theta > 0 \,, \quad \mathrm{cosec}\,\theta < 0$

As $\cos\theta$ is positive and $\mathrm{cosec}\,\theta$ is negative in 4th quadrant.

Thus θ lies in 4th quadrant.

(v) $\boldsymbol{\tan\theta < 0 \,, \quad \sec\theta > 0}$

Solution:

$\tan\theta < 0 \,, \quad \sec\theta > 0$

As $\sec\theta$ is positive and $\tan\theta$ is negative in 4th quadrant.

Thus θ lies in 4th quadrant.

(vi) $\boldsymbol{\cos\theta < 0 \,, \quad \tan\theta < 0}$

Solution:

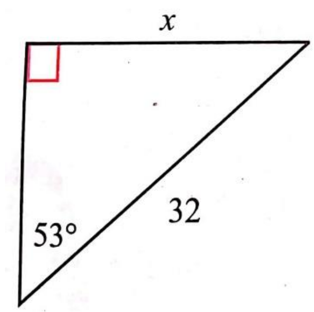
$\cos\theta < 0 \,, \quad \tan\theta < 0$

As $\cos\theta$ *and* $\tan\theta$ are negative in 2nd quadrant.

Thus θ lies in 2nd quadrant.

Q7: For each triangle, find each missing measure to two decimal places.

(i)



Solution:

To Find:

$Perpendicular = r = ?$

$As \; \sin\theta = \dfrac{perp}{hyp}$

$\sin 53^o = \dfrac{x}{32}$

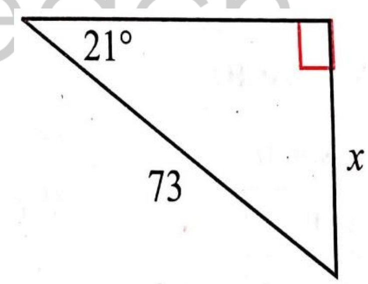
$0.7986 = \dfrac{x}{32}$

$0.7986 \times 32 = x$

$25.56 = x$

$x = 25.56$

(ii)



Solution:

To Find:

$Perpendicular = r = ?$

$As \; \sin\theta = \dfrac{perp}{hyp}$

$\sin 21^o = \dfrac{x}{73}$

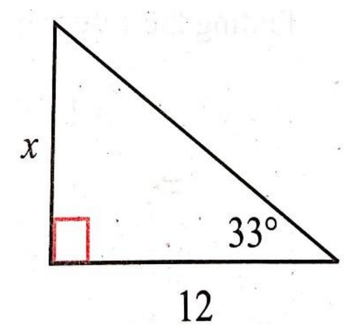
$0.3584 = \dfrac{x}{73}$

$0.3584 \times 73 = x$

$26.1632 = x$

$x = 26.1632$

(iii)



Solution:

To Find:

$Perpendicular = r = ?$

$As \; \tan\theta = \dfrac{perp}{base}$

$\tan 33^o = \dfrac{x}{12}$

$0.6494 = \dfrac{x}{12}$

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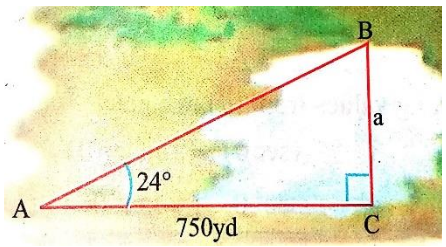
$0.6494 \times 12 = x$
$7.7928 = x$
$x = 7.7928$

Q8: The irregular blue shape in the diagram represents a lake. The distance across the lake "a" is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

Solution:
To Find:
$Perpendicular = r = ?$

$As\ \tan\theta = \frac{perp}{base}$



$\tan 24^o = \frac{a}{750}$

$0.4452 = \frac{a}{750}$

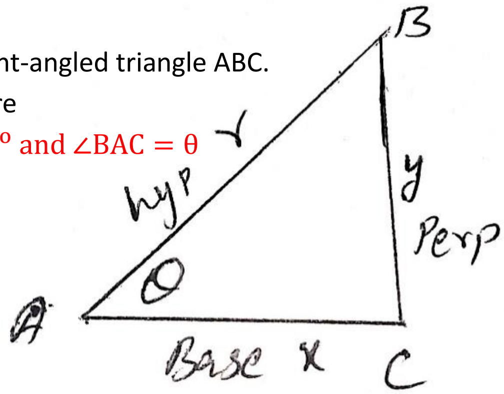
$0.4452 \times 750 = a$
$333.9 = a$
$a = 333.9$

Ex # 7.5

Trigonometric Identities
Following are the fundamental trigonometric identities.

- $sin^2\theta + cos^2\theta = 1$
- $1 + \tan^2\theta = sec^2\theta$
- $1 + \cot^2\theta = cosec^2\theta$

Proof:
Consider a right-angled triangle ABC.
From the figure



$m\angle ACB = 90^o$ and $\angle BAC = \theta$

$\sin\theta = \frac{y}{r}$

$\mathrm{cosec}\,\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$

$\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

Now by Pythagoras theorem
$x^2 + y^2 = r^2 \ldots . equ\ (i)$
Divide equ (i) by r^2

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

Put the values
$(\cos\theta)^2 + (\sin\theta)^2 = 1$
$\boldsymbol{cos^2\theta + sin^2\theta = 1}$

Divide equ (i) by x^2

$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$

$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$

Put the values
$1 + (\tan\theta)^2 = (\sec\theta)^2$
$\boldsymbol{1 + \tan^2\theta = \sec^2\theta}$

Divide equ (i) by y^2

$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$

$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2$

Put the values
$(\cot\theta)^2 + 1 = (\mathrm{cosec}\,\theta)^2$
$\cot^2\theta + 1 = \mathrm{cosec}^2\theta$
$\boldsymbol{1 + \cot^2\theta = \mathrm{cosec}^2\theta}$

Trigonometric Ratios Memorization

$\sin\theta = \frac{1}{cosec\ \theta}$ or $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{sec\ \theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{cot\ \theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

1. $cos^2\theta + sin^2\theta = 1$
   $cos^2\theta = 1 - sin^2\theta$
   $sin^2\theta = 1 - cos^2\theta$
2. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$
   $\cot^2\theta = \mathrm{cosec}^2\theta - 1$
3. $1 + \tan^2\theta = \sec^2\theta$
   $\tan^2\theta = \sec^2\theta - 1$

Example # 16

Show that $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Solution:
$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$
L.H.S
$(\sin\theta + \cos\theta)^2$
$= sin^2\theta + cos^2\theta + 2\,sin\,\theta\,cos\,\theta$
$\boldsymbol{As\ \ sin^2\theta + cos^2\theta = 1}$
$= sin^2\theta + cos^2\theta + 2\,sin\,\theta\,cos\,\theta$
$= 1 + 2\,sin\,\theta\,cos\,\theta$

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$= \mathbf{R.H.S}$

Hence proved

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Prove that $\boldsymbol{\sin\theta = \sqrt{1-\cos^2\theta}}$

Solution:

$\sin\theta = \sqrt{1-\cos^2\theta}$

R.H.S

$\sqrt{1-\cos^2\theta}$

As $\boldsymbol{1 - cos^2\theta = \sin^2\theta}$

$= \sqrt{sin^2\theta}$

$= \sin\theta$

$= \mathbf{L.H.S}$

Hence proved

$\sin\theta = \sqrt{1-\cos^2\theta}$

Example # 17

Prove that $\boldsymbol{\sec^2\theta + \tan^2\theta = \dfrac{1+\sin^2\theta}{1-\sin^2\theta}}$

Solution:

$\sec^2\theta + \tan^2\theta = \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

L.H.S

$\sec^2\theta + \tan^2\theta$

As $\boldsymbol{\sec\theta = \dfrac{1}{\cos\theta}}$ **And** $\boldsymbol{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{1}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1+\sin^2\theta}{\cos^2\theta}$

As $\boldsymbol{\cos^2\theta = 1 - \sin^2\theta}$

$= \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

$= \mathbf{R.H.S}$

Hence proved

$\sec^2\theta + \tan^2\theta = \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

Example # 18

Prove that $\boldsymbol{\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot\theta}$

Solution:

$\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot\theta$

L.H.S

$\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

As $\boldsymbol{1 - \sin^2\theta = cos^2\theta}$

$= \dfrac{\sqrt{cos^2\theta}}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= \mathbf{L.H.S}$

Hence proved

$\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot\theta$

Exercise # 7.5

Page # 186

Prove the following trigonometric identities.

1. $\boldsymbol{(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta}$

Solution:

$(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

L.H.S

$(\sec^2\theta - 1)\cos^2\theta$

As $\boldsymbol{\sec^2\theta - 1 = \tan^2\theta}$

$= (\tan^2\theta)\cos^2\theta$

$= \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$

$= \sin^2\theta$

$= \mathbf{R.H.S}$

Hence proved

$(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

---

2. $\boldsymbol{\tan\theta + \sec\theta = \dfrac{1+\sin\theta}{\cos\theta}}$

Solution:

$\tan\theta + \sec\theta = \dfrac{1+\sin\theta}{\cos\theta}$

L.H.S

$\tan\theta + \sec\theta$

As $\boldsymbol{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$ **And** $\boldsymbol{\sec\theta = \dfrac{1}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$

$= \dfrac{\sin\theta + 1}{\cos\theta}$

$\dfrac{1+\sin\theta}{\cos\theta}$

$= \mathbf{R.H.S}$

Hence proved

$\tan\theta + \sec\theta = \dfrac{1+\sin\theta}{\cos\theta}$

---

3. $\boldsymbol{(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta}$

Solution:

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$(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$

L.H.S

$(\cos\theta - \sin\theta)^2$

$= \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

As $\cos^2\theta + \sin^2\theta = 1$

$= 1 - 2\cos\theta\sin\theta$

$=$ **R.H.S**

Hence proved

$(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$

---

4. $\mathbf{\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1}$

Solution:

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

L.H.S

$\cos^2\theta - \sin^2\theta$

As $\sin^2\theta = 1 - \cos^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= \cos^2\theta - 1 + \cos^2\theta$

$= \cos^2\theta . \cos^2\theta - 1$

$= 2\cos^2\theta - 1$

$=$ **R.H.S**

Hence proved

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

---

5. $\mathbf{\tan\theta + \cot = \sec\theta \operatorname{cosec}\theta}$

Solution:

$\tan\theta + \cot = \sec\theta \operatorname{cosec}\theta$

L.H.S

$\tan\theta + \cot$

As $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **And** $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta\sin\theta + \cos\theta\cos\theta}{\cos\theta\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

As $\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{1}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta} . \dfrac{1}{\sin\theta}$

As $\dfrac{1}{\cos\theta} = \sec\theta$ **And** $\dfrac{1}{\sin\theta} = \operatorname{cosec}\theta$

$= \sec\theta \operatorname{cosec}\theta$

$=$ **R.H.S**

Hence proved

$\tan\theta + \cot = \sec\theta \operatorname{cosec}\theta$

---

6. $\mathbf{\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}}$

Solution:

$\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}$

L.H.S

$\dfrac{1 - \sin\theta}{\cos\theta}$

Multilply and divide by $1 + \sin\theta$

$= \dfrac{1 - \sin\theta}{\cos\theta} \times \dfrac{1 + \sin\theta}{1 + \sin\theta}$

$= \dfrac{1^2 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

As $1 - \sin^2\theta = \cos^2\theta$

$= \dfrac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{\cos\theta}{1 + \sin\theta}$

$=$ **R.H.S**

Hence proved

$\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}$

---

7. $\mathbf{\sin\theta\sqrt{1 + \tan^2\theta} = \tan\theta}$

Solution:

$\sin\theta\sqrt{1 + \tan^2\theta} = \tan\theta$

L.H.S

$\sin\theta\sqrt{1 + \tan^2\theta}$

As $1 + \tan^2\theta = \sec^2\theta$

$= \sin\theta\sqrt{\sec^2\theta}$

$= \sin\theta\sec\theta$

$= \sin\theta \times \dfrac{1}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ **R.H.S**

Hence proved

$\sin\theta\sqrt{1 + \tan^2\theta} = \tan\theta$

---

8. $\mathbf{\cos\theta = \sqrt{1 - \sin^2\theta}}$

Solution:

$\cos\theta = \sqrt{1 - \sin^2\theta}$

R.H.S

$\sqrt{1 - \sin^2\theta}$

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As $\mathbf{1-\sin^2\theta=\cos^2\theta}$

$=\sqrt{\cos^2\theta}$

$=\cos\theta$

$=\mathbf{L.H.S}$

Hence proved

$\cos\theta=\sqrt{1-\sin^2\theta}$

---

9. $\mathbf{(1+\cos\theta)(1-\cos\theta)=\dfrac{1}{\mathrm{cosec}^2\theta}}$

Solution:

$(1+\cos\theta)(1-\cos\theta)=\dfrac{1}{\mathrm{cosec}^2\theta}$

L.H.S

$(1+\cos\theta)(1-\cos\theta)$

$=1^2-\cos^2\theta$

$=1-\cos^2\theta$

As $\mathbf{1-\cos^2\theta=\sin^2\theta}$

$=\sin^2\theta$

As $\mathbf{\sin^2\theta=\dfrac{1}{\mathrm{cosec}^2\theta}}$

$=\dfrac{1}{\mathrm{cosec}^2\theta}$

$=\mathbf{R.H.S}$

Hence proved

$(1+\cos\theta)(1-\cos\theta)=\dfrac{1}{\mathrm{cosec}^2\theta}$

---

10. $\mathbf{\cos x-\cos x\sin^2 x=\cos^3 x}$

Solution:

$\cos x-\cos x\sin^2 x=\cos^3 x$

L.H.S

$\cos x-\cos x\sin^2 x$

$=\cos x(1-\sin^2 x)$

As $\mathbf{1-\sin^2 x=\cos^2 x}$

$=\cos x(\cos^2 x)$

$=\cos^3 x$

$=\mathbf{R.H.S}$

Hence proved

$\cos x-\cos x\sin^2 x=\cos^3 x$

---

11. $\mathbf{\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\mathrm{cosec}\, x}$

Solution:

$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\mathrm{cosec}\, x$

L.H.S

$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$

$=\dfrac{\sin x.\sin x+(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$

$=\dfrac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$

$=\dfrac{\sin^2 x+1+\cos^2 x+2(1)(\cos x)}{\sin x(1+\cos x)}$

$=\dfrac{\sin^2 x+1+\cos^2 x+2\cos x}{\sin x(1+\cos x)}$

$=\dfrac{\sin^2 x+\cos^2 x+1+2\cos x}{\sin x(1+\cos x)}$

As $\mathbf{\sin^2 x+\cos^2 x=1}$

$=\dfrac{1+1+2\cos x}{\sin x(1+\cos x)}$

$=\dfrac{2+2\cos x}{\sin x(1+\cos x)}$

$=\dfrac{2(1+\cos x)}{\sin x(1+\cos x)}$

$=\dfrac{2}{\sin x}$

$=2\times\dfrac{1}{\sin x}$

As $\mathbf{\dfrac{1}{\sin x}=\mathrm{cosec}\, x}$

$=2\mathrm{cosec}\, x$

$=\mathbf{R.H.S}$

Hence proved

$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\mathrm{cosec}\, x$

---

12. $\mathbf{\dfrac{\sin x}{1+\cos x}=\dfrac{1-\cos x}{\sin x}}$

Solution:

$\dfrac{\sin x}{1+\cos x}=\dfrac{1-\cos x}{\sin x}$

L.H.S

$\dfrac{\sin x}{1+\cos x}$

Multilply and divide by $\mathbf{1-\cos x}$

$=\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}$

$=\dfrac{\sin x(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$=\dfrac{\sin x(1-\cos x)}{1-\cos^2\theta}$

As $\mathbf{1-\cos^2\theta=\sin^2\theta}$

$=\dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$=\dfrac{\sin x(1-\cos x)}{\sin x.\sin x}$

$=\dfrac{1-\cos x}{\sin x}$

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$= \mathbf{R.H.S}$

Hence proved

$\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

13. $\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = 2 + 2\cot^2 a$

Solution:

$\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = 2 + 2\cot^2 a$

L.H.S

$\frac{1}{1+\cos a} + \frac{1}{1-\cos a}$

$= \frac{1(1-\cos a) + 1(1+\cos a)}{(1+\cos a)(1-\cos a)}$

$= \frac{1+1-\cos a+\cos a}{1-\cos^2 a}$

As $\mathbf{1-\cos^2 a = \sin^2 a}$

$= \frac{2}{\sin^2 a}$

$= 2 \times \frac{1}{\sin^2 a}$

As $\mathbf{\frac{1}{\sin^2 a} = cosec^2 a}$

$= 2\,cosec^2 a$

As $\mathbf{cosec^2 a = 1 + cot^2 a}$

$= 2(1 + cot^2 a)$

$= 2 + 2\,cot^2 a$

$= \mathbf{R.H.S}$

Hence proved

$\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = 2 + 2\cot^2 a$

14. $\mathbf{\cos^4 b - \sin^4 b = 1 - 2\sin^2 b}$

Solution:

$\cos^4 b - \sin^4 b = 1 - 2\sin^2 b$

L.H.S

$\cos^4 b - \sin^4 b$

$= \cos^4 b - \sin^4 b$

$= (\cos^2 b)^2 - (\sin^2 b)^2$

As $\mathbf{(a)^2 - (b)^2 = (a+b)(a-b)}$

$= (\cos^2 b + \sin^2 b)(\cos^2 b - \sin^2 b)$

As $\mathbf{\cos^2 b + \sin^2 b = 1}$

$= (1)(\cos^2 b - \sin^2 b)$

$= \cos^2 b - \sin^2 b$

As $\mathbf{\cos^2 b = 1 - \sin^2 b}$

$= 1 - \sin^2 b - \sin^2 b$

$= 1 - 2\sin^2 b$

$= \mathbf{R.H.S}$

Hence proved

$\cos^4 b - \sin^4 b = 1 - 2\sin^2 b$

15. $\mathbf{\frac{\sin y + \cos y}{\sin y} - \frac{\cos y - \sin y}{\cos y} = \sec y\, cosec\, y}$

Solution:

$\frac{\sin y + \cos y}{\sin y} - \frac{\cos y - \sin y}{\cos y} = \sec y\, cosec\, y$

L.H.S

$\frac{\sin y + \cos y}{\sin y} - \frac{\cos y - \sin y}{\cos y}$

$= \frac{\cos y(\sin y + \cos y) - \sin y(\cos y - \sin y)}{\sin y \cos y}$

$= \frac{\cos y \sin y + \cos^2 y - \sin y \cos y + \sin^2 y}{\sin y \cos y}$

$= \frac{\cos y \sin y - \sin y \cos y + \cos^2 y + \sin^2 y}{\sin y \cos y}$

As $\mathbf{\cos^2 y + \sin^2 y = 1}$

$= \frac{1}{\sin y \cos y}$

$= \frac{1}{\sin y} \cdot \frac{1}{\cos y}$

As $\mathbf{\frac{1}{\sin y} = cosec\, y}$ **And** $\mathbf{\frac{1}{\cos y} = \sec y}$

$= cosec\, y \sec y$

$= \sec y\, cosec\, y$

$= \mathbf{R.H.S}$

Hence proved

$\frac{\sin y + \cos y}{\sin y} - \frac{\cos y - \sin y}{\cos y} = \sec y\, cosec\, y$

16. $\mathbf{(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}}$

Solution:

$(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

L.H.S

$(\sec x - \tan x)^2$

$= (\sec x - \tan x)^2$

As $\mathbf{\sec x = \frac{1}{\cos x}}$ **And** $\mathbf{\tan x = \frac{\sin x}{\cos x}}$

$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$

$= \left(\frac{1-\sin x}{\cos x}\right)^2$

$= \frac{(1-\sin x)^2}{\cos^2 x}$

As $\mathbf{\cos^2 x = 1 - \sin^2 x}$

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$$= \frac{(1-\sin x)^2}{1-\sin^2 x}$$
$$= \frac{(1-\sin x)^2}{1^2-\sin^2 x}$$
$$= \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}$$
$$= \frac{(1-\sin x)}{(1+\sin x)}$$
$= \mathbf{R.H.S}$

Hence proved

$$(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$$

---

17. $\sin x \tan x + \cos x = \sec x$

Solution:

$\sin x \tan x + \cos x = \sec x$

L.H.S

$\sin x \tan x + \cos x$

$$As \ \tan x = \frac{\sin x}{\cos x}$$
$$= \sin x \frac{\sin x}{\cos x} + \cos x$$
$$= \frac{\sin^2 x}{\cos x} + \cos x$$
$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$
$$As \ \sin^2 x + \cos^2 x = 1$$
$$= \frac{1}{\cos x}$$
$$As \ \frac{1}{\cos x} = \sec x$$
$= \sec x$

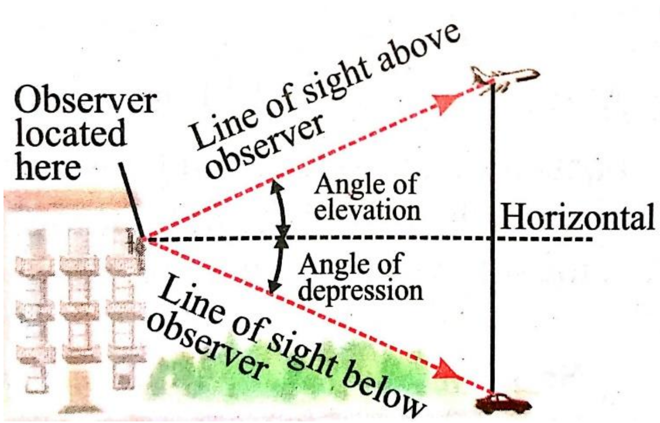
$= \mathbf{R.H.S}$

Hence proved

$\sin x \tan x + \cos x = \sec x$

Ex # 7.6

Angle of Elevation and Depression



Example # 20

An aerial photographer who photographs a farm house for a company has determined from experience that the best photo is taken at a height of approximately 475 ft and a distance of 850 ft from the farmhouse. What is the angle of depression from the plane to the house?

Solution:

From the figure

Let the height of aerial photographer $= m\overline{AC} = 475\ ft$

And distance from the farmhouse $= m\overline{AB} = 850\ ft$

To Find:

Angle of Depression $= \theta = ?$

Thus $\angle ABC = \theta = ?$ (Alternate angle)

$$\sin\theta = \frac{perp}{hyp}$$
$$\sin\theta = \frac{475}{850}$$
$\sin\theta = 0.5588$

$\theta = \sin^{-1} 0.5\,588$

$\theta = 33.97^o$

$\theta = 34^o$

Example # 21: To measure cloud height at night, a vertical beam of light is directed on a spot on the cloud. From a point 135 ft away from the light source, the angle of elevation to the spot is found to be 67.35°. Find the height of the cloud.

Solution:

From the figure

Let the distance between point & *light source* $= 135\ ft$

And Angle of elevation $= 67.35^o$

To Find:

Height of cloud $= h = ?$

As we have

$$\tan\theta = \frac{perp}{base}$$
$$\tan 6735^o = \frac{h}{135}$$
$$2.396 = \frac{h}{135}$$
$2.396 \times 135 = h$

$323.46 = h$

$h = 323.46$

Thus

Height of cloud $= 323.46\ ft$

Example # 22: A light house is 300 m above the sea level. Angles of depression of two boats from the top of light house are 30° and 45° respectively. If line joining

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the boats passes through the foot of light house. Find the distance between the boats when they are on the same side of the light house.

Solution:

From the figure

Let the height of light house $= m\overline{AB} = 300\ m$

Let Boats are point C and D

As angle of depressions are 30^o and 45^o

As $\angle EBD = 30^o$ *Then* $\angle BDA = 30^o$ (Alternate angle)

As $\angle EBC = 45^o$ *Then* $\angle BCA = 45^o$ (Alternate angle)

To Find:

Distance between two boats $= m\overline{CD} = ?$

Now

From Right angled ΔABD

$$\tan\theta = \frac{perp}{base}$$

$$\tan 30^o = \frac{m\overline{AB}}{m\overline{AD}}$$

$$\frac{1}{\sqrt{3}} = \frac{300}{m\overline{AD}}$$

$$m\overline{AD} = 300\sqrt{3}$$

From Right angled ΔABC

$$\tan\theta = \frac{perp}{base}$$

$$\tan 45^o = \frac{m\overline{AB}}{m\overline{AC}}$$

$$1 = \frac{300}{m\overline{AC}}$$

$$m\overline{AC} = 300$$

As we have

$$m\overline{AC} + m\overline{CD} = m\overline{AD}$$

As $m\overline{AD} = 300\sqrt{3}$ and $m\overline{AC} = 300$

So

$$300 + m\overline{CD} = 300\sqrt{3}$$

$$m\overline{CD} = 300\sqrt{3} - 300$$

$$m\overline{CD} = 300\left(\sqrt{3} - 1\right)$$

Thus

Distance between two boats $= 300\left(\sqrt{3} - 1\right) m$

Exercise # 7.6

Page # 189

Q1: A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to nearest degree.

Solution:

From the figure

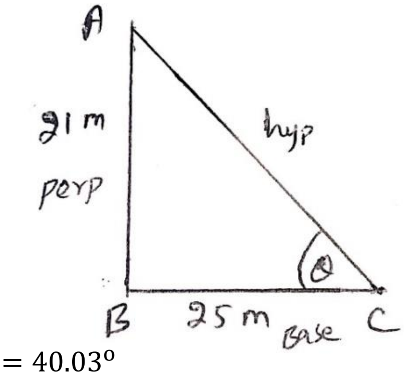
Let the height of building $= m\overline{AB} = 21m$

And shadow of building $= m\overline{BC} = 25m$

To Find:

Angle of Elevation $= \angle ACB = \theta = ?$

As we have



$$\tan\theta = \frac{perp}{base}$$

$$\tan\theta = \frac{21}{25}$$

$$\tan\theta = 0.84$$

$$\theta = \tan^{-1} 0.84$$

$$\theta = 40.03^o$$

Thus

Angle of Elevation $= 40.03^o$

Q2: A light house is 150 m above the sea level. Angle of depression of a boat from its top is 60°. find the distance between the boat and the lighthouse.

Solution:

From the figure

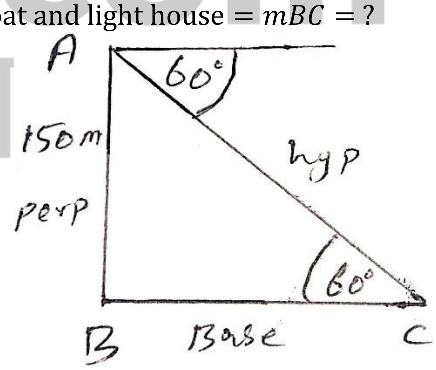
Let the height of light house $= m\overline{AB} = 150\ m$

And Angle of depression $= 60^o$

To Find:

Distance b/w boat and light house $= m\overline{BC} = ?$

As we have



$$\tan\theta = \frac{perp}{base}$$

$$\tan 60^o = \frac{m\overline{AB}}{m\overline{BC}}$$

$$\sqrt{3} = \frac{150}{m\overline{BC}}$$

$$m\overline{BC} = \frac{150}{\sqrt{3}}$$

Multiply and divide by $\sqrt{3}$

$$m\overline{BC} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$m\overline{BC} = \frac{150\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$m\overline{BC} = \frac{150\sqrt{3}}{3}$$

$$m\overline{BC} = 50\sqrt{3}$$

Thus

Distance b/w boat and light house $= 50\sqrt{3}\ m$

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Unit # 7

Q3: A tree is 50 m high. Find the angle of elevation of its top to a point, on the ground 100 m away from the foot of a tree.

Solution:

From the figure

Let the height of tree $= m\overline{AB} = 50\ m$

And distance between a point and tree $= m\overline{BC} = 100\ m$

To Find:

Angle of Elevation $= \angle ACB = \theta = ?$

As we have

$\tan\theta = \dfrac{perp}{base}$

$\tan\theta = \dfrac{50}{100}$

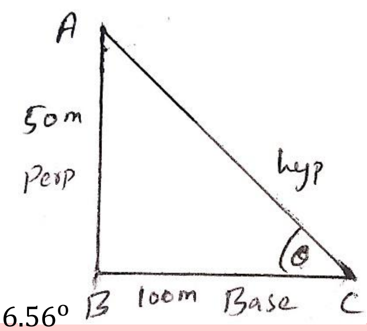
$\tan\theta = 0.5$

$\theta = \tan^{-1} 0.5$

$\theta = 26.56^o$

Thus

Angle of Elevation $= 26.56^o$



Q4: From top of hill 240 m high, measure of angles of depression of top and bottom of minaret are 30° and 60° respectively. Find height of minaret.

Solution:

From the figure

Let the height of hill $= m\overline{AB} = 240\ m$

As $\angle DAE = 30^o$

Also $\angle DAC = 60^o$

Also $\angle ACB = 60^o$ (Alternate angle)

To Find:

Height of minaret $= m\overline{CE} = ?$

Now

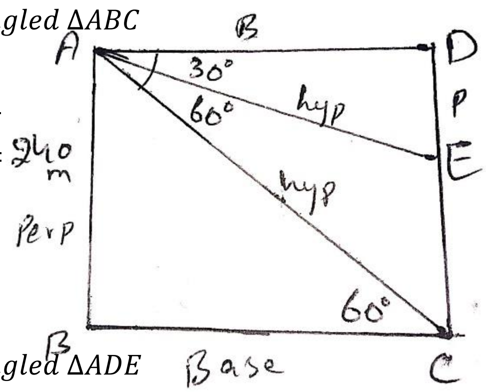
From Right angled ΔABC

$\tan\theta = \dfrac{perp}{base}$

$\tan 60^o = \dfrac{m\overline{AB}}{m\overline{BC}}$

$\sqrt{3} = \dfrac{240}{m\overline{BC}}$

$m\overline{BC} = \dfrac{240}{\sqrt{3}}$



From Right angled ΔADE

$\tan\theta = \dfrac{perp}{base}$

$\tan 30^o = \dfrac{m\overline{DE}}{m\overline{AD}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{m\overline{DE}}{m\overline{AD}}$

$\dfrac{1}{\sqrt{3}} m\overline{AD} = m\overline{DE}$

$\boldsymbol{As\ m\overline{AD} = m\overline{BC} = \dfrac{240}{\sqrt{3}}}$

$\dfrac{1}{\sqrt{3}} \times \dfrac{240}{\sqrt{3}} = m\overline{DE}$

$\dfrac{240}{\left(\sqrt{3}\right)^2} = m\overline{DE}$

$\dfrac{240}{3} = m\overline{DE}$

$80 = m\overline{DE}$

$m\overline{DE} = 80$

As we have

$m\overline{DC} = m\overline{DE} + m\overline{EC}$

As $m\overline{DC} = m\overline{AB} = 240$ and $m\overline{DE} = 80$

So

$240 = 80 + m\overline{EC}$

$240 - 80 = m\overline{EC}$

$160 = m\overline{EC}$

$m\overline{EC} = 160$

Thus

Height of minaret $= 160\ m$

Q5: A Police helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of 72°. Find the distance of stolen car, to the nearest foot, from a point directly below the helicopter.

Solution:

From the figure

Let the height of helicopter $= m\overline{AB} = 800\ ft$

And Angle of depression $= 72^o$

To Find:

Distance from foot of helicopter to car $= m\overline{BC} = ?$

As we have

$\tan\theta = \dfrac{perp}{base}$

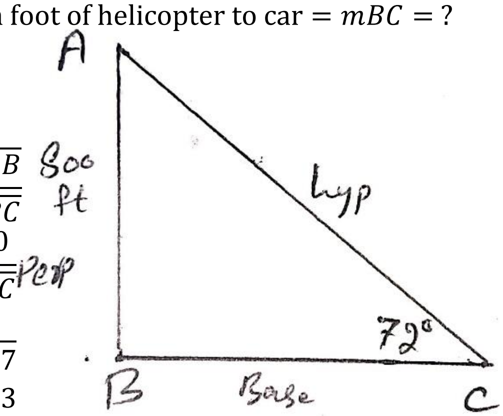
$\tan 72^o = \dfrac{m\overline{AB}}{m\overline{BC}}$

$3.0777 = \dfrac{800}{m\overline{BC}}$

$m\overline{BC} = \dfrac{800}{3.0777}$

$m\overline{BC} = 259.93$

Thus



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Distance from foot of helicopter to car $= 259.93\ ft$

Q6: A light house is 300 m above the sea level. The angle of depression of two boats from the top light house are 30° and 45° respectively. If the line of joining of the boats passes through the foot of light house. Find the distance between two boats when they are on the opposite side of light house.

Solution:

From the figure

Let the height of light house $= m\overline{AB} = 300\ m$

As angle of depressions are 30^o and 45^o

Thus $\angle ACB = 30^o$ (Alternate angle)

Also $\angle ADB = 45^o$ (Alternate angle)

To Find:

Distance between two boats $= m\overline{CD} = ?$

Now

From Right angled ΔABC

$$\tan\theta = \frac{perp}{base}$$

$$\tan 30^o = \frac{m\overline{AB}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{300}{m\overline{BC}}$$

$$m\overline{BC} = 300\sqrt{3}$$

From Right ΔABD

$$\tan\theta = \frac{perp}{base}$$

$$\tan 45^o = \frac{m\overline{AB}}{m\overline{BD}}$$

$$1 = \frac{300}{m\overline{BD}}$$

$$m\overline{BD} = 300$$

As we have

$$m\overline{CD} = m\overline{CB} + m\overline{BD}$$

As $m\overline{CB} = 300\sqrt{3}$ and $m\overline{BD} = 300$

So

$$m\overline{CD} = 300\sqrt{3} + 300$$

$$m\overline{CD} = 300(\sqrt{3}+1)$$

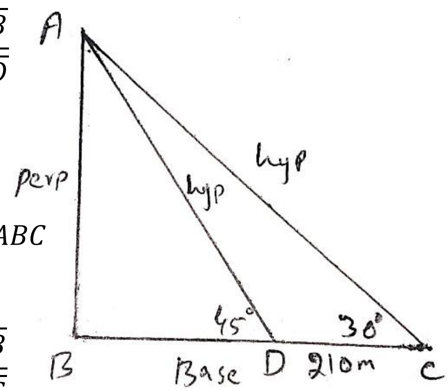
Thus

Distance between two boats $= 300(\sqrt{3}+1)\ m$

Q7: The angle of elevation of the top of a cliff is 30°. Walking 210 meter from the point towards the cliff, the angle of elevation becomes 45°. Find the height of the cliff.

Solution:

From the figure



Angle of elevation at C $= 30^o$

Angle of elevation at D $= 45^o$

Distance between D & C $= m\overline{CD} = 210\ m$

And $m\overline{BC} = m\overline{BD} + m\overline{DC}$

To Find:

Height of cliff $= m\overline{AB} = ?$

Now

From Right angled ΔABD

$$\tan\theta = \frac{perp}{base}$$

$$\tan 45^o = \frac{m\overline{AB}}{m\overline{BD}}$$

$$1 = \frac{m\overline{AB}}{m\overline{BD}}$$

$$m\overline{BD} = m\overline{AB}$$

From Right ΔABC

$$\tan\theta = \frac{perp}{base}$$

$$\tan 30^o = \frac{m\overline{AB}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{m\overline{AB}}{m\overline{BD} + m\overline{DC}} \quad \therefore m\overline{BC} = m\overline{BD} + m\overline{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{m\overline{AB}}{m\overline{AB} + 210} \quad \therefore m\overline{BD} = m\overline{AB}$$

By Cross Multiplication

$$m\overline{AB} + 210 = m\overline{AB}\sqrt{3}$$

$$210 = m\overline{AB}\sqrt{3} - m\overline{AB}$$

$$m\overline{AB}\sqrt{3} - m\overline{AB} = 210$$

$$m\overline{AB}(\sqrt{3}-1) = 210$$

$$m\overline{AB} = \frac{210}{(\sqrt{3}-1)}$$

Thus

Height of cliff $= \frac{210}{(\sqrt{3}-1)}\ m$

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MATHEMATICS

Class 10th

Unit # 13 PRACTICAL GEOMETRY CIRCLE

NAME: ______________________________

F.NAME: ______________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

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UNIT # 13

PRACTICAL GEOMETRY CIRCLE

<u>Circumscribe</u>

Above the figure

<u>Inscribe</u>

Between or in the figure

<u>Circumcircle or Circumscribe a circle</u>

The circle passes through the vertices of polygon (triangle, square, hexagon).

<u>Incircle or inscribe a circle</u>

The circle which touches the sides of polygon (triangle, square, hexagon)

<u>Escribed circle</u>

The circle touching one side of the triangle externally and two produced sides internally is called escribed circle.

<u>Ex # 13.1</u>

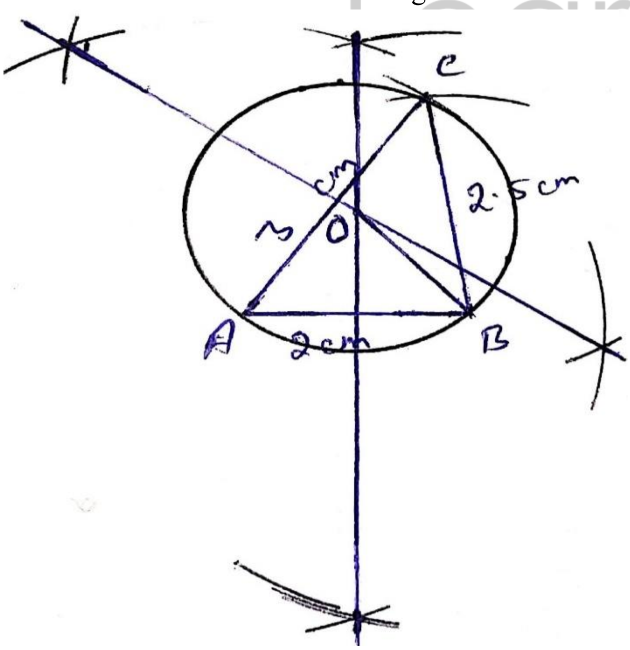
1. Construct a triangle with sides 2 cm, 2.5 cm and 3 cm. also draw its circumcircle.

Given

Let $m\,\overline{AB} = 2\,cm,\ m\,\overline{BC} = 2.5\,cm,$ and $m\,\overline{AC} = 3\,cm$

Required

To circumscribe a circle about the given $\Delta\,ABC$



Steps of construction

1. Draw a line $m\,\overline{AB} = 2\,cm$
2. With B as centre, draw an arc of radius 2.5 cm.
3. With A as centre, draw another arc of radius 3 cm.
4. Both the arcs meet at point C.
5. Thus, ABC is the triangle according to data
6. Draw the perpendicular bisectors of $\overline{AB}$ and $m\overline{AC}$ which intersect each other at O.
7. Join O to B.
8. With centre O and radius $m\,\overline{OB}$ draw a circle.
9. This is the required circumscribed circle about the given $\Delta\,ABC$

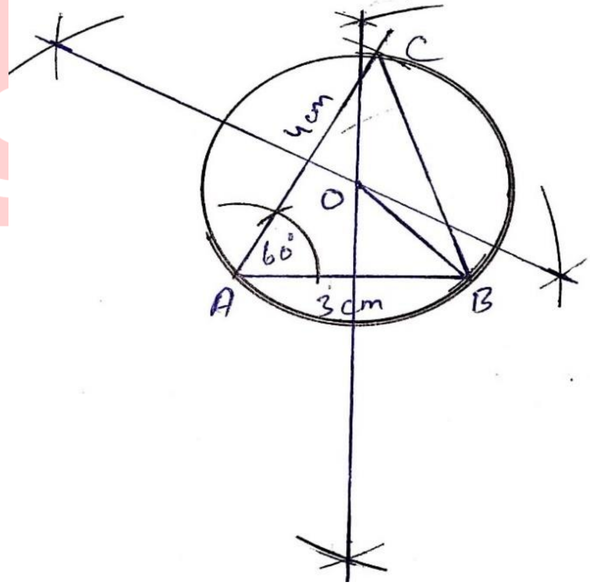
2. Construct a triangle ABC such that $m\overline{AB} = 3"$ $m\overline{AC} = 4"$ and $m\angle A = 60^0$. draw circumcircle to this triangle.

Given

Let $m\,\overline{AB} = 3\,cm,\ m\,\overline{AC} = 4\,cm,$ and $m\angle A = 60^0$

Required

To circumscribe a circle about the given $\Delta\,ABC$



Steps of Construction

1. Draw a line $m\overline{AB} = 3\,cm$
2. At point A, draw an angle of 60^0
3. With A as centre, draw an arc of radius 4 cm which cuts angle 60^0 at points C.
4. Join B to C.
5. Thus, ABC is the required triangle.
6. Draw the perpendicular bisectors of $\overline{AB}$ and $m\overline{AC}$ which intersect each other at O.
7. Join O to B.
8. With centre O and radius $m\,\overline{OB}$ draw a circle which touches the vertices of triangle.
9. This is the required circumscribed circle about the given $\Delta\,ABC$

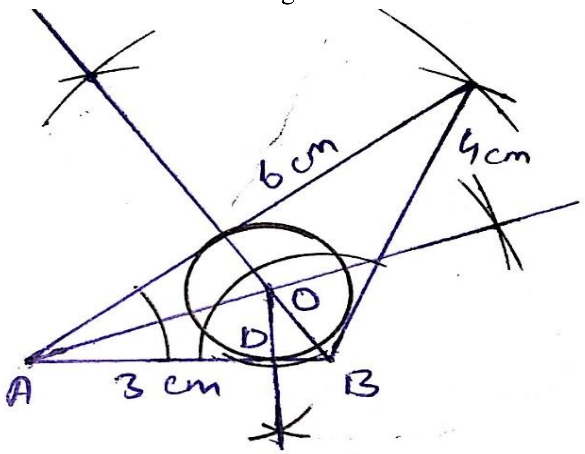
3. Suppose we have a triangle whose sides are 3 cm, 4 cm and 6 cm respectively. Draw its inscribed circle.

Given

Let $m\overline{AB} = 3\ cm,\ m\overline{BC} = 4\ cm,$ and $m\overline{AC} = 6\ cm$

Required

To inscribe a circle in the given $\Delta\ ABC$



Steps of construction

1. Draw a line $m\overline{AB} = 3\ cm$
2. With B as centre, draw an arc of radius 4 cm.
3. With A as centre, draw another arc of radius 6 cm.
4. Both the arcs meet at point C.
5. Thus, ABC is the triangle according to data
6. Draw the bisectors of angles A and B which pass through same point O.
7. From O, draw $\overline{OD} \perp \overline{AB}$
8. With centre O and radius $m\overline{OD}$ draw a circle which touches the sides of triangle.
9. This is the required inscribed circle in the given $\Delta\ ABC$

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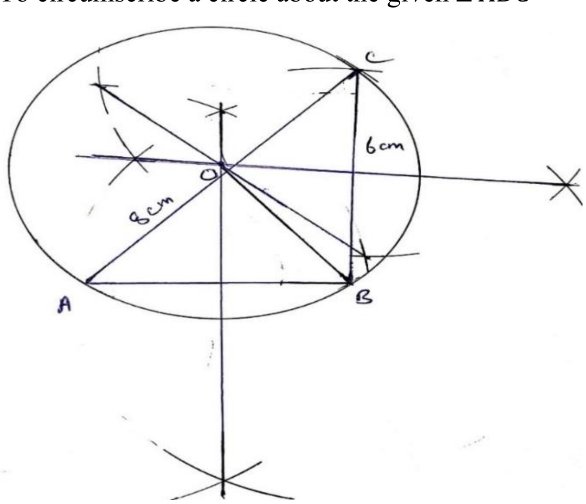
4. Construct a triangle ABC with sides $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ and $m\overline{CA} = 8\ cm$. Draw perpendicular bisectors of its sides and then circumscribe a circle.

Given

$m\overline{AB} = 5\ cm,\ m\overline{BC} = 6\ cm,$ and $m\overline{CA} = 8\ cm$

Required

To circumscribe a circle about the given $\Delta\ ABC$



Steps of construction

1. Draw a line $m\overline{AB} = 5\ cm$
2. With B as centre, draw an arc of radius 6 cm.
3. With A as centre, draw another arc of radius 8 cm.
4. Both the arcs meet at point C.
5. Thus, ABC is the triangle according to data
6. Draw the perpendicular bisectors of $\overline{AB}$, $\overline{BC}$ and $m\overline{AC}$ which intersect each other at O.
7. Join O to B.
8. With centre O and radius $m\overline{OB}$ draw a circle which touches the vertices of triangle.
9. This is the required circumscribed circle about the given $\Delta\ ABC$

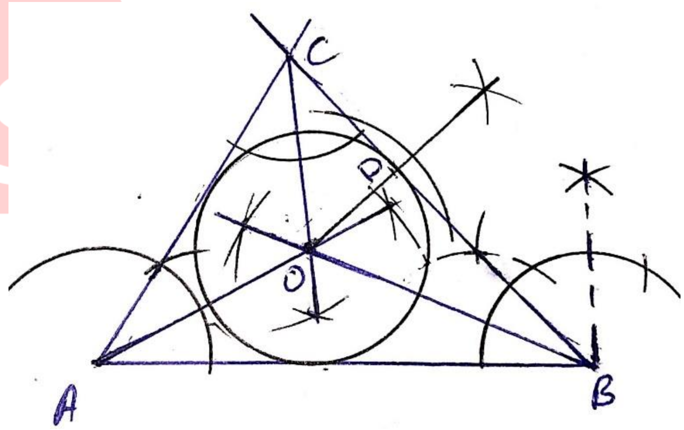
5. Draw a triangle ABC with $m\angle A = 60^o$ and $m\angle B = 45^o$. draw three angle bisectors and then inscribe a circle in it.

Given

Let $m\overline{AB} = 5\ cm,\ m\angle A = 60^o$ and $m\angle B = 45^o$

Required

To inscribe a circle in the given $\Delta\ ABC$



Steps of construction

1. Draw a suitable line $m\overline{AB} = 5\ cm$
2. At point A, draw an angle of 60^0
3. At point B, draw another angle of 45^0
4. Both the angles meet at point C
5. Thus, ABC is the required triangle.
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. From O, draw $\overline{OD} \perp \overline{BC}$
8. With centre O and radius $m\overline{OD}$ draw a circle which touches the sides of triangle.
9. This is the required inscribed circle in the given $\Delta\ ABC$

6. An equilateral triangle in inscribed in a circle. Find the altitude of the triangle if the radius of the circle varies as under.

$r = 3\ units,\ \ r = 4\ units,\ r = 6\ units, r = 12\ units.$

Can you deduce some result from this?

Unit # 13

$\boldsymbol{r = 3\ units}$

<u>Solution</u>:

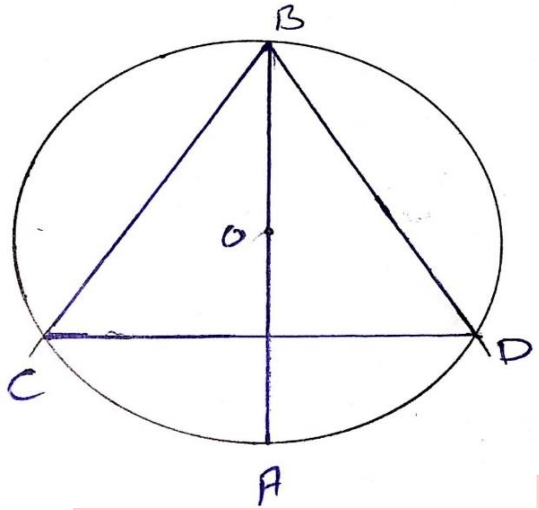
Let $1\ unit = 1\ cm$ Then $3\ units = 3\ cm$

Given

A circle with centre O and radius 3 cm.

Required

Inscribe an equilateral triangle in the circle.



Steps of construction

1. At point O, draw a circle of radius 3 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. With centre A and radius 3 cm, draw two arcs which cut the circle at C and D.
4. Join B, C and D.
5. This is the required equilateral triangle.

Note:

The altitude of a triangle is 4.5 cm.

$\boldsymbol{r = 4\ units}$

<u>Solution</u>:

Let $1\ unit = 1\ cm$

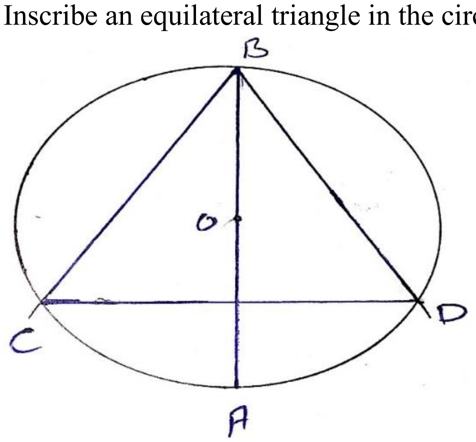
Then $4\ units = 4\ cm$

Given

A circle with centre O and radius 4 cm.

Required

Inscribe an equilateral triangle in the circle.



Steps of construction

1. At point O, draw a circle of radius 4 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. With centre A and radius 4 cm, draw two arcs which cut the circle at C and D.
4. Join B, C and D.
5. This is the required equilateral triangle.

Note:

The altitude of a triangle is 4.5 cm.

$\boldsymbol{r = 6\ units}$

<u>Solution</u>:

Let $2\ units = 1\ cm$

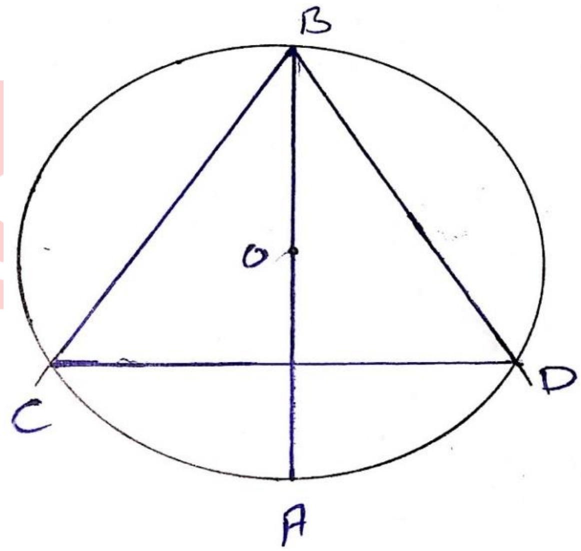
Then $6\ units = 3\ cm$

Given

A circle with centre O and radius 3 cm.

Required

Inscribe an equilateral triangle in the circle.



Steps of construction

1. At point O, draw a circle of radius 3 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. With centre A and radius 3 cm, draw two arcs which cut the circle at C and D.
4. Join B, C and D.
5. This is the required equilateral triangle.

Note:

The altitude of a triangle is 4.5 cm.

$\boldsymbol{r = 12\ units}$

<u>Solution</u>:

Let $3\ units = 1\ cm$

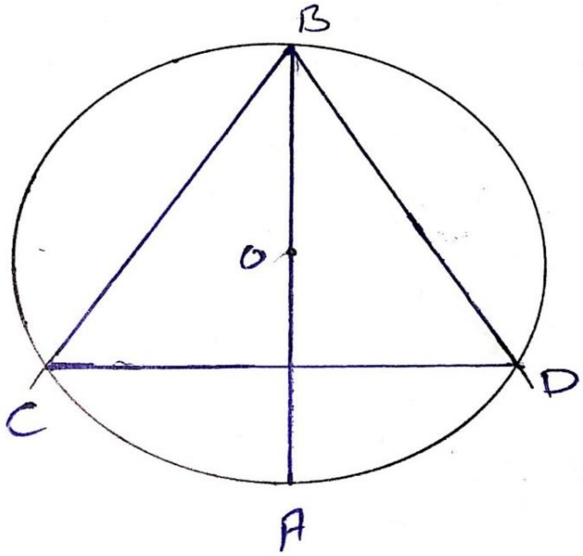
Then $12\ units = 4\ cm$

Given

A circle with centre O and radius 4 cm.

Required

Inscribe an equilateral triangle in the circle.



Steps of construction

1. At point O, draw a circle of radius 4 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. With centre A and radius 4 cm, draw two arcs which cut the circle at C and D.
4. Join B, C and D.
5. This is the required equilateral triangle.

Note:

The altitude of a triangle is 4.5 cm.

---

7. An equilateral triangle is circumscribed about a circle. Find the altitude of the triangle if radius r of the circle varies as $r = 2\ units$, $r = 5\ units$, $r = 10\ units$. Can you deduce some result from this?

$r = 2\ units$

Solution:

Let 1 $unit = 1\ cm$

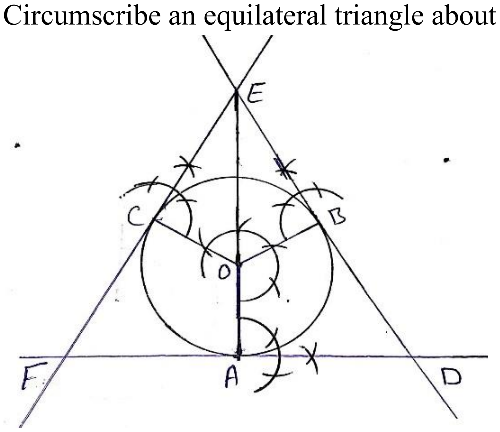
Then 2 $units = 2\ cm$

Given

A circle with centre O and radius 2 cm.

Required

Circumscribe an equilateral triangle about the circle.



Steps of construction

1. At point O, draw a circle of radius 2 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120°.
4. Draw another angle $\angle BOC$ of measure 120°.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

Note:

The altitude of a triangle is 4.5 cm.

$r = 5\ units$

Solution:

Let 1 $unit = 1\ cm$

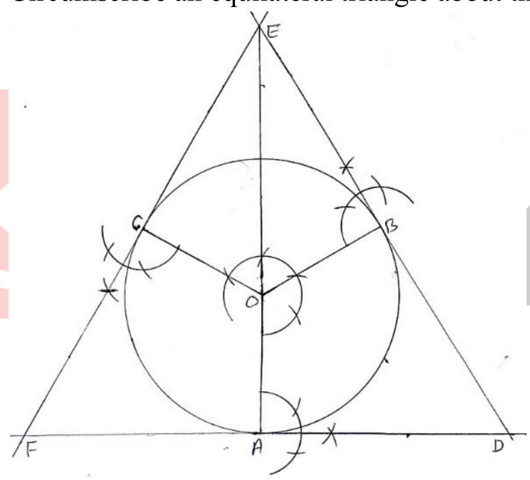
Then 5 $units = 5\ cm$

Given

A circle with centre O and radius 5 cm.

Required

Circumscribe an equilateral triangle about the circle.



Steps of construction

1. At point O, draw a circle of radius 5 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120°.
4. Draw another angle $\angle BOC$ of measure 120°.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

Note:

The altitude of a triangle is 4.5 cm.

$r = 10\ units$

Solution:

Let 2 $units = 1\ cm$

Then 10 $units = 5\ cm$

Given

A circle with centre O and radius 5 cm.

Required

Circumscribe an equilateral triangle about the circle.

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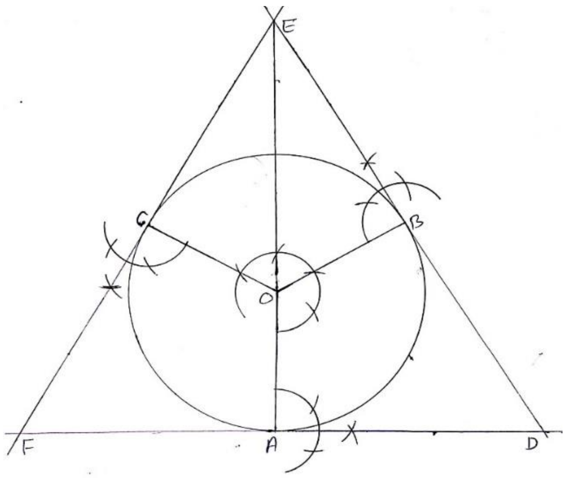


Steps of construction

1. At point O, draw a circle of radius 5 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120^o.
4. Draw another angle $\angle BOC$ of measure 120^o.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

Note:
The altitude of a triangle is 4.5 cm.

---

8. Circumcircle an equilateral triangle about a circle of radius 2", 3" and 1".

$r = 2''$

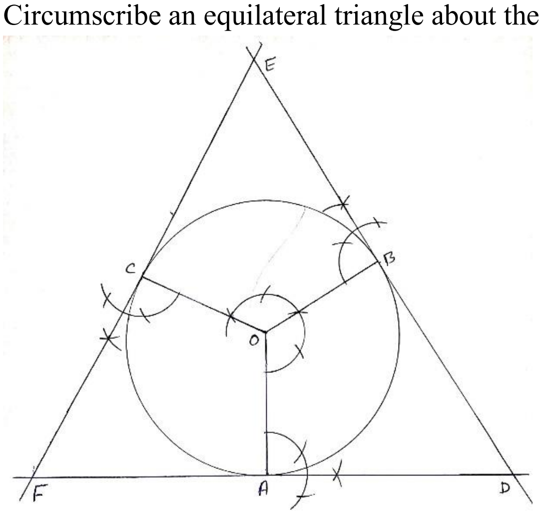
Solution:

Given

A circle with centre O and radius 2 cm.

Required

Circumscribe an equilateral triangle about the circle.



Steps of construction

1. At point O, draw a circle of radius 2 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120^o.
4. Draw another angle $\angle BOC$ of measure 120^o.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

$r = 3''$

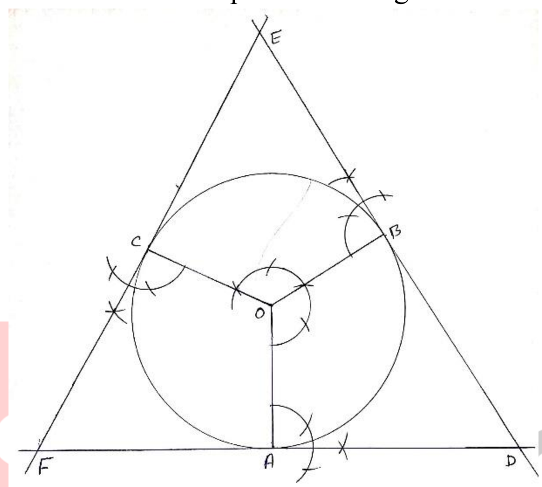
Solution:

Given

A circle with centre O and radius 3 cm.

Required

Circumscribe an equilateral triangle about the circle.



Steps of construction

1. At point O, draw a circle of radius 3 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120^o.
4. Draw another angle $\angle BOC$ of measure 120^o.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

$r = 1''$

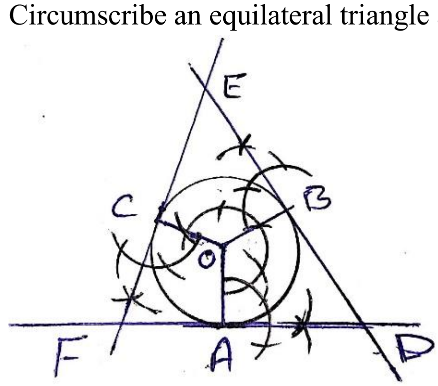
Solution:

Given

A circle with centre O and radius 1 cm.

Required

Circumscribe an equilateral triangle about the circle.



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Steps of construction

1. At point O, draw a circle of radius 1 cm
2. Take any point A on the circle and join to O.
3. Draw an angle $\angle AOB$ of measure 120^o.
4. Draw another angle $\angle BOC$ of measure 120^o.
5. Draw perpendiculars at points A, B and C which cut each other at points D, E and F.
6. Thus, DEF is the required equilateral triangle about the circle.

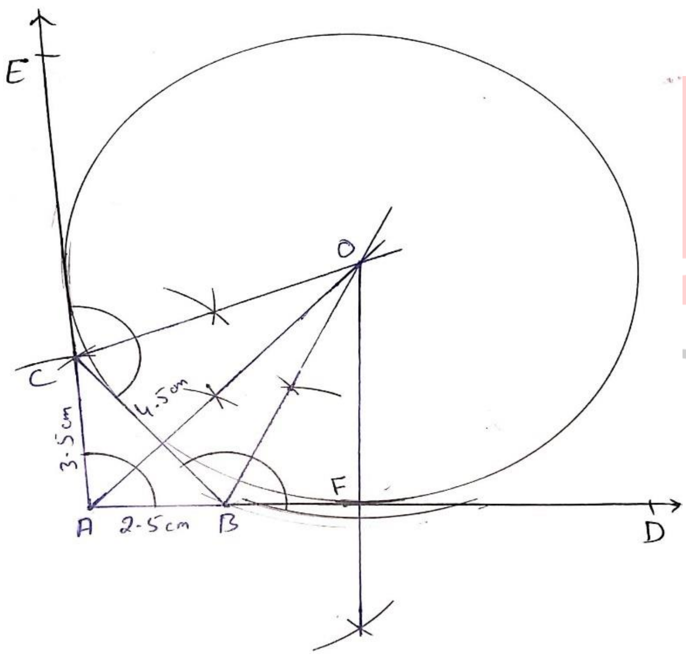
9. Draw a triangle with sides 2.5 cm, 3.5 cm and 4.5 cm long. Draw an escribed circle to the triangle touching the longest side of the triangle.

Given

Let $m\,\overline{AB} = 2.5\,cm$, $m\,\overline{BC} = 4.5\,cm$, and $m\,\overline{AC} = 3.5\,cm$

Required

An escribed circle touching the longest side of $\Delta\,ABC$



Steps of construction

1. Draw a line $m\,\overline{AB} = 2.5\,cm$
2. With B as centre, draw an arc of radius 4.5 cm.
3. With A as centre, draw another arc of radius 3.5 cm.
4. Both the arcs meet at point C.
5. Thus, ABC is the triangle according to data.
6. Produce $\overline{AB}$ and $m\overline{AC}$ to form exterior angles $\angle CBD$ and $\angle BCE$
7. Draw bisectors of $\angle BAC$, $\angle CBD$ and $\angle BCE$.
8. All the angle bisectors intersect at point O.
9. Draw $\overline{OF} \perp \overline{AD}$
10. With centre O and radius $m\,\overline{OF}$ draw a circle which touches $\overline{BC}$, $\overline{BD}$ and $m\overline{CE}$
11. This is the required escribed circle touches the longest side of given $\Delta\,ABC$

10. For the problem in Q9, draw an escribed circle to the triangle touching the smallest side.

OR

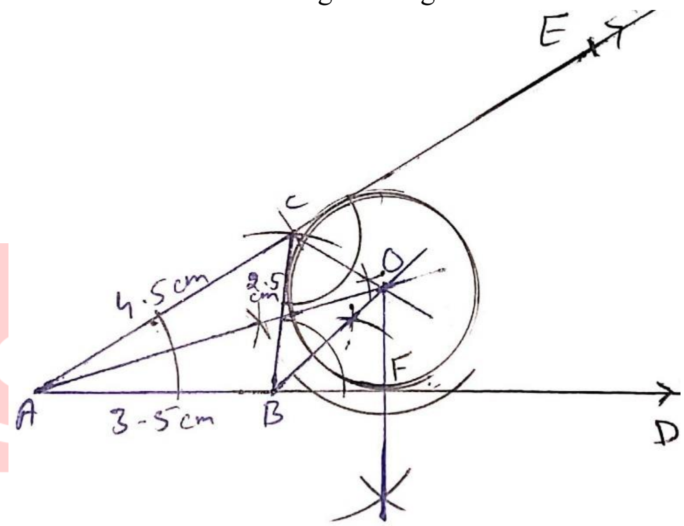
Draw a triangle with sides 2.5 cm, 3.5 cm and 4.5 cm long. Draw an escribed circle to the triangle touching the shortest side of the triangle.

Given

Let $m\,\overline{AB} = 3.5\,cm$, $m\,\overline{BC} = 2.5\,cm$, and $m\,\overline{AC} = 4.5\,cm$

Required

An escribed circle touching the longest side of $\Delta\,ABC$



Steps of construction

1. Draw a line $m\,\overline{AB} = 3.5\,cm$
2. With B as centre, draw an arc of radius 2.5 cm.
3. With A as centre, draw another arc of radius 4.5 cm.
4. Both the arcs meet at point C.
5. Thus, ABC is the triangle according to data.
6. Produce $\overline{AB}$ and $m\overline{AC}$ to form exterior angles $\angle CBD$ and $\angle BCE$
7. Draw bisectors of $\angle BAC$, $\angle CBD$ and $\angle BCE$.
8. All the angle bisectors intersect at point O.
9. Draw $\overline{OF} \perp \overline{AD}$
10. With centre O and radius $m\,\overline{OF}$ draw a circle which touches $\overline{BC}$, $\overline{BD}$ and $m\overline{CE}$
11. This is the required escribed circle touches the smallest side of given $\Delta\,ABC$

Ex # 13.2

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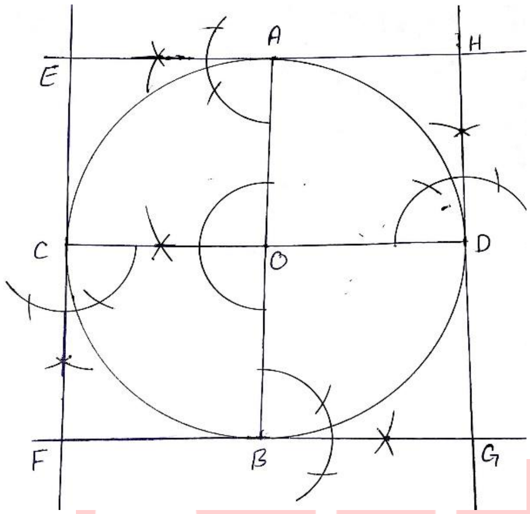
1. Circumscribe a square about a circle of radius 5 cm.

Given

A circle with centre O and radius 5 cm.

Required

Circumscribed a square about a circle.



Steps of construction

1. At point O, draw a circle of radius 5 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. Draw another diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$
4. Draw perpendiculars at the extremities A, C, B and D which cut each other at points E, F, G and H.
5. Thus, EFGH is the required square circumscribed about that circle

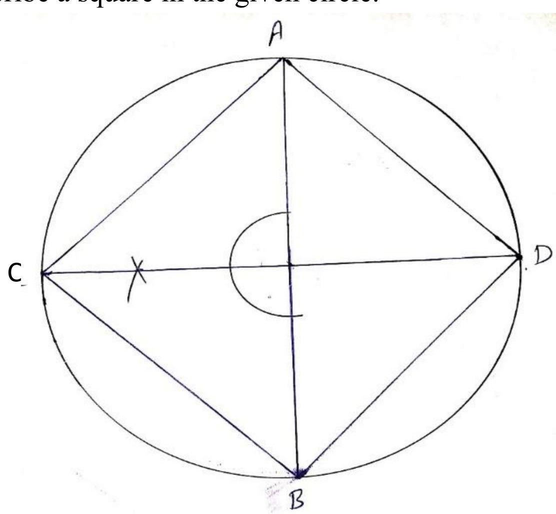
2. Inscribe a square in a circle of radius 6 cm.

Given

A circle with centre O and radius 6 cm.

Required

Inscribe a square in the given circle.



Steps of construction

1. At point O, draw a circle of radius 6 cm
2. Draw diameter $\overline{AB}$ of the circle.
3. Draw another diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$
4. Join A to D, B and C.
5. This ABCD is the required square.
6. This is the required inscribe square in the circle.

Q3: Draw a square of side 6cm. Circumscribe a circle about that square and then inscribe a circle in the same square. Measure the radii of these circles.

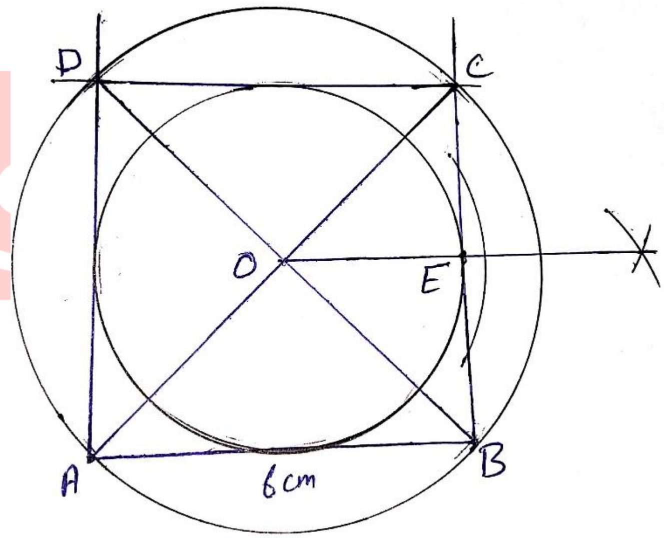
Solution

Given

A square of side 6cm

Required

Circumscribe and inscribe a circle of the given square and measure the radii of these circles.



Steps of construction

1. Draw a line $m\overline{AB} = 6cm.$
2. At points A and B, draw 90°.
3. At points A and B, draw two arcs of radius 6cm which cut both 90° at points D and C.
4. Join C and D.
5. This ABCD is the required square.

For Circumscribe Circle

1. Join A to C and B to D, which cut each other at point O.
2. With centre O and radius $m\overline{OA}$, draw a circle which intersect the square at A, B, C and D.
3. This is the required circumscribe circle.

Radius

As ΔABC is the right-angled triangle.

By Pythagoras Theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (6)^2 + (6)^2$
$(AC)^2 = 36 + 36$
$(AC)^2 = 72$
$\sqrt{(AC)^2} = \sqrt{72}$
$AC = 8.48$
As $\overline{AC}$ is the diameter of circle. So $\overline{AC} = d = 8.48$
$r = \frac{d}{2}$
$r = \frac{8.48}{2}$
$r = 4.24cm$

For inscribe circle

1. Draw $\overline{OE} \perp \overline{BC}$
2. With centre O and radius $m\overline{OE}$, draw a circle which touches the sides $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$
3. This is the required inscribe circle.

Radius

As $\overline{OE}$ is half of 6cm. Then
$r = \frac{6}{2}$
$r = 3cm$

4: First draw a circle of suitable radius, so that the square circumscribed about that circle has sides of length 8 units.

Solution

Given

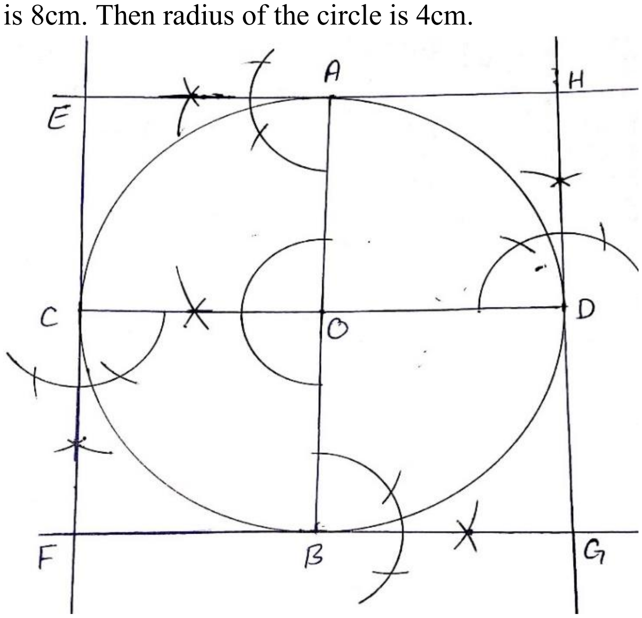
A circle and a square

Required

A circle and a square about that circle

Note

Let 1 unit = 1cm then 8 units = 8 cm
As diameter of circle is equal to the side of a square which is 8cm. Then radius of the circle is 4cm.



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Steps of Construction

1. At point O, draw a circle of radius 4cm
2. Draw diameter $\overline{AB}$ of the circle.
3. Draw another diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$
4. Draw perpendiculars the extremities A, C, B and D which cut each other at points E, F, G and H.
5. Thus, EFGH is the required square circumscribed about that circle

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Q5: Inscribe a square of side 10 cm in a circle. What will be the size of radius?

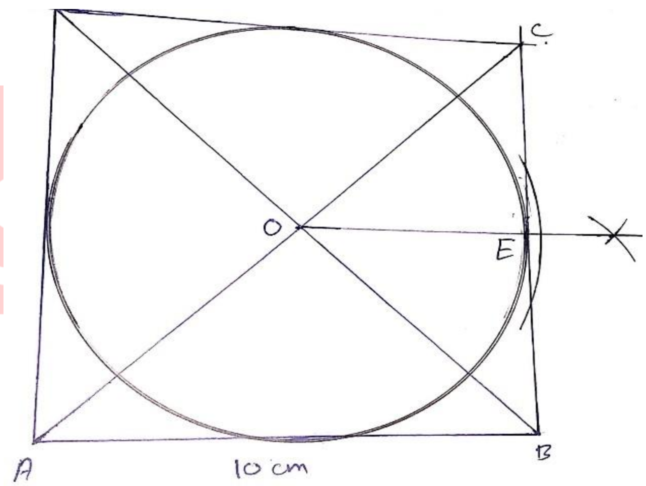
Solution

Given

A square of side 10cm

Required

Inscribe a circle of the given square and also find its radius.



Steps of construction

1. Draw a line $m\overline{AB} = 10cm$.
2. At points A and B, draw 90°.
3. At points A and B, draw two arcs of radius 10 cm which cut both 90° at points D and C.
4. Join C and D.
5. Draw $\overline{OE} \perp \overline{BC}$
6. With centre O and radius $m\overline{OE}$, draw a circle which touches the sides $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$
7. This is the required inscribe circle.

Radius

By Pythagoras Theorem
$(AC)^2 = (AB)^2 + (BC)^2$
$(AC)^2 = (10)^2 + (10)^2$
$(AC)^2 = 100 + 100$
$(AC)^2 = 200$
$\sqrt{(AC)^2} = \sqrt{200}$
$AC = 14.14$

As $\overline{AC}$ is the diameter of circle. So $\overline{AC} = d = 14.14$

$r = \frac{d}{2}$

$r = \frac{14.14}{2}$

$r = 7.07\ cm$

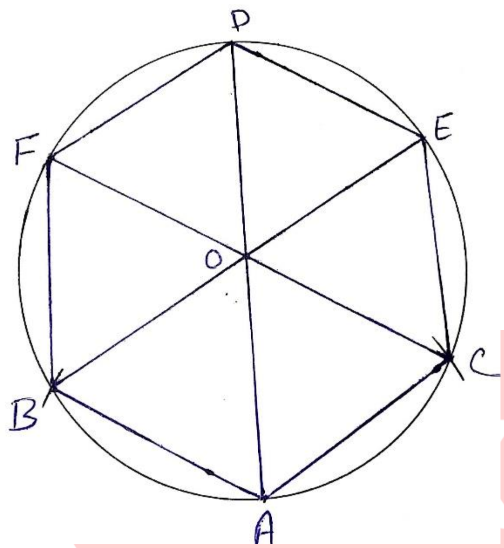
6: Inscribe a regular hexagon in a circle of radius 4 cm

Given

A circle with radius 4 cm.

Required

To inscribe a regular hexagon in the given circle.



Steps of Construction

1. With centre O, draw a circle of radius 4 cm.
2. Take a point A on the circumference of the circle.
3. At point A, draw two arcs of 4cm which cut the circumference of the circle at point B and C.
4. Through O, draw $\overline{AD},\ \overline{BE}\ and\ \overline{CF}$.
5. Draw $\overline{AB},\ \overline{BF},\ \overline{FD},\ \overline{DE},\ \overline{EC},\ and\ \overline{CA}$.
6. Thus, ABFDEC is the required inscribe hexagon.

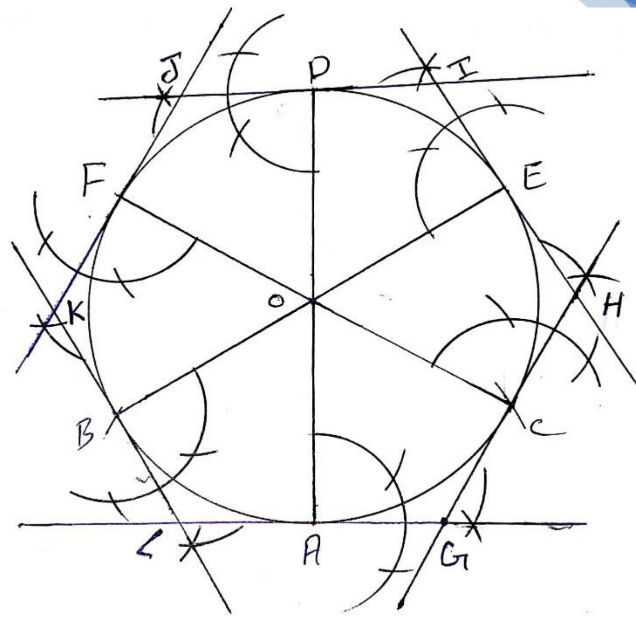
7: Construct a circle of radius 4 cm and draw a regular hexagon about the circle.

Given

A circle with radius 4 cm.

Required

To draw a regular hexagon about the given circle.



Steps of Construction

1. With centre O, draw a circle of radius 4 cm.
2. Take a point A on the circumference of the circle.
3. At point A, draw two arcs of 4cm which cut the circumference of the circle at point B and C.
4. Through O, draw $\overline{AD},\ \overline{BE}\ and\ \overline{CF}$.
5. Draw perpendiculars at the extremities A, C, E, D, F and B which cut each other at points G, H, I, J, K and L.
6. Thus, GHIJKL is the required circumscribe hexagon.

8: Draw a circle of radius 8 cm. Circumscribe a regular hexagon about that circle and also inscribe a regular in the same circle. Find the areas of these geometrical figures. Comment on the values of these areas.

<u>Solution</u>:

Let $2\ cm = 1\ cm$

Then $8\ units = 4\ cm$

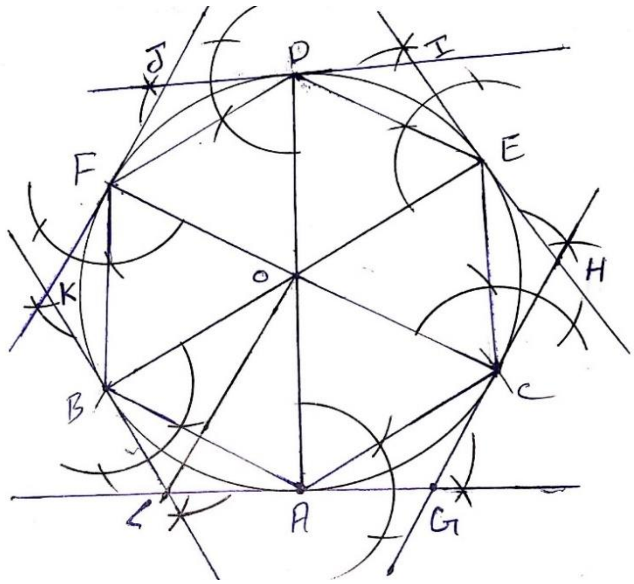
Given

A circle with radius 4 cm.

Required

To draw circumscribe and inscribe a hexagon of the given circle and also find their areas.

Unit # 13



Steps of Construction

1. With centre O, draw a circle of radius 4 cm.
2. Take a point A on the circumference of the circle.
3. At point A, draw two arcs of 4cm which cut the circumference of the circle at point B and C.
4. Through O, draw $\overline{AD},\ \overline{BE}\ and\ \overline{CF}$.
5. Draw perpendiculars at the extremities A, C, E, D, F and B which cut each other at points G, H, I, J, K and L.
6. Thus, GHIJKL is the required circumscribe hexagon.
7. Draw $\overline{AC},\ \overline{BF},\ \overline{FD},\ \overline{DE},\ \overline{EC},\ and\ \overline{CA}$.
8. Thus, ABFDEC is the required inscribe hexagon.

Area of circumscribe hexagon

As we know that GHIJKL is circumscribe hexagon.
As hexagon has six equal sectors with angle 60^0 each. Let take one sector OALB. Join O to L such that $\angle AOB = 30^0$
Now ΔAOB is the Right-angled triangle.

As $\overline{OL} = hyp, \overline{AL} = perp, \overline{OA} = base = 4cm$

$$\tan 30^0 = \frac{\overline{AL}}{\overline{OA}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{AL}}{4}$$

$$\frac{4}{\sqrt{3}} = \overline{AL}$$

$$\overline{AL} = \frac{4}{\sqrt{3}}$$

Now for side of hexagon

$$s = \overline{GL} = 2\overline{AL}$$

$$s = 2\left(\frac{4}{\sqrt{3}}\right)$$

$$s = \frac{8}{\sqrt{3}}$$

So, Area of circumscribe hexagon $= \frac{3\sqrt{3}}{2} \times s^2$

$$= \frac{5.2}{2} \times \left(\frac{8}{\sqrt{3}}\right)^2$$

$$= \frac{5.2}{2} \times \frac{64}{3}$$

$$= 55.47\ cm^2$$

Area of inscribe hexagon

Since ABFDEC is inscribe hexagon and consists of six equilateral triangles with angle 60^0 each.
As the angles and sides of an equilateral triangle are equal so each side equal to 4 cm.

So, Area of inscribe hexagon $= \frac{3\sqrt{3}}{2} \times s^2$

$$= \frac{5.2}{2} \times (4)^2$$

$$= \frac{5.2}{2} \times 16$$

$$= 41.6\ cm^2$$

9: Draw two regular hexagons of perimeters 6 cm and 30 cm respectively. Determine their centres. From their centres draw perpendicular to any of their sides. What is the relation of these two perpendiculars?

Hexagon of perimeter 6cm

Given

Perimeters of hexagon 6cm

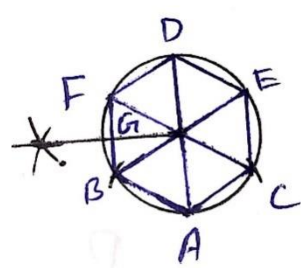
Required

Regular Hexagon

Note

As hexagon has six equal sides, then each side is 1cm.

$$\frac{6}{6} = 1$$



Steps of construction

1. Draw a circle of radius 1 cm
2. Take a point A on the circumference of a circle.
3. With centre A, draw an arc of radius 1 cm which cuts the circumference of a circle at point B.
4. Similarly draw successive arcs of 1cm which cut the circumference of the circle at C, D, E and F.
5. Draw $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE},\ and\ \overline{EF}$.
6. Thus, ABCDEF is the required hexagon of perimeter 6cm.

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Hexagon of perimeter 30cm

Given

Perimeters of hexagon 30cm

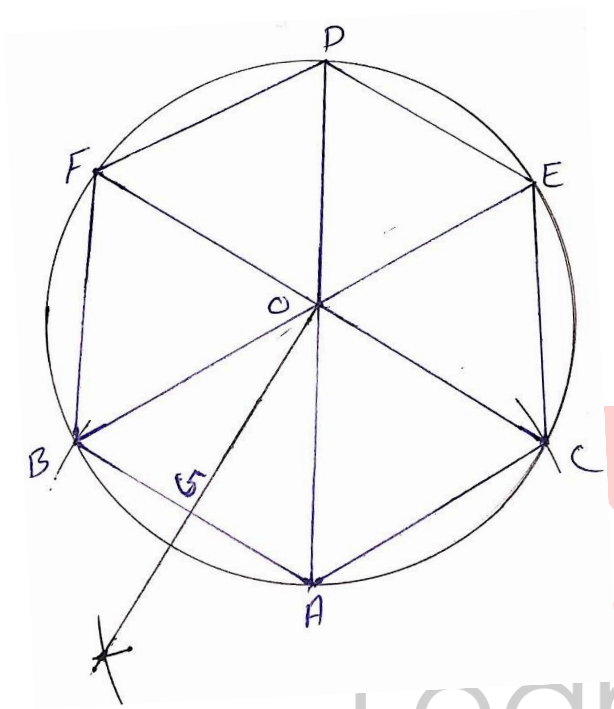
Required

Regular Hexagon

Note

As hexagon has six equal sides, then each side is 1cm.

$\frac{30}{6} = 5$



Steps of construction

1. Draw a circle of radius 5 cm
2. Take a point A on the circumference of a circle.
3. With centre A, draw an arc of radius 1 cm which cuts the circumference of a circle at point B.
4. Similarly draw successive arcs of 1cm which cut the circumference of the circle at C, D, E and F.
5. Draw $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE},$ and $\overline{EF}$.
6. Thus, ABCDEF is the required hexagon of perimeter 30cm.

10: Can you construct a square whose area equal to the areas of a given circle? Discuss in detail

Solution:

As we know that

Area of circle $= \pi r^2$

Area of square $= s^2$ $\quad \therefore$ s = side

As area of square = area of circle

$s^2 = \pi r^2$

Taking square root on B.S

$\sqrt{s^2} = \sqrt{\pi r^2}$

$s = \sqrt{\pi} r$

$s = 1.77r$

From the above equation, when radius is multiplied with $\sqrt{\pi}$ which is approximately equal to 1.77, in such case, the area of square is equal to area of given circle.

Tangent to the circle.

A line that intersects a circle at exactly one point is called a tangent, and the point of intersection is called the point of tangency or point of contact.

Ex # 13.3

Page # 267

1. Draw an arc of length 7 cm. Without using the center, draw a tangent through a given point A when A is:

(i) The middle point of the arc.

As length of arc = 7cm

As we know that

$l = r\theta$

Here we take $\theta = 90^o = \frac{\pi}{2}$

So, $l = r\theta$

$7 = r\left(\frac{\pi}{2}\right)$

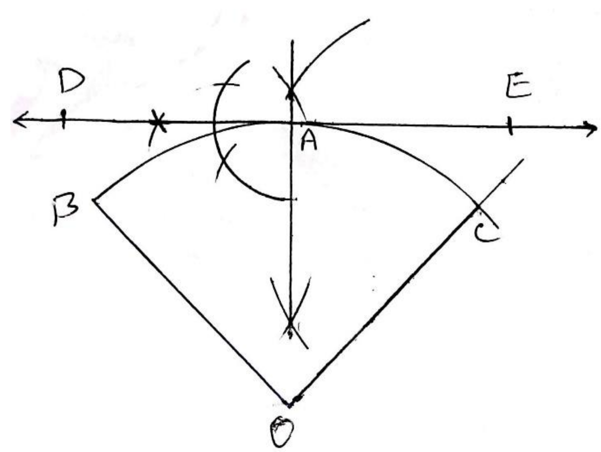
$7\left(\frac{2}{\pi}\right) = r$

$\frac{14}{3.14159} = r$

$4.46 = r$

$r = 4.46$

$r = 4.5\ cm$



Steps of construction

1. Draw a line $\overline{OB} = 4.5\ cm$
2. At point O, draw an angle of 90^o
3. With center O and radius 4.5 cm, draw an arc from Point B and cuts 90^o at point C.
4. The arc $\widehat{BC}$ will be 7cm.

5. Bisect $\widehat{BC}$ at point A.
6. At point A, draw the perpendicular $\overline{DE}$ which is the required tangent at the middle point A of the arc.

(ii) End point of the arc.

As length of arc = 7cm

As we know that

$$l = r\theta$$

Here we take $\theta = 90^o = \frac{\pi}{2}$

So, $l = r\theta$

$7 = r\left(\frac{\pi}{2}\right)$

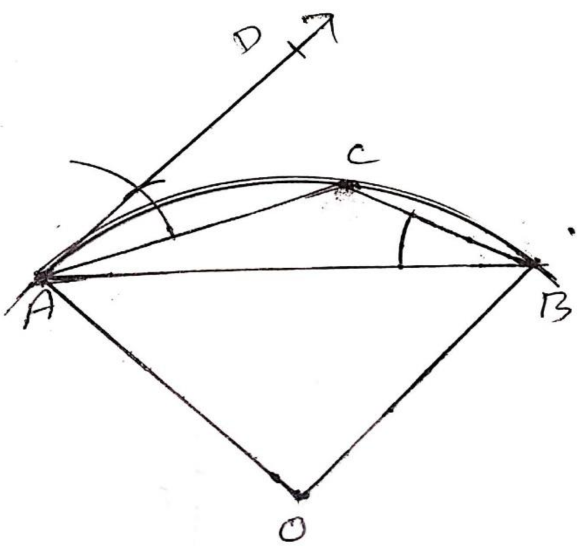
$7\left(\frac{2}{\pi}\right) = r$

$\frac{14}{3.14159} = r$

$4.46 = r$

$r = 4.46$

$r = 4.5\ cm$



Steps of construction

1. Draw a line $\overline{OA} = 4.5\ cm$
2. At point O, draw an angle of 90^o
3. With center O and radius 4.5 cm, draw an arc from Point A and cuts 90^o at point B.
4. The arc $\widehat{AB}$ will be 7cm.
5. Take any point C on the arc $\widehat{AB}$ and join C to A and B.
6. Join A to B.
7. Construct $\angle CAD \cong \angle ABC$.
8. $\overrightarrow{AD}$ is the required tangent at point A.

(iii) Outside the arc.

As length of arc = 7cm

As we know that

$$l = r\theta$$

Here we take $\theta = 90^o = \frac{\pi}{2}$

So, $l = r\theta$

$7 = r\left(\frac{\pi}{2}\right)$

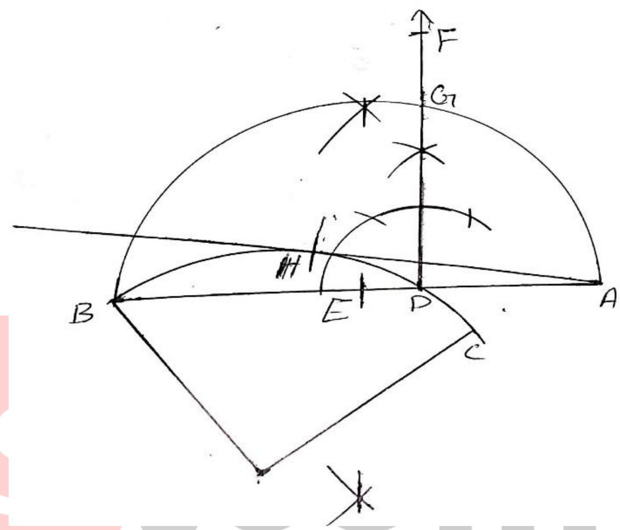
$7\left(\frac{2}{\pi}\right) = r$

$\frac{14}{3.14159} = r$

$4.46 = r$

$r = 4.46$

$r = 4.5\ cm$



Steps of construction

1. Draw a line $\overline{OB} = 4.5\ cm$
2. At point O, draw an angle of 90^o
3. With center O and radius 4.5 cm, draw an arc from Point B and cuts 90^o at point C.
4. The arc $\widehat{BC}$ will be 7cm.
5. Take a point A outside the arc.
6. Join B and A which cuts the arc at point D.
7. Bisect $\overline{BA}$ at point E.
8. With centre E and radius $m\overline{BE}$, draw the semi-circle.
9. Draw $\overline{DF} \perp \overline{BA}$, which cuts the semi-circle at point G.
10. With centre A and radius $m\overline{AG}$, draw an arc which cuts the arc $\widehat{BC}$ at point H.
11. Join $\overline{AH}$ which is the required tangent.

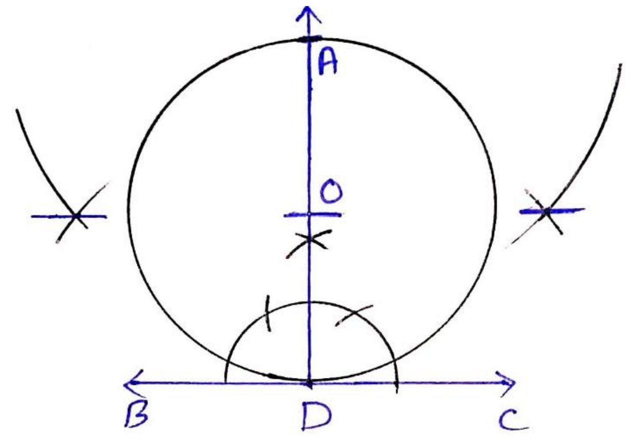
2. Draw a circle passing through a point A and touching a given line $\overleftrightarrow{BC}$ at point D.

Given

A line $\overleftrightarrow{BC}$ and a point A

Required

A circle passing through a point A and touching a given line $\overleftrightarrow{BC}$ at point D



Steps of construction

1. Draw a line $\overleftrightarrow{BC}$
2. Take a point D on $\overleftrightarrow{BC}$
3. Draw $\overline{AD} \perp \overleftrightarrow{BC}$
4. Take O the midpoint of $\overline{AD}$
5. With centre O and radius $m\overline{OA}$, draw a circle which is passing through a point A and touching a given line $\overleftrightarrow{BC}$ at point D

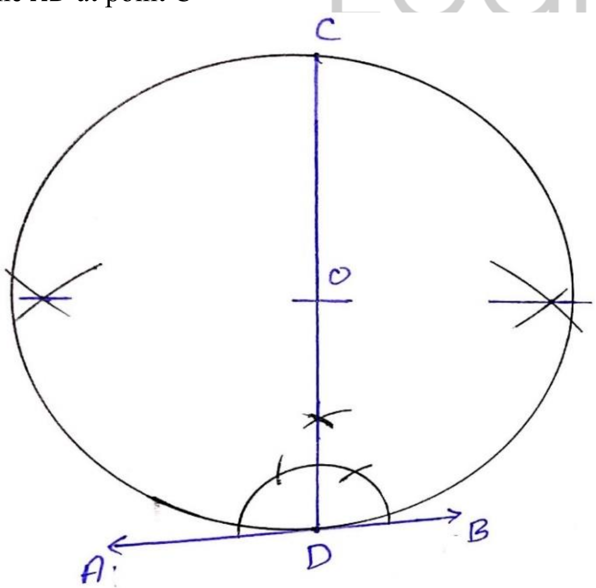
3. Describe a circle of radius 4cm, passing through a given point C and touching a given straight line $\overleftrightarrow{AB}$.

Given

A line $\overleftrightarrow{AB}$ and a point C

Required

A circle passing through a point C and touching a given line $\overleftrightarrow{AB}$ at point C



Steps of construction

1. Draw a line $\overleftrightarrow{AB}$
2. Take a point D on $\overleftrightarrow{AB}$
3. Draw $\overline{DC} \perp \overleftrightarrow{AB}$ such that $\overline{CD} = 8cm$
4. Take O the midpoint of $\overline{CD}$
5. With centre O and radius $m\overline{OC} = 4cm$, draw a circle which is passing through a point C and touching a given line $\overleftrightarrow{AB}$ at point D.

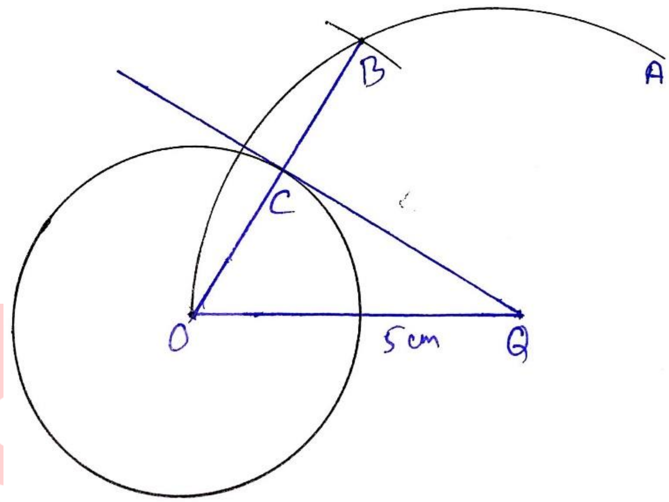
4. Radius of a circle is 2.5 cm. A point Q is at a distance of 5cm from the centre. Draw tangent to the circle from the point Q.

Given

A circle with radius 2.5cm and a point Q 5cm outside from the centre.

Required

Tangent to a circle from point Q



Steps of construction

1. With centre O and radius 2.5cm, draw a circle
2. Take a point Q at 5cm from point O
3. With centre Q and radius $m\overline{OQ} = 5cm$, draw an arc OA
4. With center O and radius of 5cm which is diameter of circle (diameter $= 2.5 + 2.5$), draw another arc intersecting the arc OA at point B.
5. Draw $\overline{OB}$ which intersects the circle at point C.
6. Through C, draw $\overleftrightarrow{QC}$ which is the required tangent.

5. Radii of two circles are 2cm and 3cm and their centres are 8cm apart. Draw direct common tangents to the circles.

Given

Radii of two circles are 2cm and 3cm and their centres are 8cm apart.

Required

Direct common tangents to the circles.

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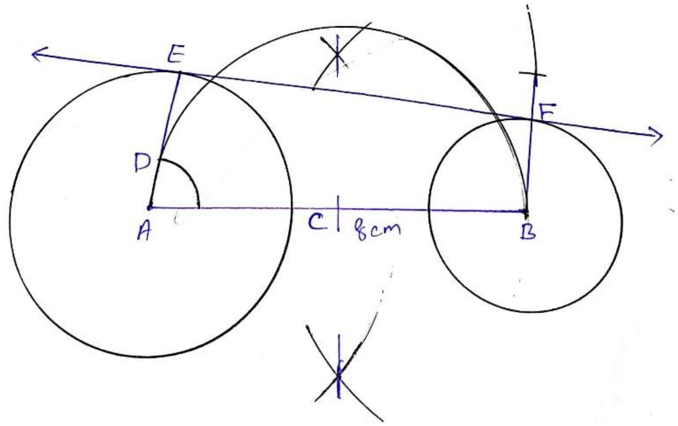

Steps of construction

1. Draw a line $m\overline{AB} = 8\ cm$.
2. With centre A, draw a circle of radius 3cm.
3. With centre B, draw another circle of radius 2cm.
4. Bisect $\overline{AB}$ at point C.
5. With centre C and radius $m\overline{CA}$, draw a semi-circle.
6. With centre A and radius 1cm ($3cm - 2cm = 1cm$), draw an arc cutting the semi-circle at point D.
7. Draw $\overrightarrow{AD}$ to the meet the bigger circle at point E.
8. Draw $\overline{BF} || \overline{AE}$.
9. Join E and F and produce the line.
10. Thus $\overleftrightarrow{EF}$ is the required direct common tangent of the given circles.

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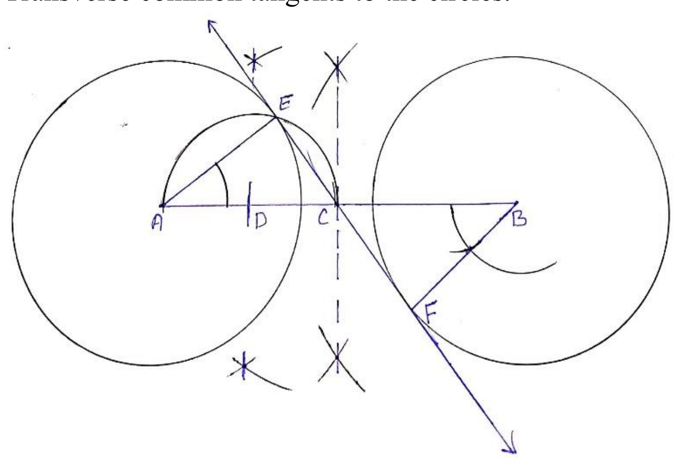
6. Two congruent circles are of radius 4cm each. Their centres are 10cm apart. Draw transverse common tangents to these circles.

Given

Two congruent circles are of radius 4cm each and their centres are 10cm apart.

Required

Transverse common tangents to the circles.



Steps of construction

1. Draw a line $m\overline{AB} = 10\ cm$.
2. With centre A, draw a circle of radius 4cm.
3. With centre B, draw another circle of radius 4cm.
4. Bisect $\overline{AB}$ at point C.
5. Bisect $\overline{AC}$ at point D.
6. With centre D and radius $m\overline{DA}$, draw a semi-circle which cuts the given circle at point E.
7. Join A to E.
8. Draw $\overline{BF} || \overline{AE}$.
9. Join E and F and produce the line.
10. Thus $\overleftrightarrow{EF}$ is the required transverse common tangent of the given circles.

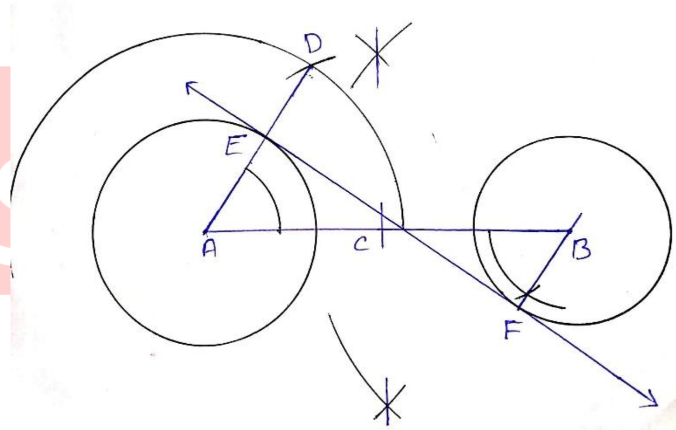
7. Radii of two circles are 2cm and 2.5cm respectively. Distance between their centres is 5.5cm. Draw transverse common tangents to the circles.

Given

Radii of two circles are 2cm and 2.5cm and distance between their centres is 5.5cm

Required

Transverse common tangents to the circles.



Steps of construction

1. Draw a line $m\overline{AB} = 8\ cm$ and produce to both directions.
2. With centre A, draw a circle of radius 2.5cm.
3. With centre B, draw another circle of radius 2cm.
4. With centre A and radius 4.5cm ($2.5cm + 2cm = 4.5cm$), draw semi-circle.
5. Bisect $\overline{AB}$ at point C.
6. With centre C and radius $m\overline{AC}$, draw an arc which cuts the semi-circle at point D.
7. Draw $\overline{AD}$ to the meet the bigger circle at point E.
8. Draw $\overline{BF} || \overline{AE}$.
9. Join E and F and produce the line.
10. Thus $\overleftrightarrow{EF}$ is the required transverse common tangent of the given circles.

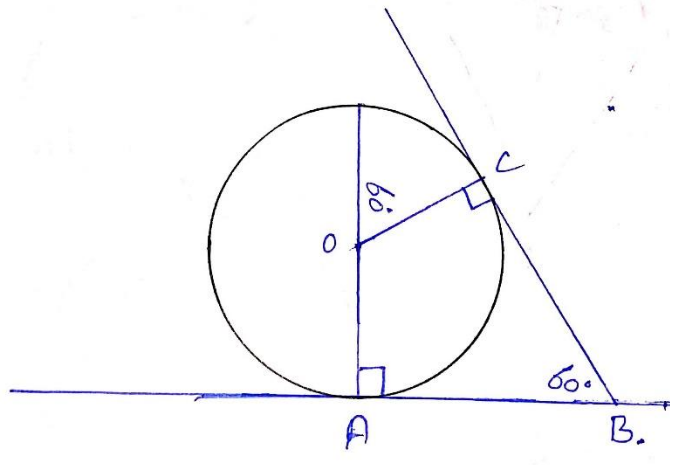
8. Draw ∠ABC of measure 60^0. Construct a circle having radius 2.5 cm and touching the arms of the angle.

Given

$\angle ABC = 60^o$ and circle of radius 2.5 cm

Required

The given circle touching the arms of angle



Steps of construction

1. With centre O, draw a circle of radius 2.5cm.
2. Draw the diameter $\overline{AB}$ of the circle.
3. At point O, draw an angle of 60^o, which intersects the circle at point C.
4. At point A and C, draw angles of 90^o and intersect each other at point B.
5. Hence $\angle ABC$ should be 60^o
6. Thus, the circle touches the arm of given angle.

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UNIT # 1

QUADRATIC EQUATIONS

Ex # 1.1

Quadratic equation

Introduction:
The name Quadratic comes from "quad" means square because the highest power of the variable is 2.

Definition:
An equation in the form of $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$

OR

An equation of degree/ exponent/ power 2 is called **quadratic equation**

$ax^2 + bx + c = 0$ is quadratic equation in one variable.

Solutions or Roots of equation:
All those values of the variable for which the given equation is true are called **solutions or roots** of the equation.

Solution Set:
The set of all solutions is called **solution set**.

Example:

$\mathbf{x^2 - 9 = 0}$

$x^2 - 3^2 = 0$

$(x+3)(x-3) = 0$

$x + 3 = 0 \quad or \quad x - 3 = 0$

$x = -3 \quad or \quad x = 3$

$x^2 - 9 = 0$ is true only for $x = 3$ *and* $x = -3$, Hence $x = 3$ *and* $x = -3$ are the solutions or roots of $x^2 - 9 = 0$ and $\{3, -3\}$ is the solution set of $x^2 - 9 = 0$

Note:

1. $ax^2 + bx + c = 0$ is called General or Standard form of Quadratic equation.
2. In Quadratic equation $a \neq 0$

Methods of Solving Quadratic equation:

(a) By Factorization
(b) By Completing the Square
(c) By Quadratic Formula

Ex # 1.1

Solving Quadratic equation by Factorization:

In this method, a quadratic equation can easily be solved by splitting it in factors.

Rules for Splitting:

Let $ax^2 + bx + c = 0$

1. Write the quadratic equation in standard form if necessary like above equation.
2. First we find the product of a(coefficient of x^2) and c (constant term) i.e. ac
3. Then find two numbers b_1 and b_2 such that b_1 and $b_2 = b$ and also $b_1 b_2 = a.c$
4. Now $ax^2 + b_1 x + b_2 x + c = 0$ can be factorized into two linear factors.
5. Equate each factor to zero by zero – product property.
6. Solve the equation for given variable.

Example # 1 (i)

$\mathbf{2x^2 + 2x - 11 = 1}$

Solution:

R.W	
$(x^2)(-6) = -6x^2$	
Add	**Multiply**
$+3x$	$+3x$
$-2x$	$-2x$
$+x$	$-6x^2$

$2x^2 + 2x - 11 = 1$

$2x^2 + 2x - 11 - 1 = 0$

$2x^2 + 2x - 12 = 0$

$2(x^2 + x - 6) = 0$

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x+3) - 2(x+3) = 0$

$x + 1 = 0 \quad or \quad x + 4 = 0$

$x = -1 \quad or \quad x = -4$

Solution Set $= \{-1, -4\}$

Example # 1 (ii)

$\mathbf{12t^2 = t + 1}$

Solution:

$12t^2 = t + 1$

$12t^2 - t - 1 = 0$

$12t^2 + 3t - 4t - 1 = 0$

$3t(4t + 1) - 1(4t + 1) = 0$

$(4t + 1)(3t - 1) = 0$

$4t + 1 = 0 \quad or \quad 3t - 1 = 0$

$4t = -1 \quad or \quad 3t = 1$

$t = \frac{-1}{4} \quad or \quad t = \frac{1}{3}$

R.W	
$(12t^2)(-1) = -12t^2$	
Add	Multiply
$+3t$	$+3t$
$-4t$	$-4t$
$-t$	$-12t^2$

Example # 2

A ball is thrown straight up, from 3 m above the ground with a velocity of 14 m/s. When does it hit the ground?

Solution:

Height starts at 3 m = 3

Velocity is 14 m/s = 14t

Gravity pulls by $5m/s^2 = -5t^2$

The height h at any time t is:

$h = 3 + 14t - 5t^2$

The height is zero when the ball hit the ground.

$0 = 3 + 14t - 5t^2$

$0 = -5t^2 + 14t + 3$

$0 = -(5t^2 - 14t - 3)$

$0 = 5t^2 - 14t - 3$

$5t^2 - 14t - 3 = 0$

$5t^2 + 1t - 15t - 3 = 0$

$t(5t + 1) - 3(5t + 1) = 0$

$(5t + 1)(t - 3) = 0$

$5t + 1 = 0 \quad or \quad t - 3 = 0$

$5t = -1 \quad or \quad t = 3$

$t = \frac{-1}{5} \quad or \quad t = 3$

R.W	
$(5t^2)(-3) = -15t^2$	
Add	Multiply
$+1t$	$+1t$
$-15t$	$-15t$
$-14t$	$-15t^2$

The $t = \frac{-1}{5}$ is negative time which is impossible

So the ball hits the ground after 3 seconds.

Ex # 1.1

2. Solving Quadratic equation by Completing square:

1. Write the quadratic equation in its standard form
2. Divide all terms by the co-efficient of x^2 if other than 1
3. Shift the constant term to the right side of the equation.
4. Multiply the co – efficient of x with $\frac{1}{2}$ then take Square of it and Add to B.S
5. Write Left – hand side of the equation as a perfect square and simplify the Right – hand side
6. Take square root on B.S of the equation and solve it.

Example # 3

$\mathbf{x^2 - 8x + 9 = 0}$

Solution:

$x^2 - 8x + 9 = 0$

Subtract 9 from B.S

$x^2 - 8x + 9 - 9 = 0 - 9$

$x^2 - 8x = -9$

Add $(4)^2$ on B.S

$x^2 - 8x + (4)^2 = -9 + (4)^2$

$(x)^2 - 2(x)(4) + (4)^2 = -9 + 16$

$(x - 4)^2 = 7$

$\sqrt{(x - 4)^2} = \pm\sqrt{7}$

$x - 4 = \pm\sqrt{7}$

$x = 4 \pm \sqrt{7}$

$8 \times \frac{1}{2} = 4$
$(4)^2$

3. Solving Quadratic equation by Quadratic Formula:

1. Write the quadratic equation in its standard form:
2. Compare the given equation with the standard quadratic equation $ax^2 + bx + c = 0$ to get the values of a, b and c.
3. Put the values of a, b and c in the given formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Now solve it for the values of variable.

Note:

By Quadrating formula we can solve all quadratic equations.

Ex # 1.1

Derivation of Quadratic Formula:

As we have standard form of quadratic equation:

$ax^2 + bx + c = 0$

Divide each term by "a"

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Shift the constant term $\frac{c}{a}$ to right side

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

Add $\left(\frac{b}{2a}\right)^2$ *of B.S*

$$\frac{b}{a} \times \frac{1}{2} = \frac{b}{2a}$$

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$(x)^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4a}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example # 4

Solve $3x^2 - 6x + 2 = 0$ by quadratic formula.

Solution:

$3x^2 - 6x + 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = -6, c = 2$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

Ex # 1.1

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{6 \pm \sqrt{4 \times 3}}{6}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$x = \frac{2(3 \pm \sqrt{3})}{6}$$

$$x = \frac{3 \pm \sqrt{3}}{3}$$

$$x = \frac{3}{3} \pm \frac{\sqrt{3}}{3}$$

$$x = 1 \pm \frac{\sqrt{3}}{3}$$

Solution Set $= \left\{1 \pm \frac{\sqrt{3}}{3}\right\}$

Example # 5

A company is making frames of a new product they are launching. The frame will be cut out of a piece of steel. To keep the weight down, the final area should be 28 cm^2. The inside of the frame has to be 11 cm by 6 cm. what should the width x of the meatal be?

Solution:

According to condition:

Length $= 11 + 2x$

Width $= 6 + 2x$

Area of steel before cutting

$Area = (11 + 2x)(6 + 2x)$

$Area = 66 + 22x + 12x + 4x^2$

$Area = 66 + 34x + 4x^2$

$Area = 4x^2 + 34x + 66$

Area of steel after cutting out the 11×6 middle

$Area = 4x^2 + 34x + 66 - 11 \times 6$

$Area = 4x^2 + 34x$

As Area is 28 cm^2

$28 = 4x^2 + 34x$

$0 = 4x^2 + 34x - 28$

$4x^2 + 34x - 28 = 0$

$2(2x^2 + 17x - 14) = 0$

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$2x^2 + 17x - 14 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = 17, c = -14$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(17) \pm \sqrt{(17)^2 - 4(2)(-14)}}{2(2)}$$

$$x = \frac{-17 \pm \sqrt{289 + 112}}{4}$$

$$x = \frac{-17 \pm \sqrt{401}}{4}$$

$$x = \frac{-17 + \sqrt{401}}{4} \quad or \quad x = \frac{-17 - \sqrt{401}}{4}$$

$x = 0.8 \quad or \quad x = -9.3$

As length cannot be negative.

So width $= 0.8$

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Ex # 1.1

Page # 8

Q1: Solve each of the following equations by factorization.

(i) $\boldsymbol{x^2 + 5x + 4 = 0}$

Solution:

$x^2 + 5x + 4 = 0$

$x^2 + 1x + 4x + 4 = 0$

$x(x + 1) + 4(x + 1) = 0$

$(x + 1)(x + 4) = 0$

$x + 1 = 0 \quad or \quad x + 4 = 0$

$x = -1 \quad or \quad x = -4$

Solution Set $= \{-1, -4\}$

R.W	
$(x^2)(4) = 4x^2$	
Add	Multiply
$+1x$	$+1x$
$+4x$	$+4x$
$+5x$	$4x^2$

(ii) $\boldsymbol{(x - 3)^2 = 4}$

Solution:

$(x - 3)^2 = 4$

$(x)^2 - 2(x)(3) + (3)^2 = 4$

$x^2 - 6x + 9 = 4$

$x^2 - 6x + 9 - 4 = 0$

$x^2 - 6x + 5 = 0$

$x^2 - 1x - 5x + 5 = 0$

$x(x - 1) - 5(x - 1) = 0$

$(x - 1)(x - 5) = 0$

$x - 1 = 0 \quad or \quad x - 5 = 0$

R.W	
$(x^2)(5) = 5x^2$	
Add	Multiply
$-1x$	$-1x$
$-5x$	$-5x$
$-6x$	$5x^2$

$x = 1 \quad or \quad x = 5$

Solution Set $= \{1, 5\}$

(iii) $\boldsymbol{x^2 + 3x - 10 = 0}$

Solution:

$x^2 + 3x - 10 = 0$

$x^2 - 2x + 5x - 10 = 0$

$x(x - 2) + 5(x - 2) = 0$

$(x - 2)(x + 5) = 0$

$x - 2 = 0 \quad or \quad x + 5 = 0$

$x = 2 \quad or \quad x = -5$

Solution Set $= \{2, -5\}$

R.W	
$(x^2)(-10) = -10x^2$	
Add	Multiply
$-2x$	$-2x$
$+5x$	$+5x$
$+3x$	$-10x^2$

(iv) $\boldsymbol{6x^2 - 13x + 5 = 0}$

Solution:

$6x^2 - 13x + 5 = 0$

$6x^2 - 3x - 10x + 5 = 0$

$3x(2x - 1) - 5(2x - 1) = 0$

$(2x - 1)(3x - 5) = 0$

$2x - 1 = 0 \quad or \quad 3x - 5 = 0$

$2x = 1 \quad or \quad 3x = 5$

$x = \frac{1}{2} \quad or \quad x = \frac{5}{3}$

Solution Set $= \left\{\frac{1}{2}, \frac{5}{3}\right\}$

R.W	
$(6x^2)(5) = 30x^2$	
Add	Multiply
$-3x$	$-3x$
$-10x$	$-10x$
$-13x$	$30x^2$

(v) $\boldsymbol{3(x^2 - 1) = 4(x - 1)}$

Solution:

$3(x^2 - 1) = 4(x - 1)$

$3x^2 - 3 = 4x - 4$

$3x^2 - 3 - 4x - 4 = 0$

$3x^2 - 4x - 3 - 4 = 0$

$3x^2 - 4x - 7 = 0$

$3x^2 + 3x - 7x - 7 = 0$

$3x(x + 1) - 7(x + 1) = 0$

$(x + 1)(3x - 7) = 0$

$x + 1 = 0 \quad or \quad 3x - 7 = 0$

$x = -1 \quad or \quad 3x = 7$

$x = -1 \quad or \quad x = \frac{7}{3}$

Solution Set $= \left\{-1, \frac{7}{3}\right\}$

R.W	
$(3x^2)(-7) = -21x^2$	
Add	Multiply
$+3x$	$+3x$
$-7x$	$-7x$
$-4x$	$-21x^2$

Ex # 1.1

(vi) $\boldsymbol{x(3x-5)=(x-6x)(x-7)}$

Solution:

$x(3x-5)=(x-6x)(x-7)$
$3x^2-5x=x^2-7x-6x+42$
$3x^2-5x=x^2-13x+42$
$3x^2-5x-x^2+13x-42=0$
$3x^2-x^2-5x+13x-42=0$
$2x^2+8x-42=0$
$2(x^2+4x-21)=0$
Divide B.S by 2, we get
$x^2+4x-21=0$
$x^2+7x-3x-21=0$
$x(x+7)-3(x+7)=0$
$(x+7)(x-3)=0$
$x+7=0 \quad or \quad x-3=0$
$x=-7 \quad or \quad x=3$
Solution Set $=\{-7,3\}$

R.W	
$(x^2)(-21)=-21x^2$	
Add	Multiply
$+7x$	$+7x$
$-3x$	$-3x$
$+4x$	$-21x^2$

Q2: **Solve each of the following equations by completing the square.**

(i) $\boldsymbol{x^2+6x-40=0}$

Solution:

$x^2+6x-40=0$
Add 40 on B.S
$x^2+6x-40+40=0+40$
$x^2+6x=40$
Add $(3)^2$ on B.S
$x^2+6x+(3)^2=40+(3)^2$
$(x)^2+2(x)(3)+(3)^2=40+9$
$(x+3)^2=49$
Taking square root on B.S
$\sqrt{(x+3)^2}=\pm\sqrt{49}$
$x+3=\pm 7$
$x+3=7 \quad or \quad x+3=-7$
$x=7-3 \quad or \quad x=-7-3$
$x=4 \quad or \quad x=-10$
Solution Set $=\{4,-10\}$

$6\times\frac{1}{2}=3$

(ii) $\boldsymbol{x^2-10x+11=0}$

Solution:

$x^2-10x+11=0$
Subtract 11 from B.S
$x^2-10x+11-11=0-11$
$x^2-10x=-11$

Ex # 1.1

Add $(5)^2$ on B.S
$x^2-10x+(5)^2=-11+(5)^2$
$(x)^2-2(x)(5)+(5)^2=-11+25$
$(x-5)^2=14$
Taking on square root on B.S
$\sqrt{(x-5)^2}=\pm\sqrt{14}$
$x-5=\pm\sqrt{14}$
$x-5=\sqrt{14} \quad or \quad x-5=-\sqrt{14}$
$x=5+\sqrt{14} \quad or \quad x=5-\sqrt{14}$
Solution Set $=\{5+\sqrt{14},5-\sqrt{14}\}$

$10\times\frac{1}{2}=5$

(iii) $\boldsymbol{4x^2+12x=0}$

Solution:

$4x^2+12x=0$
Divide all terms by 4
$\frac{4x^2}{4}+\frac{12x}{4}=\frac{0}{4}$
$x^2+3x=0$
Add $\left(\frac{3}{2}\right)^2$ on B.S
$x^2+3x+\left(\frac{3}{2}\right)^2=0+\left(\frac{3}{2}\right)^2$
$(x)^2+2(x)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2=\frac{9}{4}$
$\left(x+\frac{3}{2}\right)^2=\frac{9}{4}$
Taking square root on B.S
$\sqrt{\left(x+\frac{3}{2}\right)^2}=\pm\sqrt{\frac{9}{4}}$
$x+\frac{3}{2}=\pm\frac{3}{2}$
$x+\frac{3}{2}=\frac{3}{2} \quad or \quad x+\frac{3}{2}=-\frac{3}{2}$
$x=\frac{3}{2}-\frac{3}{2} \quad or \quad x=-\frac{3}{2}-\frac{3}{2}$
$x=0 \quad or \quad x=\frac{-3-3}{2}$
$x=0 \quad or \quad x=\frac{-6}{2}$
$x=0 \quad or \quad x=-3$
Solution Set $=\{0,-3\}$

$3\times\frac{1}{2}=\frac{3}{2}$

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Ex # 1.1

(iv) $\mathbf{5x^2 - 10x - 840 = 0}$

Solution:

$5x^2 - 10x - 840 = 0$

Divide all terms by 5

$\frac{5x^2}{5} - \frac{10x}{5} - \frac{840}{5} = \frac{0}{5}$

$2 \times \frac{1}{2} = 1$

$x^2 - 2x - 168 = 0$

Add 168 on B.S

$x^2 - 2x - 168 + 168 = 0 + 168$

$x^2 - 2x = 168$

Add $(1)^2$ on B.S

$x^2 - 2x + (1)^2 = 168 + (1)^2$

$(x)^2 - 2(x)(1) + (1)^2 = 168 + 1$

$(x - 1)^2 = 169$

Taking square root on B.S

$\sqrt{(x-1)^2} = \pm\sqrt{169}$

$x - 1 = \pm 13$

$x - 1 = 13 \quad or \quad x - 1 = -13$

$x = 13 + 1 \quad or \quad x = -13 + 1$

$x = 14 \quad or \quad x = -12$

Solution Set $= \{\mathbf{14}, \mathbf{-12}\}$

(v) $\mathbf{9x^2 - 6x + \frac{5}{9} = 0}$

Solution:

$9x^2 - 6x + \frac{5}{9} = 0$

Divide all terms by 9

$\frac{9x^2}{9} - \frac{6x}{9} + \frac{\frac{5}{9}}{9} = \frac{0}{9}$

$x^2 - \frac{2x}{3} + \frac{5}{9} \div 9 = 0$

$x^2 - \frac{2x}{3} + \frac{5}{9} \times \frac{1}{9} = 0$

$x^2 - \frac{2x}{3} + \frac{5}{81} = 0$

Subtract $\frac{5}{81}$ from B.S

$x^2 - \frac{2x}{3} + \frac{5}{81} - \frac{5}{81} = 0 - \frac{5}{81}$

Ex # 1.1

$x^2 - \frac{2x}{3} = -\frac{5}{81}$

$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

Add $\left(\frac{1}{3}\right)^2$ on B.S

$x^2 - \frac{2x}{3} + \left(\frac{1}{3}\right)^2 = -\frac{5}{81} + \left(\frac{1}{3}\right)^2$

$(x)^2 - 2(x)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 = -\frac{5}{81} + \frac{1}{9}$

$\left(x - \frac{1}{3}\right)^2 = \frac{-5+9}{81}$

$\left(x - \frac{1}{3}\right)^2 = \frac{4}{81}$

Taking square root on B.S

$\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{\frac{4}{81}}$

$x - \frac{1}{3} = \pm\frac{2}{9}$

$x - \frac{1}{3} = \frac{2}{9} \quad or \quad x - \frac{1}{3} = -\frac{2}{9}$

$x = \frac{2}{9} + \frac{1}{3} \quad or \quad x = -\frac{2}{9} + \frac{1}{3}$

$x = \frac{2+3}{9} \quad or \quad x = \frac{-2+3}{9}$

$x = \frac{5}{9} \quad or \quad x = \frac{1}{9}$

Solution Set $= \left\{\mathbf{\frac{5}{9}}, \mathbf{\frac{1}{9}}\right\}$

(vi) $\mathbf{(x-1)(x+3) = 5(x+2) - 3}$

Solution:

$(x-1)(x+3) = 5(x+2) - 3$

$x^2 + 3x - 1x - 3 = 5x + 10 - 3$

$x^2 + 2x - 3 = 5x + 7$

$x^2 + 2x - 3 = 5x + 7$

$x^2 + 2x - 3 - 5x - 7 = 0$

$x^2 + 2x - 5x - 3 - 7 = 0$

$x^2 - 3x - 10 = 0$

Add 10 on B.S

$x^2 - 3x - 10 + 10 = 0 + 10$

$x^2 - 3x = 10$

$3 \times \frac{1}{2} = \frac{3}{2}$

Add $\left(\frac{3}{2}\right)^2$ on B.S

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$

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Ex # 1.1

$$(x)^2 - 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = 10 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{40+9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{49}{4}$$

Taking square root on B.S

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \pm\sqrt{\frac{49}{4}}$$

$$x - \frac{3}{2} = \pm\frac{7}{2}$$

$$x - \frac{3}{2} = \frac{7}{2} \quad or \quad x + \frac{3}{2} = -\frac{7}{2}$$

$$x = \frac{7}{2} + \frac{3}{2} \quad or \quad x = -\frac{7}{2} + \frac{3}{2}$$

$$x = \frac{7+3}{2} \quad or \quad x = \frac{-7+3}{2}$$

$$x = \frac{7+3}{2} \quad or \quad x = \frac{-7+3}{2}$$

$$x = \frac{10}{2} \quad or \quad x = \frac{-4}{2}$$

$$x = 5 \quad or \quad x = -2$$

Solution Set $= \{\mathbf{5}, \mathbf{-2}\}$

Q3: Solve each of the following equations by quadratic formula.

(i) $\mathbf{x^2 - 8x + 15 = 0}$

Solution:

$x^2 - 8x + 15 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -8, c = 15$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{8 \pm \sqrt{4}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

$$x = \frac{8+2}{2} \quad or \quad x = \frac{8-2}{2}$$

Ex # 1.1

$$x = \frac{10}{2} \quad or \quad x = \frac{6}{2}$$

$$x = 5 \quad or \quad x = 3$$

Solution Set $= \{\mathbf{5}, \mathbf{3}\}$

(ii) $\mathbf{x^2 - 2x - 4 = 0}$

Solution:

$x^2 - 2x - 4 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -2, c = -4$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 5}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(1 \pm \sqrt{5})}{2}$$

$$x = 1 \pm \sqrt{5}$$

$$x = 1 + \sqrt{5} \quad or \quad x = 1 - \sqrt{5}$$

Solution Set $= \{\mathbf{1 + \sqrt{5}}, \mathbf{1 - \sqrt{5}}\}$

(iii) $\mathbf{4x^2 + 3x = 0}$

Solution:

$4x^2 + 3x = 0$

Or $4x^2 + 3x + 0 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = 3, c = 0$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(0)}}{2(4)}$$

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Ex # 1.1

$$x=\frac{-3\pm\sqrt{9-0}}{8}$$
$$x=\frac{-3\pm\sqrt{9}}{8}$$
$$x=\frac{-3\pm3}{8}$$
$$x=\frac{-3+3}{8}\quad or\quad x=\frac{-3-3}{8}$$
$$x=\frac{0}{8}\quad or\quad x=\frac{-6}{8}$$
$$x=0\quad or\quad x=\frac{-3}{4}$$

Solution Set $=\left\{\mathbf{0},\frac{\mathbf{-3}}{\mathbf{4}}\right\}$

(iv) $\mathbf{3x(x-2)+1=0}$

Solution:

$3x(x-2)+1=0$

$3x^2-6x+1=0$

Compare it with $ax^2+bx+c=0$

Here $a=3, b=-6, c=1$

As we have
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
Put the values
$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(1)}}{2(3)}$$
$$x=\frac{6\pm\sqrt{36-12}}{6}$$
$$x=\frac{6\pm\sqrt{24}}{6}$$
$$x=\frac{6\pm\sqrt{4\times6}}{6}$$
$$x=\frac{6\pm2\sqrt{6}}{6}$$
$$x=\frac{2(3\pm\sqrt{6})}{6}$$
$$x=\frac{3\pm\sqrt{6}}{3}$$
$$x=\frac{3+\sqrt{6}}{3}\quad or\quad x=\frac{3-\sqrt{6}}{3}$$
$$x=\frac{3}{3}+\frac{\sqrt{6}}{3}\quad or\quad x=\frac{3}{3}-\frac{\sqrt{6}}{3}$$
$$x=1+\frac{\sqrt{6}}{3}\quad or\quad x=1-\frac{\sqrt{6}}{3}$$

Solution Set $=\left\{\mathbf{1}+\frac{\sqrt{\mathbf{6}}}{\mathbf{3}},\mathbf{1}-\frac{\sqrt{\mathbf{6}}}{\mathbf{3}}\right\}$

Ex # 1.1

(v) $\mathbf{6x^2-17x+12=0}$

Solution:

$6x^2-17x+12=0$

Compare it with $ax^2+bx+c=0$

Here $a=6, b=-17, c=12$

As we have
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
Put the values
$$x=\frac{-(-17)\pm\sqrt{(-17)^2-4(6)(12)}}{2(6)}$$
$$x=\frac{17\pm\sqrt{289-288}}{12}$$
$$x=\frac{17\pm\sqrt{1}}{12}$$
$$x=\frac{17\pm1}{12}$$
$$x=\frac{17+1}{12}\quad or\quad x=\frac{17-1}{12}$$
$$x=\frac{18}{12}\quad or\quad x=\frac{16}{12}$$
$$x=\frac{3}{2}\quad or\quad x=\frac{4}{3}$$

Solution Set $=\left\{\frac{\mathbf{3}}{\mathbf{2}},\frac{\mathbf{4}}{\mathbf{3}}\right\}$

(vi) $\frac{\mathbf{x^2}}{\mathbf{3}}-\frac{\mathbf{x}}{\mathbf{12}}=\frac{\mathbf{1}}{\mathbf{24}}$

Solution:
$$\frac{x^2}{3}-\frac{x}{12}=\frac{1}{24}$$
Multiply all terms by 24
$$24\times\frac{x^2}{3}-24\times\frac{x}{12}=24\times\frac{1}{24}$$
$8x^2-2x=1$

$8x^2-2x-1=0$

Compare it with $ax^2+bx+c=0$

Here $a=8, b=-2, c=-1$

As we have
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
Put the values
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(8)(-1)}}{2(8)}$$

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Ex # 1.1

$x = \frac{2 \pm \sqrt{4+32}}{16}$

$x = \frac{2 \pm \sqrt{36}}{16}$

$x = \frac{2 \pm 6}{16}$

$x = \frac{2+6}{16}$ or $x = \frac{2-6}{16}$

$x = \frac{8}{16}$ or $x = \frac{-4}{16}$

$x = \frac{1}{2}$ or $x = \frac{-1}{4}$

Solution Set $= \left\{\frac{1}{2}, \frac{-1}{4}\right\}$

Q4: Find all the solutions to the following equations.

(i) $t^2 - 8t + 7 = 0$

Solution:

$t^2 - 8t + 7 = 0$

$t^2 - 1t - 7t + 7 = 0$

$t(t-1) - 7(t-1) = 0$

$(t-1)(t-7) = 0$

$t - 1 = 0$ or $t - 7 = 0$

$t = 1$ or $t = 7$

Solution Set $= \{1, 7\}$

R.W	
$(t^2)(7) = 7t^2$	
Add	Multiply
$-1t$	$-1t$
$-7t$	$-7t$
$-8t$	$-21x^2$

(ii) $72 + 6x = x^2$

Solution:

$72 + 6x = x^2$

$0 = x^2 - 6x - 72$

$x^2 - 6x - 72 = 0$

$x^2 + 6x - 12x - 72 = 0$

$x(x+6) - 12(x+6) = 0$

$(x+6)(x-12) = 0$

$x + 6 = 0$ or $x - 12 = 0$

$x = -6$ or $x = 12$

Solution Set $= \{-6, 12\}$

R.W	
$(x^2)(-72) = -72x^2$	
Add	Multiply
$+6x$	$+6x$
$-12x$	$-12x$
$-6x$	$-72x^2$

(iii) $r^2 + 4r + 1 = 0$

Solution:

$r^2 + 4r + 1 = 0$

Compare it with $ar^2 + br + c = 0$

Here $a = 1, b = 4, c = 1$

Ex # 1.1

As we have

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Put the values

$r = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)}$

$r = \frac{-4 \pm \sqrt{16-4}}{2}$

$r = \frac{-4 \pm \sqrt{12}}{2}$

$r = \frac{-4 \pm \sqrt{4 \times 3}}{2}$

$r = \frac{-4 \pm 2\sqrt{3}}{2}$

$r = \frac{2(-2 \pm \sqrt{3})}{2}$

$r = -2 \pm \sqrt{3}$

$r = -2 + \sqrt{3}$ or $r = -2 - \sqrt{3}$

Solution Set $= \{-2 + \sqrt{3}, -2 - \sqrt{3}\}$

(iv) $x(x+10) = 10(-10-x)$

Solution:

$x(x+10) = 10(-10-x)$

$x^2 + 10x = -100 - 10x$

$x^2 + 10x + 10x + 100 = 0$

$x(x+10) + 10(x+10) = 0$

$(x+10)(x+10) = 0$

$x + 10 = 0$ or $x + 10 = 0$

$x = -10$ or $x = -10$

Solution Set $= \{-10\}$

Q5: The equation $(y+13)(y+a)$ has no linear term. Find value of a.

Solution:

$(y+13)(y+a)$

$= y^2 + ay + 13y + 13a$

$= y^2 + (a+13)y + 13a$

A linear term is term with a degree/power of 1.

Here y is a linear term.

As there is no linear term then the co − efficient of y must be zero.

So $a + 13 = 0$

$a = -13$

Thus for $a = -13$,

the above equation has no linear term.

Ex # 1.1

Q6: The equation $ax^2 + 5x = 3$ has $x = 1$ as a solution. What is the other solution?

Solution:

$ax^2 + 5x = 3 \quad ......equ\ (i)$

As $x = 1$ is the solution

So put $x = 1$ in equ (i)

$a(1)^2 + 5(1) = 3$

$a + 5 = 3$

$a = 3 - 5$

$a = -2$

put $a = -2$ in equ (i) to become equation

$-2x^2 + 5x = 3$

$-2x^2 + 5x - 3 = 0$

$-(2x^2 - 5x + 3) = 0$

$2x^2 - 5x + 3 = 0$

$2x^2 - 2x - 3x + 3 = 0$

$2x(x - 1) - 3(x - 1) = 0$

$(x - 1)(2x - 3) = 0$

$x - 1 = 0 \quad or \quad 2x - 3 = 0$

$x = 1 \quad or \quad 2x = 3$

$x = 1 \quad or \quad x = \frac{3}{2}$

R.W	
$(2x^2)(3) = 6x^2$	
Add	**Multiply**
$-2x$	$-2x$
$-3x$	$-3x$
$-5x$	$6x^2$

Thus the other solution is $x = \frac{3}{2}$

Q7: What is the positive difference of the roots of $x^2 - 7x - 9 = 0$?

Solution:

$x^2 - 7x - 9 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -7, c = -9$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 + 36}}{2}$$

$$x = \frac{7 \pm \sqrt{85}}{2}$$

$$x = \frac{7 + \sqrt{85}}{2} \quad or \quad x = \frac{7 - \sqrt{85}}{2}$$

Now the positive difference of roots is given by

$$\frac{7 + \sqrt{85}}{2} - \frac{7 - \sqrt{85}}{2} = \frac{7 + \sqrt{85} - (7 - \sqrt{85})}{2}$$

Ex # 1.1

$$\frac{7 + \sqrt{85}}{2} - \frac{7 - \sqrt{85}}{2} = \frac{7 + \sqrt{85} - 7 + \sqrt{85}}{2}$$

$$\frac{7 + \sqrt{85}}{2} - \frac{7 - \sqrt{85}}{2} = \frac{2\sqrt{85}}{2}$$

$$\frac{7 + \sqrt{85}}{2} - \frac{7 - \sqrt{85}}{2} = \sqrt{85}$$

Ex # 1.2

Solution of Equations reducible to Quadratic form

Type 1: $\boldsymbol{ax^4 + bx^2 + c = 0}$

1. Make $(x^2)^2$
2. Put $x^2 = y$
3. We get quadratic equation.
4. Solve Quadratic equation on any method.

Note

Polynomials of degree four is called biquadratic

Example # 6

$\boldsymbol{12x^4 - 11x^2 + 2 = 0}$

Solution:

$12x^4 - 11x^2 + 2 = 0$

$12(x^2)^2 - 11x^2 + 2 = 0$

Let $x^2 = y$

$12(y)^2 - 11y + 2 = 0$

$12y^2 - 11y + 2 = 0$

$12y^2 - 3y - 8y + 2 = 0$

$3y(4y - 1) - 2(4y - 1) = 0$

$(4y - 1)(3y - 2) = 0$

R.W	
$(12y^2)(2) = 24y^2$	
Add	**Multiply**
$-3y$	$-3y$
$-8y$	$-8y$
$-11y$	$24y^2$

$4y - 1 = 0 \quad or \quad 3y - 2 = 0$

$4y = 1 \quad or \quad 3y = 2$

$y = \frac{1}{4} \quad or \quad y = \frac{2}{3}$

$But\ y = x^2$

$x^2 = \frac{1}{4} \quad or \quad x^2 = \frac{2}{3}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{4}} \quad or \quad \sqrt{x^2} = \pm\sqrt{\frac{2}{3}}$

$x = \pm\frac{1}{2} \quad or \quad x = \pm\sqrt{\frac{2}{3}}$

Solution Set $= \left\{\pm\frac{1}{2}, \pm\sqrt{\frac{2}{3}}\right\}$

Ex # 1.2

Type 2: $\boldsymbol{p(x)+\frac{1}{p(x)}=0}$

Example # 7

$\boldsymbol{2x+\frac{4}{x}=9}$

Solution:

$2x+\frac{4}{x}=9$

Multiply all terms by x

$2x.x+\frac{4}{x}.x=9.x$

$2x^2+4=9x$

$2x^2+4-9x=0$

$2x^2-9x+4=0$

This is quadratic equation and solve it by factorization method.

$2x^2-1x-8x+4=0$

$x(2x-1)-4(2x-1)=0$

$(2x-1)(x-4)=0$

$2x-1=0 \quad or \quad x-4=0$

$2x=1 \quad or \quad x=4$

$x=\frac{1}{2} \quad or \quad x=4$

R.W	
$(2x^2)(4)=8x^2$	
Add	Multiply
$-1x$	$-1x$
$-8x$	$-8x$
$-9x$	$8x^2$

Solution Set $=\left\{\frac{1}{2},\mathbf{4}\right\}$

Example # 8

$\boldsymbol{\frac{x-1}{x+3}+\frac{x+3}{x-1}=\frac{13}{6}}$

Solution:

$\frac{x-1}{x+3}+\frac{x+3}{x-1}=\frac{13}{6}$

Let $\frac{x-1}{x+3}=y$ then $\frac{x+3}{x-1}=\frac{1}{y}$

$y+\frac{1}{y}=\frac{13}{6}$

Multiply all terms by $6y$

$6y\times y+6y\times\frac{1}{y}=\frac{13}{6}\times 6y$

$6y^2+6=13y$

$6y^2+6-13y=0$

$6y^2-13y+6=0$

$6y^2-4y-9y+6=0$

$2y(3y-2)-3(3y-2)=0$

$(3y-2)(2y-3)=0$

R.W	
$(6y^2)(6)=36y^2$	
Add	Multiply
$-4y$	$-4y$
$-9y$	$-9y$
$-13y$	$36y^2$

Ex # 1.2

$3y-2=0 \quad or \quad 2y-3=0$

$3y=2 \quad or \quad 2y=3$

$y=\frac{2}{3} \quad or \quad y=\frac{3}{2}$

$But\ y=\frac{x-1}{x+3}$

$\frac{x-1}{x+3}=\frac{2}{3} \quad or \quad \frac{x-1}{x+3}=\frac{3}{2}$

By cross multiplication

$3(x-1)=2(x+3) \quad or \quad 2(x-1)=3(x+3)$

$3x-3=2x+6 \quad or \quad 2x-2=3x+9$

$3x-2x=6+3 \quad or \quad 2x-3x=9+2$

$x=9 \quad or \quad -1x=11$

$x=9 \quad or \quad x=-11$

Solution Set $=\{\mathbf{9},\mathbf{-11}\}$

Type 3: Receproccal equation

$\boldsymbol{a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0}$

Example # 9 (i)

$\boldsymbol{2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0}$

Solution:

$2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$ --equ (i)

$Let\ x+\frac{1}{x}=y$

Taking square on B.S

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2$

$x^2+\frac{1}{x^2}=y^2-2$

R.W	
$(2y^2)(10)=20y^2$	
Add	Multiply
$-4y$	$-4y$
$-5y$	$-5y$
$-9y$	$20y^2$

So equ (i) becomes

$2(y^2-2)-9(y)+14=0$

$2y^2-4-9y+14=0$

$2y^2-9y-4+14=0$

$2y^2-9y+10=0$

$2y^2-4y-5y+10=0$

$2y(y-2)-5(y-2)=0$

$(y-2)(2y-5)=0$

$y-2=0 \quad or \quad 2y-5=0$

$y=2 \quad or \quad 2y=5$

$y=2 \quad or \quad y=\frac{5}{2}$

Ex # 1.2

$But\ y = x + \frac{1}{x}$

$x + \frac{1}{x} = 2 \quad or \quad x + \frac{1}{x} = \frac{5}{2}$

Now

$x + \frac{1}{x} = 2$

Multiply all terms by x

$x.x + x.\frac{1}{x} = 2.x$

$x^2 + 1 = 2x$

$x^2 + 1 - 2x = 0$

$x^2 - 2x + 1 = 0$

R.W	
$(x^2)(1) = 1x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-1x$	$-1x$
$\mathbf{-2x}$	$\mathbf{1x^2}$

$x^2 - 1x - 1x + 1 = 0$

$x(x-1) - 1(x-1) = 0$

$(x-1)(x-1) = 0$

$x - 1 = 0 \quad or \quad x - 1 = 0$

$x = 1 \quad or \quad x = 1$

Also

$x + \frac{1}{x} = \frac{5}{2}$

Multiply all terms by $3x$

$2x.x + 2x.\frac{1}{x} = \frac{5}{2}.2x$

$2x^2 + 2 = 5x$

$2x^2 + 2 - 5x = 0$

$2x^2 - 5x + 2 = 0$

R.W	
$(2x^2)(2) = 4x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-4x$	$-4x$
$\mathbf{-5x}$	$\mathbf{4x^2}$

$2x^2 - 1x - 4x + 2 = 0$

$x(2x-1) - 2(2x-1) = 0$

$(2x-1)(x-2) = 0$

$2x - 1 = 0 \quad or \quad x - 2 = 0$

$2x = 1 \quad or \quad x = 2$

$x = \frac{1}{2} \quad or \quad x = 2$

Solution Set $= \left\{\mathbf{1}, \frac{1}{2}, \mathbf{2}\right\}$

Example # 9 (ii)

$$\mathbf{8\left(x^2 + \frac{1}{x^2}\right) - 42\left(x - \frac{1}{x}\right) + 29 = 0}$$

Solution:

$8\left(x^2 + \frac{1}{x^2}\right) - 42\left(x - \frac{1}{x}\right) + 29 = 0 \quad -$ equ (i)

Let $x - \frac{1}{x} = y$

Ex # 1.2

Taking square on B.S

$\left(x - \frac{1}{x}\right)^2 = y^2$

$x^2 + \frac{1}{x^2} - 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 + 2$

So equ (i) becomes

$8(y^2 + 2) - 42(y) + 29 = 0$

$8y^2 + 16 - 42y + 29 = 0$

$8y^2 - 42y + 45 = 0$

R.W	
$(8y^2)(45) = 360y^2$	
Add	**Multiply**
$-12y$	$-12y$
$-30y$	$-30y$
$\mathbf{-42y}$	$\mathbf{360y^2}$

$8y^2 - 12y - 30y + 45 = 0$

$4y(2y-3) - 15(2y-3) = 0$

$(2y-3)(4y-15) = 0$

$2y - 3 = 0 \quad or \quad 4y - 15 = 0$

$2y = 3 \quad or \quad 4y = 15$

$y = \frac{3}{2} \quad or \quad y = \frac{15}{4}$

$x - \frac{1}{x} = \frac{3}{2} \quad or \quad x - \frac{1}{x} = \frac{15}{4}$

Now

$x - \frac{1}{x} = \frac{3}{2}$

Multiply all terms by $2x$

$2x.x - 2x.\frac{1}{x} = \frac{3}{2}.2x$

$2x^2 - 2 = 3x$

$2x^2 - 2 - 3x = 0$

$2x^2 - 3x - 2 = 0$

R.W	
$(2x^2)(-2) = -4x^2$	
Add	**Multiply**
$+1x$	$+1x$
$-4x$	$-4x$
$\mathbf{-3x}$	$\mathbf{-4x^2}$

$2x^2 + 1x - 4x - 2 = 0$

$x(2x+1) - 2(2x+1) = 0$

$(2x+1)(x-2) = 0$

$2x + 1 = 0 \quad or \quad x - 2 = 0$

$2x = -1 \quad or \quad x = 2$

$x = \frac{-1}{2} \quad or \quad x = 2$

Also

$x - \frac{1}{x} = \frac{15}{4}$

Multiply all terms by $4x$

$4x.x - 4x.\frac{1}{x} = \frac{15}{4}.4x$

$4x^2 - 4 = 15x$

$4x^2 - 4 - 15x = 0$

$4x^2 - 15x - 4 = 0$

R.W	
$(4x^2)(-4) = -16x^2$	
Add	**Multiply**
$+1x$	$+1x$
$-16x$	$-16x$
$\mathbf{-15x}$	$\mathbf{-16x^2}$

$4x^2 + 1x - 16x - 4 = 0$

Ex # 1.2

$x(4x+1)-4(4x+1)=0$

$(4x+1)(x-4)=0$

$4x+1=0 \quad or \quad x-4=0$

$4x=-1 \quad or \quad x=4$

$x=\frac{-1}{4} \quad or \quad x=4$

Solution Set $=\left\{\frac{-1}{2}, \mathbf{2}, \frac{-1}{4}, \mathbf{4}\right\}$

Type 4: Exponential Equation

Equations that involves terms of the form a^x where $a>0, a\neq 1$ are called **exponential equations**

Steps

1. Put $a^x = y$
2. We get quadratic equation.
3. Solve Quadratic equation in term of y on any method.
4. Then put again a^x and solve for x

Note:

One – to – One Property of Exponential Functions

If $b^n = b^m$ then $n=m$

Example # 10

Solve $\mathbf{4.2^{2x}-10.2^x+4=0}$

Solution:

$4.2^{2x}-10.2^x+4=0$

$4.(2^x)^2-10.2^x+4=0$

Let $2^x=y$

$4(y)^2-10y+4=0$

$4y^2-10y+4=0$

$2(2y^2-5y+2)=0$ Divide by 2

$2y^2-5y+2=0$

$2y^2-1y-4y+2=0$

$y(2y-1)-2(2y-1)=0$

$(2y-1)(y-2)=0$

$2y-1=0 \quad or \quad y-2=0$

$2y=1 \quad or \quad y=2$

$y=\frac{1}{2} \quad or \quad y=2$

But $y=2^x$

$2^x=\frac{1}{2} \quad or \quad 2^x=2$

$2^x=2^{-1} \quad or \quad 2^x=2^1$

$x=-1 \quad or \quad x=1$

Solution Set $=\{\mathbf{-1},\mathbf{1}\}$

R.W	
$(2y^2)(2)=4y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-4y$	$-4y$
$\mathbf{-5y}$	$\mathbf{4y^2}$

Ex # 1.2

Example # 11

Solve $\mathbf{2^{2+x}+2^{2-x}=10}$

Solution:

$2^{2+x}+2^{2-x}=10$

$2^2.2^x+2^2.2^{-x}=10$

$4.2^x+\frac{4}{2^x}-10=0$

Let $2^x=y$

$4y+\frac{4}{y}-10=0$

Multiply all terms by y

$4y.y+\frac{4}{y}.y-10.y=0.y$

$4y^2+4-10y=0$

$4y^2-10y+4=0$

$2(2y^2-5y+2)=0$

$2y^2-5y+2=0$ Divide by 2

$2y^2-1y-4y+2=0$

$y(2y-1)-2(2y-1)=0$

$(2y-1)(y-2)=0$

$2y-1=0 \quad or \quad y-2=0$

$2y=1 \quad or \quad y=2$

$y=\frac{1}{2} \quad or \quad y=2$

But $y=2^x$

$2^x=\frac{1}{2} \quad or \quad 2^x=2$

$2^x=2^{-1} \quad or \quad 2^x=2^1$

$x=-1 \quad or \quad x=1$

Solution Set $=\{\mathbf{-1},\mathbf{1}\}$

R.W	
$(2y^2)(2)=4y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-4y$	$-4y$
$\mathbf{-5y}$	$\mathbf{4y^2}$

Type 4: $\mathbf{(x+a)(x+b)(x+c)(x+d)=k}$

where $a+b=c+d$

Example # 12

$\boldsymbol{(x+1)(x+3)(x-2)(x-4)=24}$

Solution:

$(x+1)(x+3)(x-2)(x-4)=24$

Re – arrange it accordingly $1+(-2)=3+(-4)$

$\{(x+1)(x-2)\}\{(x+3)(x-4)\}=24$

$(x^2-2x+1x-2)(x^2-4x+3x-12)=24$

$(x^2-1x-2)(x^2-1x-12)=24$

$(x^2-1x-2)(x^2-1x-12)-24=0$

Let $x^2-1x=y$

$(y-2)(y-12)-24=0$

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$y^2 - 12y - 2y + 24 - 24 = 0$
$y^2 - 14y = 0$
$y(y - 14) = 0$
$y = 0 \quad or \quad y - 14 = 0$
$y = 0 \quad or \quad y = 14$
But $y = x^2 - 1x$
$x^2 - 1x = 0 \quad or \quad x^2 - 1x = 14$
$x^2 - 1x = 0 \quad or \quad x^2 - 1x - 14 = 0$

Now
$x^2 - 1x = 0$
$x(x - 1) = 0$
$x = 0 \quad or \quad x - 1 = 0$
$x = 0 \quad or \quad x = 1$

Also
$x^2 - 1x - 14 = 0$
Compare it with $ax^2 + bx + c = 0$
Here $a = 1, b = -1, c = -14$
As we have
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Put the values
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 + 4(1)(-14)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{1 + 56}}{2}$$
$$x = \frac{1 \pm \sqrt{57}}{2}$$
Solution Set $= \left\{\mathbf{0}, \mathbf{1}, \frac{1 \pm \sqrt{57}}{2}\right\}$

Ex # 1.2

Page # 15

Q1: Solve the following equations.

(i) $\mathbf{x^4 - 5x^2 + 4 = 0}$

Solution:

$x^4 - 5x^2 + 4 = 0$
$(x^2)^2 - 5x^2 + 4 = 0$
Let $x^2 = y$
$(y)^2 - 5y + 4 = 0$
$y^2 - 5y + 4 = 0$
$y^2 - 1y - 4y + 4 = 0$
$y(y - 1) - 4(y - 1) = 0$
$(y - 1)(y - 4) = 0$

R.W	
$(y^2)(4) = 4y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-4y$	$-4y$
$\mathbf{-5y}$	$\mathbf{4y^2}$

$y - 1 = 0 \quad or \quad y - 4 = 0$
$y = 1 \quad or \quad y = 4$
$But\ y = x^2$
$x^2 = 1 \quad or \quad x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{1} \quad or \quad \sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 1 \quad or \quad x = \pm 2$
Solution Set $= \{\pm 1, \pm 2\}$

(ii) $\mathbf{x^4 - 7x^2 + 12 = 0}$

Solution:

$x^4 - 7x^2 + 12 = 0$
$(x^2)^2 - 7x^2 + 12 = 0$
Let $x^2 = y$
$(y)^2 - 7y + 12 = 0$
$y^2 - 7y + 12 = 0$
$y^2 - 3y - 4y + 12 = 0$
$y(y - 3) - 4(y - 3) = 0$
$(y - 3)(y - 4) = 0$
$y - 3 = 0 \quad or \quad y - 4 = 0$
$y = 3 \quad or \quad y = 4$
$But\ y = x^2$
$x^2 = 3 \quad or \quad x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{3} \quad or \quad \sqrt{x^2} = \pm\sqrt{4}$
$x = \pm\sqrt{3} \quad or \quad x = \pm 2$
Solution Set $= \{\pm\sqrt{3}, \pm 2\}$

R.W	
$(y^2)(12) = 12y^2$	
Add	**Multiply**
$-3y$	$-3y$
$-4y$	$-4y$
$\mathbf{-7y}$	$\mathbf{12y^2}$

(iii) $\mathbf{6x^4 - 13x^2 + 5 = 0}$

Solution:

$6x^4 - 13x^2 + 5 = 0$
$6(x^2)^2 - 13x^2 + 5 = 0$
Let $x^2 = y$
$6(y)^2 - 13y + 5 = 0$
$6y^2 - 13 + 5 = 0$
$6y^2 - 3y - 10y + 5 = 0$
$3y(2y - 1) - 5(2y - 1) = 0$
$(2y - 1)(3y - 5) = 0$
$2y - 1 = 0 \quad or \quad 3y - 5 = 0$
$2y = 1 \quad or \quad 3y = 5$
$y = \frac{1}{2} \quad or \quad y = \frac{5}{3}$
$But\ y = x^2$
$x^2 = \frac{1}{2} \quad or \quad x^2 = \frac{5}{3}$

R.W	
$(6y^2)(5) = 30y^2$	
Add	**Multiply**
$-3y$	$-3y$
$-10y$	$-10y$
$\mathbf{-13y}$	$\mathbf{30y^2}$

Ex # 1.2

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \quad or \quad \sqrt{x^2} = \pm\sqrt{\frac{5}{3}}$

$x = \pm\frac{1}{\sqrt{2}} \quad or \quad x = \pm\sqrt{\frac{5}{3}}$

Solution Set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{\frac{5}{3}}\right\}$

(iv) $x + 2 - \frac{1}{x+2} = \frac{3}{2}$

Solution:

$x + 2 - \frac{1}{x+2} = \frac{3}{2}$

Let $x + 2 = y$

$y - \frac{1}{y} = \frac{3}{2}$

R.W	
$(2y^2)(-2) = -4y^2$	
Add	**Multiply**
$+1y$	$+1y$
$-4y$	$-4y$
$-3y$	$-4y^2$

Multiply all terms by $2y$

$2y \times y - 2y \times \frac{1}{y} = \frac{3}{2} \times 2y$

$2y^2 - 2 = 3y$

$2y^2 - 2 - 3y = 0$

$2y^2 - 3y - 2 = 0$

$2y^2 + 1y - 4y - 2 = 0$

$y(2y + 1) - 2(2y + 1) = 0$

$(2y + 1)(y - 2) = 0$

$2y + 1 = 0 \quad or \quad y - 2 = 0$

$2y = -1 \quad or \quad y = 2$

$y = \frac{-1}{2} \quad or \quad y = 2$

But $y = x + 2$

$x + 2 = \frac{-1}{2} \quad or \quad x + 2 = 2$

$x = \frac{-1}{2} - 2 \quad or \quad x = 2 - 2$

$x = \frac{-1-4}{2} \quad or \quad x = 0$

$x = \frac{-5}{2} \quad or \quad x = 0$

Solution Set $= \left\{\frac{-5}{2}, 0\right\}$

(v) $x - \frac{4}{x} = 2$

Solution:

$x - \frac{4}{x} = 2$

Multiply all terms by x

$x.x - x.\frac{4}{x} = 2.x$

$x^2 - 4 = 2x$

$x^2 - 4 - 2x = 0$

$x^2 - 2x - 4 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -2, c = -4$

As we have

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Put the values

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 + 16}}{2}$

$x = \frac{2 \pm \sqrt{20}}{2}$

$x = \frac{2 \pm \sqrt{20}}{2}$

$x = \frac{2 \pm \sqrt{4 \times 5}}{2}$

$x = \frac{2 \pm 2\sqrt{5}}{2}$

$x = \frac{2(1 \pm \sqrt{5})}{2}$

$x = 1 \pm \sqrt{5}$

$x = 1 + \sqrt{5} \quad or \quad x = 1 - \sqrt{5}$

Solution Set $= \{1 + \sqrt{5}, 1 - \sqrt{5}\}$

(vi) $\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$

Solution:

$\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$

Let $\frac{x+2}{x-2} = y$ then $\frac{x-2}{x+2} = \frac{1}{y}$

$y - \frac{1}{y} = \frac{5}{6}$

R.W	
$(6y^2)(-6) = -36y^2$	
Add	**Multiply**
$+4y$	$+4y$
$-9y$	$-9y$
$-5y$	$-36y^2$

Multiply all terms by $6y$

$6y \times y - 6y \times \frac{1}{y} = \frac{5}{6} \times 6y$

$6y^2 - 6 = 5y$

$6y^2 - 6 - 5y = 0$

Ex # 1.2

$6y^2 - 5y - 6 = 0$

$6y^2 + 4y - 9y - 6 = 0$

$2y(3y + 2) - 3(3y + 2) = 0$

$(3y + 2)(2y - 3) = 0$

$3y + 2 = 0 \quad or \quad 2y - 3 = 0$

$3y = -2 \quad or \quad 2y = 3$

$y = \frac{-2}{3} \quad or \quad y = \frac{3}{2}$

$But\ y = \frac{x+2}{x-2}$

$\frac{x+2}{x-2} = \frac{-2}{3} \quad or \quad \frac{x+2}{x-2} = \frac{3}{2}$

By cross multiplication

$3(x + 2) = -2(x - 2) \quad or \quad 2(x + 2) = 3(x - 2)$

$3x + 6 = -2x + 4 \quad or \quad 2x + 4 = 3x - 6$

$3x + 2x = 4 - 6 \quad or \quad 2x - 3x = -6 - 4$

$5x = -2 \quad or \quad -1x = -10$

$x = \frac{-2}{5} \quad or \quad x = 10$

Solution Set $= \left\{\frac{-2}{5}, 10\right\}$

(vii) $3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$

Solution:

$3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$ ... equ (i)

$Let\ x + \frac{1}{x} = y$

Taking square on B.S

$\left(x + \frac{1}{x}\right)^2 = y^2$

$x^2 + \frac{1}{x^2} + 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 - 2$

R.W	
$(3y^2)(20) = 60y^2$	
Add	**Multiply**
$-6y$	$-6y$
$-10y$	$-10y$
$-60y$	$-60y^2$

So equ (i) becomes

$3(y^2 - 2) - 16(y) + 26 = 0$

$3y^2 - 6 - 16y + 26 = 0$

$3y^2 - 16y - 6 + 26 = 0$

$3y^2 - 16y + 20 = 0$

$3y^2 - 6y - 10y + 20 = 0$

$3y(y - 2) - 10(y - 2) = 0$

$(y - 2)(3y - 10) = 0$

$y - 2 = 0 \quad or \quad 3y - 10 = 0$

Ex # 1.2

$y = 2 \quad or \quad 3y = 10$

$y = 2 \quad or \quad y = \frac{10}{3}$

But $y = x + \frac{1}{x}$

$x + \frac{1}{x} = 2 \quad or \quad x + \frac{1}{x} = \frac{10}{3}$

Now

$x + \frac{1}{x} = 2$

Multiply all terms by x

$x.x + x.\frac{1}{x} = 2.x$

$x^2 + 1 = 2x$

$x^2 + 1 - 2x = 0$

$x^2 - 2x + 1 = 0$

$x^2 - 1x - 1x + 1 = 0$

R.W	
$(x^2)(1) = x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-1x$	$-1x$
$-2x$	x^2

$x(x - 1) - 1(x - 1) = 0$

$(x - 1)(x - 1) = 0$

$x - 1 = 0 \quad or \quad x - 1 = 0$

$x = 1 \quad or \quad x = 1$

Also

$x + \frac{1}{x} = \frac{10}{3}$

Multiply all terms by $3x$

$3x.x + 3x.\frac{1}{x} = \frac{10}{3}.3x$

$3x^2 + 3 = 10x$

$3x^2 + 3 - 10x = 0$

$3x^2 - 10x + 3 = 0$

$3x^2 - 1x - 9x + 3 = 0$

R.W	
$(3x^2)(3) = 9x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-9x$	$-9x$
$-10x$	$9x^2$

$x(3x - 1) - 3(3x - 1) = 0$

$(3x - 1)(x - 3) = 0$

$3x - 1 = 0 \quad or \quad x - 3 = 0$

$3x = 1 \quad or \quad x = 3$

$x = \frac{1}{3} \quad or \quad x = 3$

Solution Set $= \left\{1, \frac{1}{3}, 3\right\}$

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Ex # 1.2

(viii) $\left(x+\frac{1}{x}\right)^2-10\left(x+\frac{1}{x}\right)+16=0$

Solution:

$\left(x+\frac{1}{x}\right)^2-10\left(x+\frac{1}{x}\right)+16=0$

Let $x+\frac{1}{x}=y$

$(y)^2-10(y)+16=0$

$y^2-10y+16=0$

$y^2-2y-8y+16=0$

$y(y-2)-8(y-2)=0$

$(y-2)(y-8)=0$

$y-2=0 \quad or \quad y-8=0$

$y=2 \quad or \quad y=8$

R.W	
$(y^2)(16)=16y^2$	
Add	**Multiply**
$-2y$	$-2y$
$-8y$	$-8y$
$-10y$	$16y^2$

$But\ y=x+\frac{1}{x}$

$x+\frac{1}{x}=2 \quad or \quad x+\frac{1}{x}=8$

Now

$x+\frac{1}{x}=2$

Multiply all terms by x

$x.x+x.\frac{1}{x}=2.x$

$x^2+1=2x$

$x^2+1-2x=0$

$x^2-2x+1=0$

$x^2-1x-1x+1=0$

$x(x-1)-1(x-1)=0$

$(x-1)(x-1)=0$

$x-1=0 \quad or \quad x-1=0$

$x=1 \quad or \quad x=1$

R.W	
$(x^2)(1)=x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-1x$	$-1x$
$-2x$	x^2

Also

$x+\frac{1}{x}=8$

Multiply all terms by x

$x.x+x.\frac{1}{x}=8.x$

$x^2+1=8x$

$x^2-8x+1=0$

Compare it with $ax^2+bx+c=0$

$Here\ a=1, b=-8, c=1$

As we have

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Ex # 1.2

Put the values

$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(1)}}{2(1)}$

$x=\frac{8\pm\sqrt{64-4}}{2}$

$x=\frac{8\pm\sqrt{60}}{2}$

$x=\frac{8\pm\sqrt{60}}{2}$

$x=\frac{8\pm\sqrt{4\times 15}}{2}$

$x=\frac{8\pm 2\sqrt{15}}{2}$

$x=\frac{2(4\pm\sqrt{15})}{2}$

$x=4\pm\sqrt{15}$

Solution Set $=\{\mathbf{1, 4\pm\sqrt{15}}\}$

(ix) $\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-4=0$

Solution:

$\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-4=0 \quad ---$ equ (i)

Let $x-\frac{1}{x}=y$

Taking square on B. S

$\left(x-\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}-2=y^2$

$x^2+\frac{1}{x^2}=y^2+2$

So equ (i) becomes

$y^2+2-y-4=0$

$y^2-y+2-4=0$

$y^2-y-2=0$

$y^2+1y-2y-2=0$

$y(y+1)-2(y+1)=0$

$(y+1)(y-2)=0$

$y+1=0 \quad or \quad y-2=0$

$y=-1 \quad or \quad y=2$

R.W	
$(y^2)(-2)=-2y^2$	
Add	**Multiply**
$+1y$	$+1y$
$-2y$	$-2y$
$-y$	$-2y^2$

$But\ y=x-\frac{1}{x}$

$x-\frac{1}{x}=-1 \quad or \quad x-\frac{1}{x}=2$

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Ex # 1.2

Now

$x-\frac{1}{x}=-1$

Multiply all terms by x

$x.x-x.\frac{1}{x}=-1.x$

$x^2-1=-x$

$x^2-1+x=0$

$x^2+x-1=0$

Compare it with $ax^2+bx+c=0$

$Here\ a=1, b=1, c=-1$

As we have

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Put the values

$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1+4}}{2}$

$x=\frac{-1\pm\sqrt{5}}{2}$

Also

$x-\frac{1}{x}=2$

Multiply all terms by x

$x.x-x.\frac{1}{x}=2.x$

$x^2-1=2x$

$x^2-2x-1=0$

Compare it with $ax^2+bx+c=0$

$Here\ a=1, b=-2, c=-1$

As we have

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Put the values

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$x=\frac{2\pm\sqrt{4+4}}{2}$

$x=\frac{2\pm\sqrt{8}}{2}$

$x=\frac{2\pm\sqrt{4\times 2}}{2}$

$x=\frac{2\pm 2\sqrt{2}}{2}$

Ex # 1.2

$x=\frac{2(1\pm\sqrt{2})}{2}$

$x=1\pm\sqrt{2}$

Solution Set $=\left\{\frac{-1\pm\sqrt{5}}{2}, 1\pm\sqrt{2}\right\}$

(x) $\mathbf{3^{2x}-10.3^x+9=0}$

Solution:

R.W	
$(y^2)(9)=9y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-9y$	$-9y$
$\mathbf{-10y}$	$\mathbf{9y^2}$

$3^{2x}-10.3^x+9=0$

$(3^x)^2-10.3^x+9=0$

Let $3^x=y$

$(y)^2-10y+9=0$

$y^2-10y+9=0$

$y^2-1y-9y+9=0$

$y(y-1)-9(y-1)=0$

$(y-1)(y-9)=0$

$y-1=0 \quad or \quad y-9=0$

$y=1 \quad or \quad y=9$

But $y=3^x$

$3^x=1 \quad or \quad 3^x=9$

$3^x=3^0 \quad or \quad 3^x=3^2$

$x=0 \quad or \quad x=2$

Solution Set $=\{\mathbf{0,2}\}$

(xi) $\mathbf{3.3^{2x+1}-10.3^x+1=0}$

Solution:

$3.3^{2x+1}-10.3^x+1=0$

$3.3^{2x}.3^1-10.3^x+1=0$

$3.3.3^{2x}-10.3^x+1=0$

$9.3^{2x}-10.3^x+1=0$

$9.(3^x)^2-10.3^x+1=0$

R.W	
$(9y^2)(1)=9y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-9y$	$-9y$
$\mathbf{-10y}$	$\mathbf{9y^2}$

Let $3^x=y$

$9(y)^2-10y+1=0$

$9y^2-10y+1=0$

$9y^2-1y-9y+1=0$

$y(9y-1)-1(9y-1)=0$

$(9y-1)(y-1)=0$

$9y-1=0 \quad or \quad y-1=0$

$9y=1 \quad or \quad y=1$

$y=\frac{1}{9} \quad or \quad y=1$

But $y=3^x$

$3^x=\frac{1}{9} \quad or \quad 3^x=1$

Ex # 1.2

$3^x = \frac{1}{3^2}$ or $3^x = 3^0$

$3^x = 3^{-2}$ or $x = 0$

$x = -2$ or $x = 0$

Solution Set $= \{-2, 0\}$

(xii) $\mathbf{5^{x+1} + 5^{2-x} = 126}$

Solution:

$5^{x+1} + 5^{2-x} = 126$

$5^x.5^1 + 5^2.5^{-x} = 126$

$5.5^x + \frac{5^2}{5^x} = 126$

Let $5^x = y$

$5y + \frac{25}{y} = 126$

R.W	
$(5y^2)(25) = 125y^2$	
Add	Multiply
$-1y$	$-1y$
$-125y$	$-125y$
$\mathbf{-126y}$	$\mathbf{125y^2}$

Multiply all terms by y

$5y \times y + \frac{25}{y} \times y = 126 \times y$

$5y^2 + 25 = 126y$

$5y^2 + 25 - 126y = 0$

$5y^2 - 126y + 25 = 0$

$5y^2 - 1y - 125y + 25 = 0$

$y(5y - 1) - 25(5y - 1) = 0$

$(5y - 1)(y - 25) = 0$

$5y - 1 = 0$ or $y - 25 = 0$

$5y = 1$ or $y = 25$

$y = \frac{1}{5}$ or $y = 25$

But $y = 5^x$

$5^x = \frac{1}{5}$ or $5^x = 25$

$5^x = 5^{-1}$ or $5^x = 5^2$

$x = -1$ or $x = 2$

Solution Set $= \{-1, 2\}$

(xiii) $\mathbf{(x-3)(x+9)(x+5)(x-7) = 385}$

Solution:

$(x-3)(x+9)(x+5)(x-7) = 385$

Re − arrange it accordingly $-3+5 = 9-7$

$\{(x-3)(x+5)\}\{(x+9)(x-7)\} = 385$

$(x^2 + 5x - 3x - 15)(x^2 - 7x + 9x - 63) = 385$

$(x^2 + 2x - 15)(x^2 + 2x - 63) - 385 = 0$

Let $x^2 + 2x = y$

$(y - 15)(y - 63) - 385 = 0$

$y^2 - 63y - 15y + 945 - 385 = 0$

Ex # 1.2

$y^2 - 78y + 560 = 0$

$y^2 - 8y - 70y + 560 = 0$

$y(y - 8) - 70(y - 8) = 0$

$(y - 8)(y - 70) = 0$

$y - 8 = 0$ or $y - 70 = 0$

$y = 8$ or $y = 70$

R.W	
$(y^2)(560) = 560y^2$	
Add	Multiply
$-8y$	$-8y$
$-70y$	$-70y$
$\mathbf{-78y}$	$\mathbf{560y^2}$

But $y = x^2 + 2x$

$x^2 + 2x = 8$ or $x^2 + 2x = 70$

$x^2 + 2x - 8 = 0$ or $x^2 + 2x - 70 = 0$

Now

$x^2 + 2x - 8 = 0$

$x^2 - 2x + 4x - 8 = 0$

$x(x - 2) + 4(x - 2) = 0$

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

$x = 2$ or $x = -4$

R.W	
$(x^2)(-8) = -8x^2$	
Add	Multiply
$-2x$	$-2x$
$+4x$	$+4x$
$\mathbf{2x}$	$\mathbf{-8x^2}$

Also

$x^2 + 2x - 70 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 2, c = -70$

As we have

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Put the values

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 + 280}}{2}$

$x = \frac{-2 \pm \sqrt{284}}{2}$

$x = \frac{-2 \pm \sqrt{4 \times 71}}{2}$

$x = \frac{-2 \pm 2\sqrt{71}}{2}$

$x = \frac{2(-1 \pm \sqrt{71})}{2}$

$x = -1 \pm \sqrt{71}$

Solution Set $= \{\mathbf{2, -4, -1 \pm \sqrt{71}}\}$

(xiv) $\mathbf{(x+1)(x+2)(x+3)(x+4) + 1 = 0}$

Solution:

$(x+1)(x+2)(x+3)(x+4) + 1 = 0$

Re − arrange it accordingly $1 + 4 = 2 + 3$

$\{(x+1)(x+4)\}\{(x+2)(x+3)\} + 1 = 0$

Ex # 1.2

$(x^2+4x+1x+4)(x^2+3x+2x+6)+1=0$

$(x^2+5x+4)(x^2+5x+6)+1=0$

Let $x^2+5x=y$

$(y+4)(y+6)+1=0$

$y^2+6y+4y+24+1=0$

$y^2+10y+25=0$

$y^2+5y+5y+25=0$

$y(y+5)+5(y+5)=0$

$(y+5)(y+5)=0$

$y+5=0 \quad or \quad y+5=0$

$y=-5 \quad or \quad y=-5$

R.W	
$(y^2)(25)=25y^2$	
Add	Multiply
$+5y$	$+5y$
$+5y$	$+5y$
$+10y$	$25y^2$

But $y=x^2+5x$

$x^2+5x=-5$

$x^2+5x+5=0$

Now

Compare it with $ax^2+bx+c=0$

Here $a=1, b=5, c=5$

As we have

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Put the values

$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(5)}}{2(1)}$$

$$x=\frac{-5\pm\sqrt{25-20}}{2}$$

$$x=\frac{-5\pm\sqrt{5}}{2}$$

$$x=\frac{-5+\sqrt{5}}{2} \quad or \quad x=\frac{-5-\sqrt{5}}{2}$$

Solution Set $=\left\{\frac{-5+\sqrt{5}}{2},\frac{-5-\sqrt{5}}{2}\right\}$

(xv) $(x+1)(x+3)(x+5)(x+7)+16=0$

Solution:

$(x+1)(x+3)(x+5)(x+7)+16=0$

Re − arrange it accordingly $1+7=3+5$

$\{(x+1)(x+7)\}\{(x+3)(x+5)\}+16=0$

$(x^2+7x+1x+7)(x^2+5x+3x+15)+16=0$

$(x^2+8x+7)(x^2+8x+15)+16=0$

Let $x^2+8x=y$

$(y+7)(y+15)+16=0$

$y^2+15y+7y+105+16=0$

$y^2+22y+121=0$

$y^2+11y+11y+121=0$

R.W	
$(y^2)(121)=121y^2$	
Add	Multiply
$+11y$	$+11y$
$+11y$	$+11y$
$+22y$	$121y^2$

Ex # 1.2

$y(y+11)+11(y+11)=0$

$(y+11)(y+11)=0$

$y+11=0 \quad or \quad y+11=0$

$y=-11 \quad or \quad y=-11$

But $y=x^2+5x$

$x^2+8x=-11$

$x^2+8x+11=0$

Compare it with $ax^2+bx+c=0$

Here $a=1, b=8, c=11$

As we have

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Put the values

$$x=\frac{-(8)\pm\sqrt{(8)^2-4(1)(11)}}{2(1)}$$

$$x=\frac{-8\pm\sqrt{64-44}}{2}$$

$$x=\frac{-8\pm\sqrt{20}}{2}$$

$$x=\frac{-8\pm\sqrt{4\times5}}{2}$$

$$x=\frac{-8\pm2\sqrt{5}}{2}$$

$$x=\frac{2(-4\pm\sqrt{5})}{2}$$

$$x=-4\pm\sqrt{5}$$

Solution Set $=\{-4+\sqrt{5},-4-\sqrt{5}\}$

Q2: **Solve the equation**

$x^4-2x^3-2x^2+2x+1=0$

Solution:

$x^4-2x^3-2x^2+2x+1=0$

Divide each term by x^2

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

Arrange it

$$x^2+\frac{1}{x^2}-2-2x+\frac{2}{x}=0$$

$$\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)=0$$

Let $x-\frac{1}{x}=y$

$(y)^2-2(y)=0$

Ex # 1.2

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0 \quad or \quad y - 2 = 0$

$y = 0 \quad or \quad y = 2$

But $y = x - \frac{1}{x}$

$x - \frac{1}{x} = 0 \quad or \quad x - \frac{1}{x} = 2$

Now

$x - \frac{1}{x} = 0$

Multiply all terms by x

$x.x - x.\frac{1}{x} = 0.x$

$x^2 - 1 = 0$

$x^2 = 1$

$\sqrt{x^2} = \pm\sqrt{1}$

$x = \pm 1$

Also

$x - \frac{1}{x} = 2$

Multiply all terms by x

$x.x - x.\frac{1}{x} = 2.x$

$x^2 - 1 = 2x$

$x^2 - 2x - 1 = 0$

Compare it with $ax^2 + bx + c = 0$

$Here\ a = 1, b = -2, c = -1$

As we have

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Put the values

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+4}}{2}$

$x = \frac{2 \pm \sqrt{8}}{2}$

$x = \frac{2 \pm \sqrt{4 \times 2}}{2}$

$x = \frac{2 \pm 2\sqrt{2}}{2}$

$x = \frac{2(1 \pm \sqrt{2})}{2}$

Ex # 1.2

$x = 1 \pm \sqrt{2}$

Solution Set $= \{\pm\mathbf{1}, \mathbf{1} \pm \sqrt{\mathbf{2}}\}$

Ex # 1.3

Radical Equation

An equation involving expression under radical sign is called a radical equation.

Extraneous Solution:

A solution of the transformed equation that does not satisfy the original equation is called an extraneous solution.

Note:

To solve radical equation, we take square to simplify square root.

Type 1: $\sqrt{ax+b} = cx + d$

Example # 13

$$\sqrt{27 - 3x} = x - 3$$

Solution:

$\sqrt{27 - 3x} = x - 3 \quad --- \text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{27 - 3x}\right)^2 = (x-3)^2$

$27 - 3x = (x)^2 - 2(x)(3) + (3)^2$

$27 - 3x = x^2 - 6x + 9$

$0 = x^2 - 6x + 3x + 9 - 27$

$0 = x^2 - 3x - 18$

$x^2 - 3x - 18 = 0$

$x^2 + 3x - 6x - 18 = 0$

$x(x+3) - 6(x+3) = 0$

$(x+3)(x-6) = 0$

$x + 3 = 0 \quad or \quad x - 6 = 0$

$x = -3 \quad or \quad x = 6$

R.W	
$(x^2)(-11) = -18x^2$	
Add	**Multiply**
$+3x$	$+3x$
$-6x$	$-6x$
$\mathbf{-3x}$	$\mathbf{-18x^2}$

Verification:

Put $x = -3$ in equ (i)

$\sqrt{27 - 3(-3)} = -3 - 3$

$\sqrt{27 + 9} = -6$

$\sqrt{36} = -6$

$6 = -6 \quad$ (**Flase**)

Put $x = 6$ in equ (i)

$\sqrt{27 - 3(6)} = 6 - 3$

$\sqrt{27 - 18} = 3$

$\sqrt{9} = 3$

$3 = 3 \quad$ (**True**)

Hence $x = 1$ is an extraneous root.

Thus Solution Set $= \{\mathbf{6}\}$

Ex # 1.3

Type 2: $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Example # 14

$$\sqrt{x+2}+\sqrt{x+7}=\sqrt{x+23}$$

Solution:

$\sqrt{x+2}+\sqrt{x+7}=\sqrt{x+23}$ --- equ (i)

Taking square root on B.S

$\left(\sqrt{x+2}+\sqrt{x+7}\right)^2=\left(\sqrt{x+23}\right)^2$

$\left(\sqrt{x+2}\right)^2+\left(\sqrt{x+7}\right)^2+2\left(\sqrt{x+2}\right)\left(\sqrt{x+7}\right)=x+23$

$x+2+x+7+2\sqrt{(x+2)(x+7)}=x+23$

$2x+9+2\sqrt{x^2+7x+2x+14}=x+23$

$2\sqrt{x^2+9x+14}=x-2x+23-9$

$2\sqrt{x^2+9x+14}=-x+14$

$2\sqrt{x^2+9x+14}=14-x$

Taking square root on B.S

$\left(2\sqrt{x^2+9x+14}\right)^2=(14-x)^2$

$4(x^2+9x+14)=(14)^2-2(14)(x)+(x)^2$

$4x^2+36x+56=196-28x+x^2$

$4x^2-x^2+36x+28x+56-196=0$

$3x^2+64x-140=0$

$3x^2-6x+70x-140=0$

$3x(x-2)+70(x-2)=0$

$(x-2)(3x+70)=0$

$x-2=0$ or $3x+70=0$

$x=2$ or $3x=-70$

$x=2$ or $x=\frac{-70}{3}$

R.W	
$(3x^2)(-140)=-140x^2$	
Add	**Multiply**
$-6x$	$-6x$
$+70x$	$+70x$
$+64x$	$-18x^2$

As $x=\frac{-70}{3}$ is an extraneous root.

Thus Solution Set = {2}

Type 2: $\sqrt{x^2+px+m}+\sqrt{x^2+qx+n}=q$

Example # 15

$$\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$$

Solution:

$\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$

$\sqrt{x^2+3x+5}=2-\sqrt{x^2+3x+1}$

Taking square root on B.S

$\left(\sqrt{x^2+3x+5}\right)^2=\left(2-\sqrt{x^2+3x+1}\right)^2$

Ex # 1.3

$x^2+3x+5=(2)^2+\left(\sqrt{x^2+3x+1}\right)^2-2(2)\sqrt{x^2+3x+1}$

$x^2+3x+5=4+x^2+3x+1-4\sqrt{x^2+3x+1}$

$x^2+3x+5=x^2+3x+4+1-4\sqrt{x^2+3x+1}$

$5=5-4\sqrt{x^2+3x+1}$

$0=-4\sqrt{x^2+3x+1}$

Divide B.S by -4, we get

$0=\sqrt{x^2+3x+1}$

$\sqrt{x^2+3x+1}=0$

Again Take square root on B.S

$\left(\sqrt{x^2+3x+1}\right)^2=(0)^2$

$x^2+3x+1=0$

Compare it with $ax^2+bx+c=0$

$Here\ a=1, b=3, c=1$

As we have

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Put the values

$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$

$x=\frac{-3\pm\sqrt{9-4}}{2}$

$x=\frac{-3\pm\sqrt{5}}{2}$

Solution Set $=\left\{\frac{-3\pm\sqrt{5}}{2}\right\}$

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Ex # 1.3

Page # 19

Q1: Solve the following equations.

(i) $\sqrt{5x+21} = x+3$

Solution:

$\sqrt{5x+21} = x+3 \quad ---\text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{5x+21}\right)^2 = (x+3)^2$

$5x+21 = (x)^2 + 2(x)(3) + (3)^2$

$5x+21 = x^2+6x+9$

$0 = x^2+6x+9-5x-21$

$0 = x^2+6x-5x+9-21$

$0 = x^2+x-12$

$x^2+x-12 = 0$

$x^2-3x+4x-12 = 0$

$x(x-3)+4(x-3) = 0$

$(x-3)(x+4) = 0$

$x-3 = 0 \quad or \quad x+4 = 0$

$x = 3 \quad or \quad x = -4$

R.W	
$(x^2)(-12) = -12x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+4x$	$+4x$
$+x$	$-12x^2$

Verification:

Put $x = 3$ in equ (i)

$\sqrt{5(3)+21} = 3+3$

$\sqrt{15+21} = 6$

$\sqrt{36} = 6$

$6 = 6 \quad$ **(True)**

Put $x = -4$ in equ (i)

$\sqrt{5(-4)+21} = -4+3$

$\sqrt{-20+21} = -1$

$\sqrt{1} = -1$

$1 = -1 \quad$ **(False)**

Hence $x = -4$ is an extraneous root.

Thus Solution Set $= \{3\}$

(ii) $\sqrt{2x-1} = x-2$

Solution:

$\sqrt{2x-1} = x-2 \quad ---\text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$

$2x-1 = (x)^2 - 2(x)(2) + (2)^2$

$2x-1 = x^2-4x+4$

$0 = x^2-4x-2x+4+1$

Ex # 1.3

$0 = x^2-6x+5$

$x^2-6x+5 = 0$

$x^2-1x-5x+5 = 0$

$x(x-1)-5(x-1) = 0$

$(x-1)(x-5) = 0$

$x-1 = 0 \quad or \quad x-5 = 0$

$x = 1 \quad or \quad x = 5$

R.W	
$(x^2)(5) = 5x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-5x$	$-5x$
$-6x$	$5x^2$

Verification:

Put $x = 1$ in equ (i)

$\sqrt{2(1)-1} = 1-2$

$\sqrt{2-1} = -1$

$\sqrt{1} = -1$

$1 = -1 \quad$ **(Flase)**

Put $x = 5$ in equ (i)

$\sqrt{2(5)-1} = 5-2$

$\sqrt{10-1} = 3$

$\sqrt{9} = 3$

$3 = 3 \quad$ **(True)**

Hence $x = 1$ is an extraneous root.

Thus Solution Set $= \{5\}$

(iii) $\sqrt{4x+5} = 2x-5$

Solution:

$\sqrt{4x+5} = 2x-5 \quad ---\text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{4x+5}\right)^2 = (2x-5)^2$

$4x+5 = (2x)^2 - 2(2x)(5) + (5)^2$

$4x+5 = 4x^2-20x+25$

$0 = 4x^2-20x-4x+25-5$

$0 = 4x^2-24x+20$

$4x^2-24x+20 = 0$

$4(x^2-6x+5) = 0$

Divide B.S by 4, we get

$x^2-6x+5 = 0$

$x^2-1x-5x+5 = 0$

$x(x-1)-5(x-1) = 0$

$(x-1)(x-5) = 0$

$x-1 = 0 \quad or \quad x-5 = 0$

$x = 1 \quad or \quad x = 5$

R.W	
$(x^2)(5) = 5x^2$	
Add	**Multiply**
$-1x$	$-1x$
$-5x$	$-5x$
$-6x$	$5x^2$

Verification:

Put $x = 1$ in equ (i)

$\sqrt{2(1)-1} = 1-2$

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Ex # 1.3

$\sqrt{2-1}=-1$

$\sqrt{1}=-1$

$1=-1$ **(Flase)**

Put $x=5$ in equ (i)

$\sqrt{2(5)-1}=5-2$

$\sqrt{10-1}=3$

$\sqrt{9}=3$

$3=3$ **(True)**

Hence $x=1$ is an extraneous root.

Thus Solution Set $=\{5\}$

(iv) $\sqrt{29-4x}=2x+3$

Solution:

$\sqrt{29-4x}=2x+3 \quad ---\text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{29-4x}\right)^2=(2x+3)^2$

$29-4x=(2x)^2+2(2x)(3)+(3)^2$

$29-4x=4x^2+12x+9$

$0=4x^2+12x+4x+9-29$

$0=4x^2+16x-20$

$4x^2+16x-20=0$

$4(x^2+4x-5)=0$

Divide B.S by 4, we get

$x^2+4x-5=0$

R.W	
$(x^2)(-5)=-5x^2$	
Add	**Multiply**
$-1x$	$-1x$
$+5x$	$+5x$
$+4x$	$-5x^2$

$x^2-1x+5x-5=0$

$x(x-1)+5(x-1)=0$

$(x-1)(x+5)=0$

$x-1=0 \quad or \quad x+5=0$

$x=1 \quad or \quad x=-5$

Verification:

Put $x=1$ in equ (i)

$\sqrt{29-4(1)}=2(1)+3$

$\sqrt{29-4}=2+3$

$\sqrt{25}=5$

$5=5$ **(True)**

Put $x=-5$ in equ (i)

$\sqrt{29-4(-5)}=2(-5)+3$

$\sqrt{29+20}=-10+3$

$\sqrt{49}=-7$

$7=-7$ **(False)**

Hence $x=-5$ is an extraneous root.

Thus Solution Set $=\{1\}$

Ex # 1.3

(v) $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Solution:

$\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13} \quad ---\text{equ (i)}$

Taking square root on B.S

$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$

$\left(\sqrt{x+7}\right)^2+\left(\sqrt{x+2}\right)^2+2\left(\sqrt{x+7}\right)\left(\sqrt{x+2}\right)=6x+13$

$x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$

$2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$

$2\sqrt{x^2+9x+14}=6x-2x+13-9$

$2\sqrt{x^2+9x+14}=4x+4$

$2\sqrt{x^2+9x+14}=2(2x+2)$

$\sqrt{x^2+9x+14}=2x+2$

Taking square root on B.S

$\left(\sqrt{x^2+9x+14}\right)^2=(2x+2)^2$

$x^2+9x+14=(2x)^2+2(2x)(2)+(2)^2$

$x^2+9x+14=4x^2+8x+4$

$0=4x^2-x^2+8x-9x+4-14$

$0=3x^2-x-10$

$3x^2-x-10=0$

$3x^2+5x-6x-10=0$

$x(3x+5)-2(3x+5)=0$

$(3x+5)(x-2)=0$

$3x+5=0 \quad or \quad x-2=0$

$3x=-5 \quad or \quad x=2$

$x=\frac{-5}{3} \quad or \quad x=2$

R.W	
$(3x^2)(-10)=-30x^2$	
Add	**Multiply**
$+5x$	$+5x$
$-6x$	$-6x$
$-x$	$-30x^2$

Verification:

Put $x=\frac{-5}{3}$ in equ (i)

$\sqrt{\frac{-5}{3}+7}+\sqrt{\frac{-5}{3}+2}=\sqrt{6\left(\frac{-5}{3}\right)+13}$

$\sqrt{\frac{-5+21}{3}}+\sqrt{\frac{-5+6}{3}}=\sqrt{2(-5)+13}$

$\sqrt{\frac{16}{3}}+\sqrt{\frac{1}{3}}=\sqrt{-10+13}$

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Ex # 1.3

$\frac{4}{\sqrt{3}}+\frac{1}{\sqrt{3}}=\sqrt{3}$

$\frac{4+1}{\sqrt{3}}=\sqrt{3}$

$\frac{5}{\sqrt{3}}=\sqrt{3}$ **(False)**

Put $x=2$ in equ (i)

$\sqrt{2+7}+\sqrt{2+2}=\sqrt{6(2)+13}$

$\sqrt{9}+\sqrt{4}=\sqrt{12+13}$

$3+2=\sqrt{25}$

$5=5$ **(True)**

Hence $x=\frac{-5}{3}$ is an extraneous root.

Thus Solution Set $=\{2\}$

(vi) $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$

Solution:

$\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$ --- equ (i)

Taking square root on B.S

$\left(\sqrt{x}+\sqrt{3x+1}\right)^2=\left(\sqrt{5x+1}\right)^2$

$(\sqrt{x})^2+(\sqrt{3x+1})^2+2(\sqrt{x})(\sqrt{3x+1})=5x+1$

$x+3x+1+2\sqrt{x(3x+1)}=5x+1$

$4x+1+2\sqrt{3x^2+x}=5x+1$

$2\sqrt{3x^2+x}=5x-4x+1-1$

$2\sqrt{3x^2+x}=x$

Taking square root on B.S

$\left(2\sqrt{3x^2+x}\right)^2=(x)^2$

$4(3x^2+x)=x^2$

$12x^2+4x=x^2$

$12x^2-x^2+4x=0$

$11x^2+4x=0$

$x(11x+4)=0$

$x=0$ or $11x+4=0$

$x=0$ or $11x=-4$

$x=0$ or $x=\frac{-4}{11}$

Verification:

Put $x=0$ in equ (i)

$\sqrt{0}+\sqrt{3(0)+1}=\sqrt{5(0)+1}$

$0+\sqrt{0+1}=\sqrt{0+1}$

$\sqrt{1}=\sqrt{1}$

$1=1$ **(True)**

Ex # 1.3

Put $x=\frac{-4}{11}$ in equ (i)

$\sqrt{\frac{-4}{11}}+\sqrt{3\left(\frac{-4}{11}\right)+1}=\sqrt{5\left(\frac{-4}{11}\right)+1}$

$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-12}{11}+1}=\sqrt{\frac{-20}{11}+1}$

$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-12+11}{11}}=\sqrt{\frac{-20+11}{11}}$

$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-1}{11}}=\sqrt{\frac{-9}{11}}$

$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-1}{11}}=\sqrt{\frac{-9}{11}}$ **(False)**

Hence $x=\frac{-4}{11}$ is an extraneous root.

Thus Solution Set $=\{0\}$

(vii) $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$

Solution:

$\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$ --- equ (i)

Taking square root on B.S

$\left(\sqrt{6x+40}-\sqrt{x+21}\right)^2=\left(\sqrt{x+5}\right)^2$

$(\sqrt{6x+40})^2+(\sqrt{x+21})^2-2\sqrt{6x+40}.\sqrt{x+21}=x+5$

$6x+40+x+21-2\sqrt{(6x+40)(x+21)}=x+5$

$7x+61-2\sqrt{6x^2+126x+40x+840}=x+5$

$-2\sqrt{6x^2+166x+840}=x-7x+5-61$

$-2\sqrt{6x^2+166x+840}=-6x-56$

$-2\sqrt{6x^2+166x+840}=-2(3x+28)$

$\sqrt{6x^2+166x+840}=3x+28$

Taking square root on B.S

$\left(\sqrt{6x^2+166x+840}\right)^2=(3x+28)^2$

$6x^2+166x+840=(3x)^2+2(3x)(28)+(28)^2$

$6x^2+166x+840=9x^2+168x+784$

$0=9x^2-6x^2+168x-166x+784-840$

$0=3x^2+2x-56$

$3x^2+2x-56=0$

$3x^2-12x+14x-56=0$

$3x(x-4)+14(x-4)=0$

$(x-4)(3x+14)=0$

$x-4=0$ or $3x+14=0$

R.W	
$(3x^2)(-56)=-168x^2$	
Add	**Multiply**
$-12x$	$-12x$
$+14x$	$+14x$
$+2x$	$-30x^2$

Ex # 1.3

$x = 4 \quad or \quad 3x = -14$

$x = 4 \quad or \quad x = \frac{-14}{3}$

By Checking , $x = \frac{-14}{3}$ is an extraneous root.

Thus Solution Set = {4}

(viii) $\sqrt{2x-3} + \sqrt{2x+4} = \sqrt{6x+13}$

Solution:

$\sqrt{2x-3} + \sqrt{2x+4} = \sqrt{6x+13}$ −−equ (i)

Taking square root on B.S

$\left(\sqrt{2x-3} + \sqrt{2x+4}\right)^2 = \left(\sqrt{6x+13}\right)^2$

$\left(\sqrt{2x-3}\right)^2 + \left(\sqrt{2x+4}\right)^2 + 2\sqrt{2x-3}.\sqrt{2x+4} = 6x+13$

$2x - 3 + 2x + 4 + 2\sqrt{(2x-3)(2x+4)} = 6x + 13$

$4x + 1 + 2\sqrt{4x^2 + 8x - 6x - 12} = 6x + 13$

$2\sqrt{4x^2 + 2x - 12} = 6x - 4x + 13 - 1$

$2\sqrt{4x^2 + 2x - 12} = 2x + 12$

$2\sqrt{4x^2 + 2x - 12} = 2(x + 6)$

$\sqrt{4x^2 + 2x - 12} = x + 6$

Taking square root on B.S

$\left(\sqrt{4x^2 + 2x - 12}\right)^2 = (x+6)^2$

$4x^2 + 2x - 12 = (x)^2 + 2(x)(6) + (6)^2$

$4x^2 + 2x - 12 = x^2 + 12x + 36$

$4x^2 - x^2 + 2x - 12x - 12 - 36 = 0$

$3x^2 - 10x - 48 = 0$

$3x^2 - 10x - 48 = 0$

$3x^2 + 8x - 18x - 48 = 0$

$x(3x + 8) - 6(x + 8) = 0$

$(3x + 8)(x - 6) = 0$

$3x + 8 = 0 \quad or \quad x - 6 = 0$

$3x = -8 \quad or \quad x = 6$

$x = \frac{-8}{3} \quad or \quad x = 6$

R.W	
$(3x^2)(-48) = -144x^2$	
Add	Multiply
$+8x$	$+8x$
$-18x$	$-18x$
$-10x$	$-144x^2$

By Checking , $x = \frac{-8}{3}$ is an extraneous root.

Thus Solution Set = {6}

(ix) $\sqrt{x^2 + 2x + 4} + \sqrt{x^2 + 2x + 9} = 5$

Solution:

$\sqrt{x^2 + 2x + 4} + \sqrt{x^2 + 2x + 9} = 5$

$\sqrt{x^2 + 2x + 4} = 5 - \sqrt{x^2 + 2x + 9}$

Taking square root on B.S

Ex # 1.3

$\left(\sqrt{x^2 + 2x + 4}\right)^2 = \left(5 - \sqrt{x^2 + 2x + 9}\right)^2$

$x^2 + 2x + 4 = (5)^2 + \left(\sqrt{x^2 + 2x + 9}\right)^2 - 2(5)\sqrt{x^2 + 2x + 9}$

$x^2 + 2x + 4 = 25 + x^2 + 2x + 9 - 10\sqrt{x^2 + 2x + 9}$

$4 = 25 + 9 - 10\sqrt{x^2 + 2x + 9}$

$4 = 34 - 10\sqrt{x^2 + 2x + 9}$

$4 - 34 = -10\sqrt{x^2 + 2x + 9}$

$-30 = -10\sqrt{x^2 + 2x + 9}$

Divide B.S by − 10 , we get

$3 = \sqrt{x^2 + 2x + 9}$

$\sqrt{x^2 + 2x + 9} = 3$

Again Take square root on B.S

$\left(\sqrt{x^2 + 2x + 9}\right)^2 = (3)^2$

$x^2 + 2x + 9 = 9$

$x^2 + 2x + 9 - 9 = 0$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0 \quad or \quad x + 2 = 0$

$x = 0 \quad or \quad x = -2$

Thus Solution Set = {0 , −2}

(x) $\sqrt{2x^2 + 3x + 5} + \sqrt{2x^2 + 3x + 1} = 2$

Solution:

$\sqrt{2x^2 + 3x + 5} + \sqrt{2x^2 + 3x + 1} = 2$

$\sqrt{2x^2 + 3x + 5} = 2 - \sqrt{2x^2 + 3x + 1}$

Taking square root on B.S

$\left(\sqrt{2x^2 + 3x + 5}\right)^2 = \left(2 - \sqrt{2x^2 + 3x + 1}\right)^2$

$2x^2 + 3x + 5 = (2)^2 + \left(\sqrt{2x^2 + 3x + 1}\right)^2 - 2(2)\sqrt{2x^2 + 3x + 1}$

$2x^2 + 3x + 5 = 4 + 2x^2 + 3x + 1 - 4\sqrt{2x^2 + 3x + 1}$

$5 = 4 + 1 - 4\sqrt{2x^2 + 3x + 1}$

$5 = 5 - 4\sqrt{2x^2 + 3x + 1}$

$5 - 5 = -4\sqrt{2x^2 + 3x + 1}$

$0 = -4\sqrt{2x^2 + 3x + 1}$

Divide B.S by − 4 , we get

$0 = \sqrt{2x^2 + 3x + 1}$

$\sqrt{2x^2 + 3x + 1} = 0$

Again Take square root on B.S

$\left(\sqrt{2x^2 + 3x + 1}\right)^2 = (0)^2$

Ex # 1.3

$2x^2+3x+1=0$

$2x^2+1x+2x+1=0$

$x(2x+1)+1(2x+1)=0$

$(x+1)(2x+1)=0$

$x+1=0 \quad or \quad 2x+1=0$

$x=-1 \quad or \quad 2x=-1$

$x=-1 \quad or \quad x=\frac{-1}{2}$

R.W	
$(2x^2)(1)=2x^2$	
Add	Multiply
$+1x$	$+1x$
$+2x$	$+2x$
$+3x$	$2x^2$

Thus Solution Set $=\left\{-1,\frac{-1}{2}\right\}$

Q2: **Find $2x+5$ if x satisfies**
$\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$

Solution:

$\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$

$\sqrt{40-9x}-\sqrt{-x}=2\sqrt{7-x}$

Taking square root on B.S

$\left(\sqrt{40-9x}-\sqrt{-x}\right)^2=\left(2\sqrt{7-x}\right)^2$

$\left(\sqrt{40-9x}\right)^2+\left(\sqrt{-x}\right)^2-2\sqrt{40-9x}.\sqrt{-x}=4(7-x)$

$40-9x+(-x)-2\sqrt{(40-9x)(-x)}=28-4x$

$40-9x-x-2\sqrt{-40x+9x^2}=28-4x$

$40-10x-2\sqrt{9x^2-40x}=28-4x$

$40-28-10x+4x-2\sqrt{9x^2-40x}=0$

$12-6x=2\sqrt{9x^2-40x}$

$2(6-3x)=2\sqrt{9x^2-40x}$

Divide B.S by 2, we get

$6-3x=\sqrt{9x^2-40x}$

Again Take square root on B.S

$(6-3x)^2=\left(\sqrt{9x^2-40x}\right)^2$

$(6)^2+(3x)^2-2(6)(3x)=9x^2-40x$

$36+9x^2-36x=9x^2-40x$

Now

$9x^2-9x^2-36x+40x+36=0$

$4x+36=0$

$4x=-36$

$x=\frac{-36}{4}$

$x=-9$

$2x+5=2(-9)+5$

$2x+5=-18+5$

$2x+5=-13$

Thus $2x+5=-13$

Review Ex # 1

Page # 15

Q2: **Solve $2w^4-5w^2+2=0$**

Solution:

$2w^4-5w^2+2=0$

$2(w^2)^2-5w^2+2=0$

Let $w^2=y$

$2(y)^2-5y+2=0$

$2y^2-5y+2=0$

$2y^2-1y-4y+2=0$

$y(2y-1)-2(2y-1)=0$

$(2y-1)(y-2)=0$

$2y-1=0 \quad or \quad y-2=0$

$2y=1 \quad or \quad y=2$

$y=\frac{1}{2} \quad or \quad y=2$

R.W	
$(2y^2)(2)=4y^2$	
Add	Multiply
$-1y$	$-1y$
$-4y$	$-4y$
$-5y$	$4y^2$

But $y=w^2$

$w^2=\frac{1}{2} \quad or \quad w^2=2$

$\sqrt{w^2}=\pm\sqrt{\frac{1}{2}} \quad or \quad \sqrt{w^2}=\pm\sqrt{2}$

$w=\pm\frac{1}{\sqrt{2}} \quad or \quad w=\pm\sqrt{2}$

Solution Set $=\left\{\pm\frac{1}{\sqrt{2}},\pm\sqrt{2}\right\}$

Q3: **Find the constant a and b such that $x=-1$ and $x=1$ are both solutions of the equation $ax^2+bx+2=0$.**

Solution:

$ax^2+bx+2=0$ .... equ (i)

Put $x=-1$ in equ (i)

$a(-1)^2+b(-1)+2=0$

$a(1)-b+2=0$

$a-b+2=0$ .... equ (ii)

Put $x=1$ in equ (i)

$a(1)^2+b(1)+2=0$

$a(1)+b+2=0$

$a+b+2=0$ .... equ (iii)

Add equ (ii) and equ (iii)

$(a-b+2)+(a+b+2)=0+0$

$a-b+2+a+b+2=0$

$2a+4=0$

$2a=-4$

Review Ex # 1

$a = \frac{-4}{2}$

$a = -2$

Put $a = -2$ in equ (iii)

$-2 + b + 2 = 0$

$b - 2 + 2 = 0$

$b = 0$

Answers:

$\boldsymbol{a = -2}$

$\boldsymbol{b = 0}$

Q4: Find all values of x such that $x^2 + 5x + 6$ and $x^2 + 19x + 34$ are equal.

Solution:

As $x^2 + 5x + 6$ *and* $x^2 + 19x + 34$ are equal

So

$x^2 + 5x + 6 = x^2 + 19x + 34$

$5x + 6 = 19x + 34$

$5x - 19x = 34 - 6$

$-14x = 28$

Divide B.S by -14

$\frac{-14x}{-14} = \frac{28}{-14}$

$x = -2$

Challenge!

Q5: Find the solutions to the equation $49x^2 - 316x + 132 = 0$

Solution:

$49x^2 - 316x + 132 = 0$

$49x^2 - 294x - 22x + 132 = 0$

$49x(x - 6) - 22(x - 6) = 0$

$(x - 6)(49x - 22) = 0$

$x - 6 = 0 \quad or \quad 49x - 22 = 0$

$x = 6 \quad or \quad 49x = 22$

$x = 6 \quad or \quad x = \frac{22}{49}$

Solution Set $= \{\mathbf{1}, \mathbf{5}\}$

R.W	
$(49x^2)(132) = 6468x^2$	
Add	**Multiply**
$-294x$	$-294x$
$-22x$	$-22x$
$\boldsymbol{-316x}$	$\boldsymbol{6468x^2}$

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UNIT # 2

THEORY OF QUADRATIC EQUATIONS

Ex # 2.1

Quadratic Equation

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant of a Quadratic equation

In quadratic formula, the expression $b^2 - 4ac$ is called Discriminant of quadratic equation.

Nature of quadratic equation

Case 1:

If $b^2 - 4ac = 0$, then the roots are real, equal and rational.

Case 2:

If $b^2 - 4ac < 0$, then the roots are unequal and imaginary.

Case 3:

If $b^2 - 4ac > 0$, then:

If $b^2 - 4ac$ is a perfect square, then roots are real, unequal and rational.

If $b^2 - 4ac$ is not a perfect square, then roots are real, unequal and irrational.

Example 1:

Find discriminant of the following equation

$\mathbf{x^2 + 9x + 2 = 0}$

Solution:

$x^2 + 9x + 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 9, c = 2$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (9)^2 - 4(1)(2)$

Discriminant $= 81 - 8$

Discriminant $= 73$

Example 2:

Examine the nature of the roots of the following quadratic equations.

(i) $\mathbf{x^2 - 8x + 16 = 0}$

Solution:

Ex # 2.1

$x^2 - 8x + 16 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -8, c = 16$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-8)^2 - 4(1)(16)$

Discriminant $= 64 - 64$

Discriminant $= 0$

Thus the roots are real, equal and rational

(ii) $\mathbf{x^2 + 9x + 2 = 0}$

Solution:

$x^2 + 9x + 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 9, c = 2$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (9)^2 - 4(1)(2)$

Discriminant $= 81 - 8$

Discriminant $= 73 > 0$

Thus the roots are real, unequal and irrational

(iii) $\mathbf{6x^2 - x - 15 = 0}$

Solution:

$6x^2 - x - 15 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 6, b = -1, c = -15$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-1)^2 - 4(6)(-15)$

Discriminant $= 1 + 360$

Discriminant $= 361$

Discriminant $= 19^2 > 0$

Thus the roots are real, unequal and rational

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Ex # 2.1

(iv) $\mathbf{4x^2 + x + 1 = 0}$

Solution:

$4x^2 + x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = 1, c = 1$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (1)^2 - 4(4)(1)$

Discriminant $= 1 - 16$

Discriminant $= -15 < 0$

Thus the roots are unequal and imaginary

Example 3:

Determine the nature of roots of the following equations and verify the results by solving the by factorization.

(i) $\mathbf{x^2 - 6x + 9 = 0}$

Solution:

$x^2 - 6x + 9 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -6, c = 9$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-6)^2 - 4(1)(9)$

Discriminant $= 36 - 36$

Discriminant $= 0$

Hence the roots are real, equal and rational

Verification by Solving equation

Using Factorizatin Method

$x^2 - 6x + 9 = 0$

$x^2 - 3x - 3x + 9 = 0$

$x(x - 3) - 3(x - 3) = 0$

$(x - 3)(x - 3) = 0$

$x - 3 = 0 \quad or \quad x - 3 = 0$

$x = 3 \quad or \quad x = 3$

Thus the roots are real, equal and rational

Hence the result is verified

(ii) $\mathbf{x^2 + 5x + 6 = 0}$

Solution:

$x^2 + 5x + 6 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 5, c = 6$

Ex # 2.1

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (5)^2 - 4(1)(6)$

Discriminant $= 25 - 24$

Discriminant $= 1$

Discriminant $= 1^2 > 0$

Hence the roots are real, unequal and rational

Verification by Solving equation

Using Factorizatin Method

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x + 2) + 3(x + 2) = 0$

$(x + 2)(x + 3) = 0$

$x + 2 = 0 \quad or \quad x + 3 = 0$

$x = -2 \quad or \quad x = -3$

Thus the roots are real, unequal and rational

Hence the result is verified

Example # 4:

Without solving, determine the nature of the roots of the quadratic equation.

$\mathbf{3x^2 - 4x + 6 = 0}$

Solution:

$3x^2 - 4x + 6 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = -4, c = 6$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-4)^2 - 4(3)(6)$

Discriminant $= 16 - 72$

Discriminant $= -56 < 0$

Thus the roots are unequal and imaginary

Example # 5:

Without solving, determine the nature of the roots of the quadratic equation.

$\mathbf{2x^2 - 7x = -1}$

Solution:

$2x^2 - 7x = -1$

$2x^2 - 7x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = -7, c = 1$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-7)^2 - 4(2)(1)$

Discriminant $= 49 - 8$

Discriminant $= 41$

Hence the roots are real, unequal and irrational

Example # 6 (i):

Determine the set of values of k for which the given quadratic equations have real roots.

$\mathbf{kx^2 + 4x + 1 = 0}$

Solution:

$kx^2 + 4x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = k, b = 4, c = 1$

If roots are Real

Discriminant $= b^2 - 4ac \geq 0$

$b^2 - 4ac \geq 0$

$(4)^2 - 4(k)(1) \geq 0$

$16 - 4k \geq 0$

$16 \geq 4k$

$\frac{16}{4} \geq k$

$4 \geq k$

$k \leq 4$

(ii) $\mathbf{2x^2 + kx + 3 = 0}$

Solution:

$2x^2 + kx + 3 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = k, c = 3$

If roots are Real

Discriminant $= b^2 - 4ac \geq 0$

$b^2 - 4ac \geq 0$

$(k)^2 - 4(2)(3) \geq 0$

$k^2 - 24 \geq 0$

$k^2 \geq 24$

Taking square root on B.S

$\sqrt{k^2} \geq \pm\sqrt{24}$

$k \geq \pm\sqrt{4 \times 6}$

$k \geq \pm\sqrt{4}.\sqrt{6}$

$k \geq \pm 2\sqrt{6}$

Therefore

$k \geq 2\sqrt{6}$

$k \leq -2\sqrt{6}$

Ex # 2.1

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Q1: Find the discriminant of the following quadratic equations:

(i) $\mathbf{x^2 - 4x + 13 = 0}$

Solution:

$x^2 - 4x + 13 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -4, c = 13$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-4)^2 - 4(1)(13)$

Discriminant $= 16 - 52$

Discriminant $= -36$

(ii) $\mathbf{4x^2 - 5x + 1 = 0}$

Solution:

$4x^2 - 5x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = -5, c = 1$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-5)^2 - 4(4)(1)$

Discriminant $= 25 - 16$

Discriminant $= 9$

(iii) $\mathbf{x^2 + x + 1 = 0}$

Solution:

$x^2 + x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 1, c = 1$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (1)^2 - 4(1)(1)$

Discriminant $= 1 - 4$

Discriminant $= -3$

Q2: Examine the nature of the roots of the following equations:

(i) $\mathbf{3x^2 - 5x + 1 = 0}$

Solution:

$3x^2 - 5x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = -5, c = 1$

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Ex # 2.1

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-5)^2 - 4(3)(1)$

Discriminant $= 25 - 12$

Discriminant $= 13 > 0$

Hence the roots are real, unequal and irrational

(ii) $\mathbf{6x^2 + x - 2 = 0}$

Solution:

$6x^2 + x - 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 6, b = 1, c = -2$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (1)^2 - 4(6)(-2)$

Discriminant $= 1 + 48$

Discriminant $= 49$

Discriminant $= 7^2 > 0$

Hence the roots are real, unequal and rational

(iii) $\mathbf{3x^2 + 2x + 1 = 0}$

Solution:

$3x^2 + 2x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = 2, c = 1$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (2)^2 - 4(3)(1)$

Discriminant $= 4 - 12$

Discriminant $= -8 < 0$

Thus the roots are unequal and imaginary

Q3: For what value of k the roots of the following equations are equal.

(i) $\mathbf{x^2 + kx + 9 = 0}$

Solution:

$x^2 + kx + 9 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = k, c = 9$

As roots are equal then

Discriminant $= b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$(k)^2 - 4(1)(9) = 0$

$k^2 - 36 = 0$

$k^2 = 36$

Ex # 2.1

Taking square root on B. S

$\sqrt{k^2} = \pm\sqrt{36}$

$k = \pm 6$

(ii) $\mathbf{12x^2 + kx + 3 = 0}$

Solution:

$12x^2 + kx + 3 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 12, b = k, c = 3$

As roots are equal then

Discriminant $= b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$(k)^2 - 4(12)(3) = 0$

$k^2 - 144 = 0$

$k^2 = 144$

Taking square root on B. S

$\sqrt{k^2} = \pm\sqrt{144}$

$k = \pm 12$

(iii) $\mathbf{x^2 - 5x + k = 0}$

Solution:

$x^2 - 5x + k = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -5, c = k$

As roots are equal then

Discriminant $= b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$(-5)^2 - 4(1)(k) = 0$

$25 - 4k = 0$

$-4k = -25$

$4k = 25$

$k = \dfrac{25}{4}$

Q4: Determine whether the following quadratic equations and verify the results by solving them.

(i) $\mathbf{x^2 + 5x + 5 = 0}$

Solution:

$x^2 + 5x + 5 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 5, c = 5$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (5)^2 - 4(1)(5)$

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Ex # 2.1

Discriminant $= 25 - 20$

Discriminant $= 5 > 0$

As the roots are real

Now find the roots

Using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{-5 \pm \sqrt{5}}{2}$$

$$x = \frac{-5 + \sqrt{5}}{2} \quad or \quad x = \frac{-5 - \sqrt{5}}{2}$$

Thus the roots are $\boldsymbol{\frac{-5 + \sqrt{5}}{2}, \frac{-5 - \sqrt{5}}{2}}$

(ii) $\boldsymbol{4x^2 + 12x + 9 = 0}$

Solution:

$4x^2 + 12x + 9 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = 12, c = 9$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (12)^2 - 4(4)(9)$

Discriminant $= 144 - 144$

Discriminant $= 0 = 0$

As the roots are real

Now find the roots

Using Factorizatin Method

$4x^2 + 12x + 9 = 0$

$4x^2 + 6x + 6x + 9 = 0$

$2x(2x + 3) + 3(2x + 3) = 0$

$(2x + 3)(2x + 3) = 0$

$2x + 3 = 0 \quad or \quad 2x + 3 = 0$

$2x = -3 \quad or \quad 2x = -3$

$x = \frac{-3}{2} \quad or \quad x = \frac{-3}{2}$

Solution Set $= \left\{\boldsymbol{\frac{-3}{2}}\right\}$

Ex # 2.1

(iii) $\boldsymbol{6x^2 + x - 2 = 0}$

Solution:

$6x^2 + x - 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 6, b = 1, c = -2$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (1)^2 - 4(6)(-2)$

Discriminant $= 1 + 48$

Discriminant $= 49$

Discriminant $= 7^2 > 0$

Verification by Solving equation

Using Factorizatin Method

$6x^2 + x - 2 = 0$

$6x^2 - 3x + 4x - 2 = 0$

$3x(2x - 1) + 2(2x - 1) = 0$

$(2x - 1)(3x + 2) = 0$

$2x - 1 = 0 \quad or \quad 3x + 2 = 0$

$2x = 1 \quad or \quad 3x = -2$

$x = \frac{1}{2} \quad or \quad x = \frac{-2}{3}$

Solution Set $= \left\{\frac{1}{2}, \frac{-2}{3}\right\}$

Q5: Determine the nature of roots of the following quadratic equations and verify the results by solving them.

(i) $\boldsymbol{3x^2 - 10x + 3 = 1}$

Solution:

$3x^2 - 10x + 3 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = -10, c = 3$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-10)^2 - 4(3)(3)$

Discriminant $= 100 - 36$

Discriminant $= 64$

Discriminant $= 8^2 > 0$

Verification by Solving equation

Using Factorizatin Method

$3x^2 - 10x + 3 = 0$

$3x^2 - 1x - 9x + 3 = 0$

$x(3x - 1) - 3(3x - 1) = 0$

$(3x - 1)(x - 3) = 0$

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Ex # 2.1

$3x - 1 = 0 \quad or \quad x - 3 = 0$

$3x = 1 \quad or \quad x = 3$

$x = \frac{1}{3} \quad or \quad x = 3$

Solution Set $= \left\{\frac{1}{3}, 3\right\}$

(ii) $x^2 - 6x + 4 = 0$

Solution:

$x^2 - 6x + 4 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -6, c = 4$

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (-6)^2 - 4(1)(4)$

Discriminant $= 36 - 16$

Discriminant $= 20 > 0$

Verification by Solving equation

Using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$x = \frac{6 \pm \sqrt{4 \times 5}}{2}$$

$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(3 \pm \sqrt{5})}{2}$$

$x = 3 \pm \sqrt{5}$

$x = 3 + \sqrt{5} \quad or \quad x = 3 - \sqrt{5}$

Solution Set $= \{3 + \sqrt{5}, 3 - \sqrt{5}\}$

(iii) $x^2 - 3 = 0$

Solution:

$x^2 - 3 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 0, c = -3$

Ex # 2.1

As we have

Discriminant $= b^2 - 4ac$

Discriminant $= (0)^2 - 4(1)(-3)$

Discriminant $= 0 + 12$

Discriminant $= 12 > 0$

Verification by Solving equation

$x^2 - 3 = 0$

$x^2 = 3$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

$x = \sqrt{3} \quad or \quad x = -\sqrt{3}$

Solution Set $= \{\sqrt{3}, -\sqrt{3}\}$

Q6: For what value of k the roots of the following equations are :

(a) real (b) imaginary

(i) $2x^2 + 3x + k = 0$

Solution:

$2x^2 + 3x + k = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = 3, c = k$

a) If roots are Real

Discriminant $= b^2 - 4ac \geq 0$

$b^2 - 4ac \geq 0$

$(3)^2 - 4(2)(k) \geq 0$

$9 - 8k \geq 0$

$9 \geq 8k$

$\frac{9}{8} \geq k$

$k \leq \frac{9}{8}$

b) If roots are Imaginary

Discriminant $= b^2 - 4ac < 0$

$b^2 - 4ac < 0$

$(3)^2 - 4(2)(k) < 0$

$9 - 8k < 0$

$9 < 8k$

$\frac{9}{8} < k$

$k > \frac{9}{8}$

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Ex # 2.1

(ii) $\boldsymbol{kx^2 + 2x + 1 = 0}$

Solution:

$kx^2 + 2x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = k, b = 2, c = 1$

$a)$ **If roots are Real**

Discriminant $= b^2 - 4ac \geq 0$

$b^2 - 4ac \geq 0$

$(2)^2 - 4(k)(1) \geq 0$

$4 - 4k \geq 0$

$4 \geq 4k$

$\frac{4}{4} \geq k$

$1 \geq k$

$k \leq 1$

$b)$ **If roots are Imaginary**

Discriminant $= b^2 - 4ac < 0$

$b^2 - 4ac < 0$

$(3)^2(2)^2 - 4(k)(1) < 0$

$4 - 4k < 0$

$4 < 4k$

$\frac{4}{4} < k$

$1 < k$

$k > 1$

(iii) $\boldsymbol{x^2 + 5x + k = 0}$

Solution:

$x^2 + 5x + k = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 5, c = k$

$a)$ **If roots are Real**

Discriminant $= b^2 - 4ac \geq 0$

$b^2 - 4ac \geq 0$

$(5)^2 - 4(1)(k) \geq 0$

$25 - 4k \geq 0$

$25 \geq 4k$

$\frac{25}{4} \geq k$

$k \leq \frac{25}{4}$

Ex # 2.1

$b)$ **If roots are Imaginary**

Discriminant $= b^2 - 4ac < 0$

$b^2 - 4ac < 0$

$(5)^2 - 4(1)(k) < 0$

$25 - 4k < 0$

$25 < 4k$

$\frac{25}{4} < k$

$k > \frac{25}{4}$

Ex # 2.2

Cube root of unity

Let x by the cube root of 1

$x = (1)^{\frac{1}{3}}$

Taking cube on B.S

$(x)^3 = \left[(1)^{\frac{1}{3}}\right]^3$

$x^3 = 1$

$x^3 - 1 = 0$

$(x)^3 - (1)^3 = 0$

As $\boldsymbol{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$

$(x - 1)(x^2 + (x)(1) + (1)^2) = 0$

$(x - 1)(x^2 + x + 1) = 0$

$x - 1 = 0 \quad or \quad x^2 + x + 1 = 0$

$x = 1 \quad or \quad x^2 + x + 1 = 0$

Now

$x^2 + x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 1, c = 1$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

Ex # 2.2

$$x = \frac{-1 \pm \sqrt{-1 \times 3}}{2}$$

$$x = \frac{-1 \pm \sqrt{-1}.\sqrt{3}}{2}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x = \frac{-1 + i\sqrt{3}}{2} \quad or \quad x = \frac{-1 - i\sqrt{3}}{2}$$

Thus the cube root of unity are $1, \frac{-1 + i\sqrt{3}}{2}$ and $\frac{-1 - i\sqrt{3}}{2}$

Here 1 is the real root and

$\frac{-1 + i\sqrt{3}}{2}$ *and* $\frac{-1 - i\sqrt{3}}{2}$ are complex roots

Let $\omega = \frac{-1 + i\sqrt{3}}{2}$ and $\omega^2 \frac{-1 - i\sqrt{3}}{2}$

$x = \omega \quad or \quad x = \omega^2$

Solution Set $= \{\mathbf{1}, \boldsymbol{\omega}, \boldsymbol{\omega^2}\}$

Properties of the cube root of unity

The sum of cube roots of unity is zero

$$1 + \omega + \omega^2 = 0$$

As $\omega = \frac{-1 + i\sqrt{3}}{2}$ and $\omega^2 \frac{-1 - i\sqrt{3}}{2}$

$$1 + \omega + \omega^2 = 1 + \frac{-1 + i\sqrt{3}}{2} + \frac{-1 - i\sqrt{3}}{2}$$

$$1 + \omega + \omega^2 = \frac{2 + (-1 + i\sqrt{3}) + (-1 - i\sqrt{3})}{2}$$

$$1 + \omega + \omega^2 = \frac{2 - 1 + i\sqrt{3} - 1 - i\sqrt{3}}{2}$$

$$1 + \omega + \omega^2 = \frac{2 - 1 - 1 + i\sqrt{3} - i\sqrt{3}}{2}$$

$$1 + \omega + \omega^2 = \frac{1 - 1}{2}$$

$$1 + \omega + \omega^2 = \frac{0}{2}$$

$$1 + \omega + \omega^2 = 0$$

Other properties are:

$$1 + \omega = -\omega^2$$

$$1 + \omega^2 = -\omega$$

$$\omega + \omega^2 = -1$$

Ex # 2.2

The Product of cube roots of unity is one

$$1.\omega.\omega^2 = 1$$

As $\omega = \frac{-1 + i\sqrt{3}}{2}$ and $\omega^2 \frac{-1 - i\sqrt{3}}{2}$

$$1.\omega.\omega^2 = 1.\left(\frac{-1 + i\sqrt{3}}{2}\right)\left(\frac{-1 - i\sqrt{3}}{2}\right)$$

$$1.\omega.\omega^2 = \frac{(-1 + i\sqrt{3})(-1 - i\sqrt{3})}{2 \times 2}$$

$$1.\omega.\omega^2 = \frac{(-1)^2 - (i\sqrt{3})^2}{4}$$

$$1.\omega.\omega^2 = \frac{1 - i^2(3)}{4}$$

$$1.\omega.\omega^2 = \frac{1 - 3i^2}{4}$$

$$1.\omega.\omega^2 = \frac{1 - 3(-1)}{4} \qquad \therefore i^2 = 1$$

$$1.\omega.\omega^2 = \frac{1 + 3}{4}$$

$$1.\omega.\omega^2 = \frac{4}{4}$$

$$1.\omega.\omega^2 = 1$$

OR

$\omega^3 = 1$

Reciprocal of the cube root of unity

$$\omega = \frac{1}{\omega^2} \quad and \quad \omega^2 = \frac{1}{\omega}$$

As $\omega^3 = 1$

We can write it as:

$$\omega.\omega^2 = 1$$

$$\text{Thus } \omega = \frac{1}{\omega^2}$$

And also

$$\omega^2 = \frac{1}{\omega}$$

Example # 7:

Show that

$$\boldsymbol{x^3 + y^3 = (x + y)(x - \omega y)(x - \omega^2 y)}$$

Solution:

$x^3 - y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$

R.H.S

$(x + y)(x + \omega y)(x + \omega^2 y)$

$= (x + y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$

$= (x + y)[x^2 + xy(\omega^2 + \omega) + \omega^3 y^2]$

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Ex # 2.2

As $\boldsymbol{\omega^2 + \omega = -1}$

$= (x+y)[x^2 + xy(-1) + (1)^3y^2]$

$= (x+y)(x^2 - xy + y^2)$

$= x^3 + y^3 =$ **L.H.S**

Example # 8

Evaluate $\boldsymbol{\omega^{15}, \omega^{24}, \omega^{90}, \omega^{101}, \omega^{-2}, \omega^{-13}}$

Solutions:

(i) $\omega^{15} = \omega^{3\times5}$

$= (\omega^3)^5$

$= (1)^5$

$= 1$

(ii) $\omega^{24} = \omega^{3\times8}$

$= (\omega^3)^8$

As $\boldsymbol{\omega^3 = 1}$

$= (1)^8$

$= 1$

(iii) $\omega^{90} = \omega^{3\times30}$

$= (\omega^3)^{30}$

As $\boldsymbol{\omega^3 = 1}$

$= (1)^{30}$

$= 1$

(iv) $\omega^{101} = \omega^{99}.\omega^2$

$\omega^{3\times33}.\omega^2$

$(\omega^3)^{33}.\omega^2$

As $\boldsymbol{\omega^3 = 1}$

$= (1)^{33}.\omega^2$

$= 1.\omega^2$

$= \omega^2$

(v) $\omega^{-2} = \frac{1}{\omega^2}$

As $\boldsymbol{\frac{1}{\omega^2} = \omega}$

$\omega^{-2} = \omega$

(vi) $\omega^{-13} = \frac{1}{\omega^{13}}$

$\omega^{-13} = \frac{1}{\omega^{12}.\omega}$

$\omega^{-13} = \frac{1}{\omega^{3\times4}.\omega}$

$\omega^{-13} = \frac{1}{(\omega^3)^4.\omega}$

As $\boldsymbol{\omega^3 = 1}$

$\omega^{-13} = \frac{1}{(1)^5.\omega}$

Ex # 2.2

(vii) $\omega^{-13} = \frac{1}{1.\omega}$

$\omega^{-13} = \frac{1}{\omega}$

As $\boldsymbol{\frac{1}{\omega} = \omega^2}$

$\omega^{-13} = \omega^2$

Example # 9

$\boldsymbol{(-1 + i\sqrt{3})^3 + (-1 + i\sqrt{3})^3 = 16}$

Solution:

$(-1 + i\sqrt{3})^3 + (-1 + i\sqrt{3})^3 = 16$

L.H.S

$(-1 + i\sqrt{3})^3 + (-1 + i\sqrt{3})^3$

As

$\boldsymbol{\omega = \frac{-1 + i\sqrt{3}}{2}}$ **and** $\boldsymbol{\omega^2 = \frac{-1 - i\sqrt{3}}{2}}$

$\boldsymbol{2\omega = -1 + i\sqrt{3}}$ **and** $\boldsymbol{2\omega^2 = -1 - i\sqrt{3}}$

So

$= (2\omega)^3 + (2\omega^2)^3$

$= (2^3\omega^3) + (2^3\omega^6)$

$= (8\omega^3) + (8\omega^{3\times2})$

$= (8\omega^3) + [8(\omega^3)^2]$

As $\boldsymbol{\omega^3 = 1}$

$= 8(1) + 8(1)^2$

$= 8 + 8(1)$

$= 8 + 8$

$= 16 =$ **R.H.S**

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Ex # 2.2

Page # 31

Q1: **Find the cube roots of the following numbers.**

(i) -1

Solution:

-1

Let x by the cube root of -1

$x = (-1)^{\frac{1}{3}}$

Taking cube on B.S

$(x)^3 = \left[(-1)^{\frac{1}{3}}\right]^3$

$x^3 = -1$

$x^3 + 1 = 0$

$(x)^3 + (1)^3 = 0$

As $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$(x+1)(x^2 - (x)(1) + (1)^2) = 0$

$(x+1)(x^2 - x + 1) = 0$

$x + 1 = 0 \quad or \quad x^2 - x + 1 = 0$

$x = -1 \quad or \quad x^2 - x + 1 = 0$

Now

$x^2 - x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -1, c = 1$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm \sqrt{-1 \times 3}}{2}$$

$$x = \frac{1 \pm \sqrt{-1}.\sqrt{3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

$$x = \frac{1 + i\sqrt{3}}{2} \quad or \quad x = \frac{1 - i\sqrt{3}}{2}$$

$$x = -\left(\frac{-1 - i\sqrt{3}}{2}\right) \quad or \quad x = -\left(\frac{-1 + i\sqrt{3}}{2}\right)$$

$x = -(\omega^2) \quad or \quad x = -(\omega)$

$x = -\omega^2 \quad or \quad x = -\omega$

Solution Set $= \{-1, -\omega, -\omega^2\}$

(ii) **8**

Solution:

8

Let x by the cube root of 8

$x = (8)^{\frac{1}{3}}$

Taking cube on B.S

$(x)^3 = \left[(8)^{\frac{1}{3}}\right]^3$

$x^3 = 8$

$x^3 - 8 = 0$

$(x)^3 - (2)^3 = 0$

As $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x-2)(x^2 + (x)(2) + (2)^2) = 0$

$(x-2)(x^2 + 2x + 4) = 0$

$x - 2 = 0 \quad or \quad x^2 + 2x + 4 = 0$

$x = 2 \quad or \quad x^2 + 2x + 4 = 0$

Now

$x^2 + 2x + 4 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = 2, c = 4$

As we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the values

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm \sqrt{-1 \times 4 \times 3}}{2}$$

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Ex # 2.2

$$x = \frac{-2 \pm \sqrt{-1}.\sqrt{4}.\sqrt{3}}{2}$$
$$x = \frac{-2 \pm 2i\sqrt{3}}{2}$$
$$x = \frac{2(-1 \pm i\sqrt{3})}{2}$$
$$x = 2\left(\frac{-1 \pm i\sqrt{3}}{2}\right)$$
$$x = 2\left(\frac{-1 + i\sqrt{3}}{2}\right) \quad or \quad x = 2\left(\frac{-1 - i\sqrt{3}}{2}\right)$$
$$x = 2(\omega) \quad or \quad x = 2(\omega^2)$$
$$x = 2\omega \quad or \quad x = 2\omega^2$$
Solution Set $= \{2, 2\omega, 2\omega^2\}$

(iii) -27

Solution:

-27

Let x by the cube root of -27

$$x = (-27)^{\frac{1}{3}}$$
Taking cube on B.S
$$(x)^3 = \left[(-27)^{\frac{1}{3}}\right]^3$$
$$x^3 = -27$$
$$x^3 + 27 = 0$$
$$(x)^3 + (3)^3 = 0$$
As $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$$(x+3)(x^2 - (x)(3) + (3)^2) = 0$$
$$(x+3)(x^2 - 3x + 9) = 0$$
$$x + 3 = 0 \quad or \quad x^2 - 3x + 9 = 0$$
$$x = -3 \quad or \quad x^2 - 3x + 9 = 0$$
Now
$$x^2 - 3x + 9 = 0$$
Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -3, c = 9$

As we have
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Put the values
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

Ex # 2.2

$$x = \frac{3 \pm \sqrt{9 - 36}}{2}$$
$$x = \frac{3 \pm \sqrt{-27}}{2}$$
$$x = \frac{3 \pm \sqrt{-1 \times 9 \times 3}}{2}$$
$$x = \frac{3 \pm \sqrt{-1}.\sqrt{9}.\sqrt{3}}{2}$$
$$x = \frac{3 \pm 3i\sqrt{3}}{2}$$
$$x = \frac{3(1 \pm i\sqrt{3})}{2}$$
$$x = 3\left(\frac{1 \pm i\sqrt{3}}{2}\right)$$
$$x = 3\left(\frac{1 + i\sqrt{3}}{2}\right) \quad or \quad x = 3\left(\frac{1 - i\sqrt{3}}{2}\right)$$
$$x = -3\left(\frac{-1 - i\sqrt{3}}{2}\right) \quad or \quad x = -3\left(\frac{-1 + i\sqrt{3}}{2}\right)$$
$$x = -3(\omega^2) \quad or \quad x = -3(\omega)$$
$$x = -3\omega^2 \quad or \quad x = -3\omega$$
Solution Set $= \{-3, -3\omega, -3\omega^2\}$

Q2: **Evaluate:**

(i) $\omega^{12} + \omega^{58} + \omega^{95}$

Solution:

$$\omega^{12} + \omega^{58} + \omega^{95}$$
$$= \omega^{12} + \omega^{57}.\omega^1 + \omega^{93}.\omega^2$$
$$= \omega^{3\times4} + \omega^{3\times19}.\omega + \omega^{3\times31}.\omega^2$$
$$= (\omega^3)^4 + (\omega^3)^{19}.\omega + (\omega^3)^{31}.\omega^2$$
As $\omega^3 = 1$
$$= (1)^4 + (1)^{19}.\omega + (1)^{31}.\omega^2$$
$$= 1 + 1.\omega + 1.\omega^2$$
$$= 1 + \omega + \omega^2$$
$$= 0 \qquad \therefore \mathbf{1 + \omega + \omega^2 = 0}$$

(ii) $(1 + \omega - \omega^2)^7$

Solution:

$$= (1 + \omega - \omega^2)^7$$
$$= (-\omega^2 - \omega^2)^7$$
$$= (-2\omega^2)^7$$
$$= (-2)^7(\omega^2)^7$$
$$= -128\omega^{14}$$
$$= -128.\omega^{12}.\omega^2$$

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Ex # 2.2

$= -128.\omega^{3\times4}.\omega^2$

$= -128(\omega^3)^4.\omega^2$

As $\omega^3 = 1$

$= -128(1)^4.\omega^2$

$= -128(1).\omega^2$

$= -128\,\omega^2$

(iii) $(1+3\omega-\omega^2)(1+\omega-2\omega^2)$

Solution:

$(1+3\omega-\omega^2)(1+\omega-2\omega^2)$

$= (1+\omega+2\omega-\omega^2)(1+\omega-2\omega^2)$

As $1+\omega = -\omega^2$

$= (-\omega^2+2\omega-\omega^2)(-\omega^2-2\omega^2)$

$= (2\omega-\omega^2-\omega^2)(-3\omega^2)$

$= (2\omega-2\omega^2)(-3\omega^2)$

$= -6\omega^3+6\omega^4$

$= -6\omega^3+6\omega^3\omega$

As $\omega^3 = 1$

$= -6(1)+6(1)\omega$

$= -6+6\omega$

$= -6(1-\omega)$

Q3: **Prove that:**

(i) $(1+2\omega)(1+2\omega^2)(1-\omega-\omega^2) = 6$

Solution:

$(1+2\omega)(1+2\omega^2)(1-\omega-\omega^2) = 6$

L. H. S:

$(1+2\omega)(1+2\omega^2)(1-\omega-\omega^2)$

$= (1+2\omega^2+2\omega+4\omega^3)[1-(\omega+\omega^2)]$

$= [1+2(\omega^2+\omega)+4\omega^3][1-(\omega+\omega^2)]$

As $\omega^2+\omega = -1$

$= [1+2(-1)+4(1)][1-(-1)]$

$= (1-2+4)(1+1)$

$= (-1+4)(2)$

$= (3)(2)$

$= 6 =$ **R. H. S**

Hence

$(1+2\omega)(1+2\omega^2)(1-\omega-\omega^2) = 6$

(ii) $\left(-1+i\sqrt{3}\right)^4\left(-1+i\sqrt{3}\right)^5 = 512\omega^2$

Solution:

$$\left(-1+i\sqrt{3}\right)^4\left(-1+i\sqrt{3}\right)^5 = 512\omega^2$$

Ex # 2.2

L. H. S

$\left(-1+i\sqrt{3}\right)^4\left(-1+i\sqrt{3}\right)^5$

As

$\omega = \dfrac{-1+i\sqrt{3}}{2}$ **and** $\omega^2 = \dfrac{-1-i\sqrt{3}}{2}$

$2\omega = -1+i\sqrt{3}$ **and** $2\omega^2 = -1-i\sqrt{3}$

So

$= (2\omega)^4(2\omega^2)^5$

$= (2^4\omega^4)(2^5\omega^{10})$

$= 16\times32\omega^{4+10}$

$= 512\omega^{14}$

$= 512\omega^{12}.\omega^2$

$= 512(\omega^3)^4.\omega^2$

As $\omega^3 = 1$

$= 512(1)^4.\omega^2$

$= 512\omega^2 =$ **R. H. S**

Q4: **Show that:**

(i) $x^3-y^3 = (x-y)(x-\omega y)(x-\omega^2 y)$

Solution:

$x^3-y^3 = (x-y)(x-\omega y)(x-\omega^2 y)$

R. H. S

$(x-y)(x-\omega y)(x-\omega^2 y)$

$= (x-y)(x^2-\omega^2 xy-\omega xy+\omega^3 y^2)$

$= (x-y)[x^2-xy(\omega^2+\omega)+\omega^3 y^2]$

As $\omega^2+\omega = -1$

$= (x-y)[x^2-xy(-1)+(1)^3y^2]$

$= (x-y)(x^2+xy+y^2)$

$= x^3-y^3 =$ **L. H. S**

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) = 1$

Solution:

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) = 1$

L. H. S

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (-\omega^2)(-\omega)(1+\omega^8+\omega^4+\omega^{12})$

As $1+\omega = -\omega^2$ and $1+\omega^2 = -\omega$

$= (\omega^3)(1+\omega^6.\omega^2+\omega^3\omega+\omega^{12})$

$= (\omega^3)[1+(\omega^3)^2.\omega^2+\omega^3\omega+(\omega^3)^4]$

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$= (1)(1 + (1)^2 . \omega^2 + (1)\omega + (1)^4)$

$= 1 + \omega^2 + \omega + 1$

As $1 + \omega^2 + \omega = 0$

$= 0 + 1$

$= 1$

Ex # 2.2

Roots and co-efficients of a Quadratic equation

Let α and β are the roots of quadratic equation $ax^2 + bx + c = 0$

Thus $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

And $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Then sum of roots:

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{(-b + \sqrt{b^2 - 4ac}\,) + (-b - \sqrt{b^2 - 4ac}\,)}{2a}$$

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{-b - b}{2a}$$

$$\alpha + \beta = \frac{-2b}{2a}$$

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$\alpha . \beta$

$$= \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) . \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$\alpha . \beta = \frac{(-b + \sqrt{b^2 - 4ac}\,) . (-b - \sqrt{b^2 - 4ac}\,)}{(2a)(2a)}$$

$$\alpha . \beta = \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2}$$

$$\alpha . \beta = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$\alpha . \beta = \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$\alpha . \beta = \frac{4ac}{4a^2}$$

$$\alpha . \beta = \frac{c}{a}$$

Thus

Sum of roots $= \alpha + \beta = \dfrac{-b}{a}$

Product of roots $= \alpha . \beta = \dfrac{c}{a}$

Example # 10

Without solving, find the sum and products of the roots of the equations.

(i) $\mathbf{2x^2 - 3x - 4 = 0}$

Solution:

$2x^2 - 3x - 4 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = -3, c = -4$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

And product of roots:

$$\alpha\,.\,\beta = \frac{c}{a} = \frac{-4}{2} = -2$$

(ii) $\mathbf{3x^2 + 6x - 2 = 0}$

Solution:

$3x^2 + 6x - 2 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = 6, c = -2$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-6}{3} = -2$$

And product of roots:

$$\alpha\,.\,\beta = \frac{c}{a} = \frac{-2}{3} = -\frac{2}{3}$$

Example # 11

Find the value of k so that the sum of the roots of the equation $2x^2 + kx + 6 = 0$ is equal to three times the product of its roots.

Solution:

$2x^2 + kx + 6 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = k, c = 6$

Let α and β be the roots of equation

Ex # 2.3

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-k}{2}$

And product of roots:

$\alpha . \beta = \frac{c}{a} = \frac{6}{2} = 3$

According to given condition

Sum of roots $=$ 3 $\times$ Product of roots

$\frac{-k}{2} = 3 \times 3$

$\frac{-k}{2} = 9$

Multiply B.S by 2

$\frac{-k}{2} \times 2 = 9 \times 2$

$-k = 18$

$k = -18$

Example # 12

Find the value of a if the sum of the square of the roots of $x^2 - 3ax + a^2 = 0$ is equal to 7.

Solution:

$$x^2 - 3ax + a^2 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -3a, c = a^2$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-3a)}{1} = 3a$

And product of roots:

$\alpha . \beta = \frac{c}{a} = \frac{a^2}{1} = a^2$

According to given condition

$\boldsymbol{\alpha^2 + \beta^2 = 7}$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$(\alpha + \beta)^2 - 2\alpha\beta = 7$

Put the values

$(3a)^2 - 2(a^2) = 7$

$9a^2 - 2a^2 = 7$

$7a^2 = 7$

Divide B.S by 7

Ex # 2.3

$\frac{7a^2}{7} = \frac{7}{7}$

$a^2 = 1$

Taking square root on B.S

$\sqrt{a^2} = \pm\sqrt{1}$

$a = \pm 1$

Example # 13

Find the value of k if the roots of $x^2 - 7x + k = 0$ differ by unity.

Solution:

$$x^2 - 7x + k = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -7, c = k$

Let α and $\alpha + 1$ be the roots of equation

Then sum of roots:

$\alpha + \alpha + 1 = \frac{-b}{a}$

$2\alpha + 1 = \frac{-(-7)}{1}$

$2\alpha + 1 = 7$

$2\alpha = 7 - 1$

$2\alpha = 6$

$\alpha = \frac{6}{2}$

$\alpha = 3$

And product of roots:

$\alpha(\alpha + 1) = \frac{c}{a}$

$\alpha^2 + \alpha = \frac{k}{1}$

$\alpha^2 + \alpha = k$

Put the value of α

$(3)^2 + 3 = k$

$9 + 3 = k$

$12 = k$

$k = 12$

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Ex # 2.3

Example # 14

If α, β are the roots of $9x^2 - 27x + k = 0$, find the k such that $2\alpha + 5\beta = 7$

Solution:

$$9x^2 - 27x + k = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 9, b = -27, c = k$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-27)}{9} = \frac{27}{9} = 3$

$\alpha + \beta = 3 \quad \ldots\ldots equ\ (i)$

And product of roots:

$\alpha\,.\beta = \frac{c}{a} = \frac{k}{9}$

$\alpha\,.\beta = \frac{k}{9} \quad \ldots\ldots equ\ (ii)$

According to given condition

$2\alpha + 5\beta = 7 \quad \ldots\ldots equ\ (iii)$

Equ (i) $\Rightarrow$

$\alpha + \beta = 3$

$\alpha = 3 - \beta \quad \ldots\ldots$ Equ (iv)

Put the value of α in Equ (iii)

$2(3 - \beta) + 5\beta = 7$

$6 - 2\beta + 5\beta = 7$

$6 + 3\beta = 7$

$3\beta = 7 - 6$

$3\beta = 1$

Divide B. S by 3

$\frac{3\beta}{3} = \frac{1}{3}$

$\beta = \frac{1}{3}$

Now put $\beta = \frac{1}{3}$ in equ (iv)

$\alpha = 3 - \frac{1}{3}$

$\alpha = \frac{9-1}{3}$

$\alpha = \frac{8}{3}$

Put the value of α and β in Equ (ii)

$\left(\frac{8}{3}\right)\left(\frac{1}{3}\right) = k$

$8 = k$

$k = 8$

Ex # 2.3

Example # 15

Find the value of m and n if both sum and products of roots of the equation $mx^2 - 5x + n = 0$ are equal to 10.

Solution:

$$mx^2 - 5x + n = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = m, b = -5, c = n$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{m} = \frac{5}{m}$

And product of roots:

$\alpha\,.\beta = \frac{c}{a} = \frac{n}{m}$

According to given conditions

Sum of roots equal to 10

$\frac{5}{m} = 10$

$5 = 10m$

$\frac{5}{10} = \frac{10m}{10}$

$\frac{1}{2} = m$

$m = \frac{1}{2}$

Product of roots equal to 10

$\frac{n}{m} = 10$

$n = 10m$

$n = 10 \times \frac{1}{2}$

$n = 5$

Thus $m = \frac{1}{2}$ and $n = 5$

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Ex # 2.3

Page # 34

Q1: **Without solving the equation, find the sum and products of the roots of the following quadratic equations.**

(i) $4x^2 - 4x - 3 = 0$

Solution:

$$4x^2 - 4x - 3 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = -4, c = -3$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{4} = \frac{4}{4} = 1$$

And product of roots:

$$\alpha \,.\, \beta = \frac{c}{a} = \frac{-3}{4} = -\frac{3}{4}$$

(ii) $2x^2 + 5x + 6 = 0$

Solution:

$$2x^2 + 5x + 6 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = 5, c = 6$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-5}{2} = -\frac{5}{2}$$

And product of roots:

$$\alpha \,.\, \beta = \frac{c}{a} = \frac{6}{2} = 3$$

(iii) $3x^2 + 2x - 5 = 0$

Solution:

$3x^2 + 2x - 5 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = 2, c = -5$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-2}{3} = -\frac{2}{3}$$

And product of roots:

$$\alpha \,.\, \beta = \frac{c}{a} = \frac{-5}{3} = -\frac{5}{3}$$

Ex # 2.3

Q2: **Find the value of k if sum of the roots of $2x^2 + kx + 6 = 0$ is equal to the product of its roots**

Solution:

$$2x^2 + kx + 6 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = k, c = 6$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-k}{2}$$

And product of roots:

$$\alpha \,.\, \beta = \frac{c}{a} = \frac{6}{2} = 3$$

According to given condition

Sum of roots = Product of roots

$$\frac{-k}{2} = 3$$

Multiply B.S by 2

$$\frac{-k}{2} \times 2 = 3 \times 2$$

$$-k = 6$$

$$k = -6$$

Q3: **Find the value of k if the sum of the square of the roots of $x^2 - 5kx + 6k^2 = 0$ is equal to 13.**

Solution:

$$x^2 - 5kx + 6k^2 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -5k, c = 6k^2$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-5k)}{1} = 5k$$

And product of roots:

$$\alpha \,.\, \beta = \frac{c}{a} = \frac{6k^2}{1} = 6k^2$$

According to given condition

$\boldsymbol{\alpha^2 + \beta^2 = 13}$

As $\boldsymbol{\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2}$

Then $\boldsymbol{\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta}$

So

$$(\alpha + \beta)^2 - 2\alpha\beta = 13$$

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Ex # 2.3

Put the values

$(5k)^2 - 2(6k^2) = 13$

$25k^2 - 12k^2 = 13$

$13k^2 = 13$

Divide B. S by 13

$\frac{13k^2}{13} = \frac{13}{13}$

$k^2 = 1$

Taking square root on B. S

$\sqrt{k^2} = \pm\sqrt{1}$

$k = \pm 1$

Q4: Find the value of k if the roots of $x^2 - 5x + k = 0$ differ by unity.

Solution:

$$x^2 - 5x + k = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -5, c = k$

Let α and $\alpha + 1$ be the roots of equation

Then sum of roots:

$\alpha + \alpha + 1 = \frac{-b}{a}$

$2\alpha + 1 = \frac{-(-5)}{1}$

$2\alpha + 1 = 5$

$2\alpha = 5 - 1$

$2\alpha = 4$

$\alpha = \frac{4}{2}$

$\alpha = 2$

And product of roots:

$\alpha\,(\alpha + 1) = \frac{c}{a}$

$\alpha^2 + \alpha = \frac{k}{1}$

$\alpha^2 + \alpha = k$

Put the value of α

$(2)^2 + 2 = k$

$4 + 2 = k$

$6 = k$

$k = 6$

Ex # 2.3

Q5: Find the value of k if the roots of $x^2 - 9x + k + 2 = 0$ differ by three.

Solution:

$$x^2 - 9x + k + 2 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -9, c = k + 2$

Let α and $\alpha + 3$ be the roots of equation

Then sum of roots:

$\alpha + \alpha + 3 = \frac{-b}{a}$

$2\alpha + 3 = \frac{-(-9)}{1}$

$2\alpha + 3 = 9$

$2\alpha = 9 - 3$

$2\alpha = 6$

$\alpha = \frac{6}{2}$

$\alpha = 3$

And product of roots:

$\alpha\,(\alpha + 3) = \frac{c}{a}$

$\alpha^2 + 3\alpha = \frac{k+2}{1}$

$\alpha^2 + 3\alpha = k + 2$

Put the value of α

$(3)^2 + 3(3) = k + 2$

$9 + 9 = k + 2$

$18 = k + 2$

$18 - 2 = k$

$16 = k$

$k = 16$

Q6: If α, β are the roots of $x^2 - 5x + k = 0$, find the k such that $3\alpha + 2\beta = 12$

Solution:

$$x^2 - 5x + k = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -5, c = k$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = \frac{5}{1}$

$\alpha + \beta = 5 \quad \ldots\ldots equ\ (i)$

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And product of roots:

$$\alpha . \beta = \frac{c}{a} = \frac{k}{1} = k$$

$\alpha . \beta = k \quad \ldots\ldots equ\ (ii)$

According to given condition

$3\alpha + 2\beta = 12 \quad \ldots\ldots equ\ (iii)$

Equ (i) $\Rightarrow$

$\alpha + \beta = 5$

$\alpha = 5 - \beta \quad \ldots\ldots\ldots$ Equ (iv)

Put the value of α in Equ (iii)

$3(5-\beta) + 2\beta = 12$

$15 - 3\beta + 2\beta = 12$

$15 - \beta = 12$

$-\beta = 12 - 15$

$-\beta = -3$

$\beta = 3$

Now put $\beta = 3$ in equ (iv)

$\alpha = 5 - 3$

$\alpha = 2$

Put the value of α and β in Equ (ii)

$(2)(3) = k$

$6 = k$

$k = 6$

Q7: Find the value of m and n if both sum and products of roots of the equation $mx^2 - 3x - n = 0$ are equal to $\frac{3}{5}$

Solution:

$$mx^2 - 3x - n = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = m, b = -3, c = -n$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{m} = \frac{3}{m}$$

And product of roots:

$$\alpha . \beta = \frac{c}{a} = \frac{-n}{m}$$

According to given conditions

Sum of roots equal to $\frac{3}{5}$

$$\frac{3}{m} = \frac{3}{5}$$

$3 \times 5 = 3 \times m$

$15 = 3m$

$$\frac{15}{3} = \frac{3m}{3}$$

$5 = m$

$m = 5$

Product of roots equal to $\frac{3}{5}$

$$\frac{-n}{m} = \frac{3}{5}$$

$$\frac{-n}{5} = \frac{3}{5} \qquad \therefore m = 5$$

$$\frac{-n}{5} \times 5 = \frac{3}{5} \times 5$$

$-n = 3$

$n = -3$

Thus $m = 5$ and $n = -3$

Ex # 2.4

Symmetric functions of roots of a Quadratic equation

Let α , β be the roots of a quadratic equation, then the expressions of the form of $\alpha + \beta$, $\alpha\beta$, $\alpha^2 + \beta^2$ are called the functions of the roots of the quadratic equation.

By symmetric function of the roots of an equation, we mean that the function remains unchanged in values when the roots are interchanged.

Example:

$f(\alpha, \beta) = \alpha^2 + \beta^2$ then

$f(\beta, \alpha) = \beta^2 + \alpha^2$

Both are symmetric functions.

Example # 16 (i)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\alpha + \beta$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

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Ex # 2.4

Example # 16 (ii)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\alpha\beta$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then Product of roots:

$$\alpha \, . \beta = \frac{c}{a}$$

Example # 16 (iii)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\alpha^2 + \beta^2$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha \, . \beta = \frac{c}{a}$$

Now to find $\alpha^2 + \beta^2$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$$\alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$\alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$$

Example # 16 (iv)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\alpha^3 + \beta^3$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha \, . \beta = \frac{c}{a}$$

Ex # 2.4

Now to find $\alpha^3 + \beta^3$

As $\alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta) = (\alpha + \beta)^3$

Then $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

So

$$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)$$

$$\alpha^3 + \beta^3 = \frac{-b^3}{a^3} + \frac{3bc}{a^2}$$

$$\alpha^3 + \beta^3 = \frac{-b^2 + 3abc}{a^3}$$

$$\alpha^3 + \beta^3 = \frac{3abc - b^2}{a^3}$$

Example # 16 (v)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\frac{1}{\alpha} + \frac{1}{\beta}$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha \, . \beta = \frac{c}{a}$$

Now to find $\frac{1}{\alpha} + \frac{1}{\beta}$

So

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{a} \div \frac{c}{a}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{a} \times \frac{a}{c}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{c}$$

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Ex # 2.4

Example # 16 (vi)

If α, β are the roots of $ax^2 + bx + c = 0$, find the values of the symmetric function $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha . \beta = \frac{c}{a}$$

Now to find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

So

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c^2}{a^2}}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{b^2 - 2ac}{a^2} \div \frac{c^2}{a^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{b^2 - 2ac}{c^2}$$

Ex # 2.4

Formation Of Quadratic Equation Whose Roots Are Given

Let α and β are the roots of equation

Then sum of roots:

$$\mathrm{S} = \alpha + \beta = \frac{-b}{a}$$

$$\mathrm{S} = -\frac{b}{a}$$

And product of roots:

$$\mathrm{P} = \alpha . \beta = \frac{c}{a}$$

$$\mathrm{P} = \frac{c}{a}$$

As $ax^2 + bx + c = 0$

Divide all terms by a

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Now we can write it as

$$x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

As $-\frac{b}{a} = \mathrm{S}$ and $\frac{c}{a} = \mathrm{P}$

Then

$$x^2 - \mathrm{S}x + \mathrm{P} = 0$$

Example # 17 (i)

Form a quadratic equation whose roots are $1 + \sqrt{5}$ and $1 - \sqrt{5}$

Solution:

$$1 + \sqrt{5}, 1 - \sqrt{5}$$

As $1 + \sqrt{5}$ and $1 - \sqrt{5}$ are the roots of required equation

Then sum of roots:

$$S = 1 + \sqrt{5} + 1 - \sqrt{5}$$

$$S = 1 + 1 + \sqrt{5} - \sqrt{5}$$

$$S = 2$$

And product of roots:

$$P = \left(1 + \sqrt{5}\right)\left(1 - \sqrt{5}\right)$$

$$P = (1)^2 - \left(\sqrt{5}\right)^2$$

$$P = 1 - 5$$

$$P = -4$$

Ex # 2.4

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - 2x + (-4) = 0$

$x^2 - 2x - 4 = 0$

Which is the required equation

Thus the given roots is the reverse process of solving an equation.

Example # 18

Form the quadratic equation whose roots are

(i) $\mathbf{2a+1\,,2b+1}$

Solution:

$$2a+1\,,2b+1$$

As 2a + 1 and 2b + 1 are the roots of required equation

Then sum of roots:

$S = 2a + 1 + 2a + 1$

$S = 2a + 2b + 1 + 1$

$S = 2a + 2b + 2$

And product of roots:

$P = (2a+1)\,(2b+1)$

$P = 4ab + 2a + 2b + 1$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - (2a + 2b + 2)x + (4ab + 2a + 2b + 1) = 0$

Which is the required equation

(ii) $\mathbf{a^2\,,b^2}$

Solution:

$$a^2\,,b^2$$

As a^2 and $b^2 + 1$ are the roots of required equation

Then sum of roots:

$S = a^2 + b^2$

And product of roots:

$P = (a^2)\,(b^2)$

$P = a^2\,b^2$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - (a^2 + b^2)x + a^2\,b^2 = 0$

Which is the required equation

Ex # 2.4

(iii) $\mathbf{\frac{1}{a},\frac{1}{b}}$

Solution:

$$\frac{1}{a},\frac{1}{b}$$

As $\frac{1}{a}$ and $\frac{1}{b}$ are the roots of required equation

Then sum of roots:

$S = \frac{1}{a} + \frac{1}{b}$

$S = \frac{b+a}{ab}$

$S = \frac{a+b}{ab}$

And product of roots:

$P = \left(\frac{1}{a}\right)\left(\frac{1}{b}\right)$

$P = \frac{1}{ab}$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - \left(\frac{a+b}{ab}\right)x + \frac{1}{ab} = 0$

Multiply all terms by ab

$ab \times x^2 - ab \times \left(\frac{a+b}{ab}\right)x + ab \times \frac{1}{ab} = 0$

$abx^2 - (a+b)x + 1 = 0$

Which is the required equation

(iv) $\mathbf{\frac{2}{5},\frac{5}{2}}$

Solution:

$$\frac{2}{5},\frac{5}{2}$$

As $\frac{2}{5}$ and $\frac{5}{2}$ are the roots of required equation

Then sum of roots:

$S = \frac{2}{5} + \frac{5}{2}$

$S = \frac{4+25}{10}$

$S = \frac{29}{10}$

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Ex # 2.4

And product of roots:

$$P = \left(\frac{2}{5}\right)\left(\frac{5}{2}\right)$$

$$P = \frac{10}{10}$$

$$P = 1$$

As required equation is:

$$x^2 - Sx + P = 0$$

Now

$$x^2 - \frac{29}{10}x + 1 = 0$$

Multiply all terms by 10

$$10 \times x^2 - 10 \times \frac{29}{10}x + 10 \times 1 = 0$$

$$10x^2 - 29x + 10 = 0$$

Which is the required equation

Ex # 2.4

Page # 39

Q1: (i) **If α, β are the roots of $ax^2 + bx + c = 0$, find the values of $\alpha^3\beta + \beta^3\alpha$**

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha . \beta = \frac{c}{a}$$

According to given condition

$$\alpha^3\beta + \beta^3\alpha = \alpha\beta(\alpha^2 + \beta^2)$$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$$\alpha^3\beta + \beta^3\alpha = \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$\alpha^3\beta + \beta^3\alpha = \frac{c}{a}\left[\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right]$$

$$\alpha^3\beta + \beta^3\alpha = \frac{c}{a}\left[\frac{b^2}{a^2} - \frac{2c}{a}\right]$$

$$\alpha^3\beta + \beta^3\alpha = \frac{c}{a}\left[\frac{b^2 - 2ac}{a^2}\right]$$

$$\alpha^3\beta + \beta^3\alpha = \frac{c\,(b^2 - 2ac)}{a^3}$$

Ex # 2.4

Q1: (ii) **If α, β are the roots of $ax^2 + bx + c = 0$, find the values of $(\alpha - \beta)^2$**

Solution:

$$ax^2 + bx + c = 0$$

As α and β are the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a}$$

And product of roots:

$$\alpha . \beta = \frac{c}{a}$$

According to given condition

$$(\alpha - \beta)^2 = ?$$

As $4\alpha\beta = (\alpha + \beta)^2 - (\alpha - \beta)^2$

Then $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

So

$$(\alpha - \beta)^2 = \left(\frac{-b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$

$$(\alpha - \beta)^2 = \frac{b^2 - 4a}{a^2}$$

Q2: (i) **Find the quadratic equation whose root are: $1, \frac{1}{2}$**

Solution:

$$1, \frac{1}{2}$$

As 1 and $\frac{1}{2}$ are the roots of required equation

Then sum of roots:

$$S = 1 + \frac{1}{2}$$

$$S = \frac{2 + 1}{2}$$

$$S = \frac{3}{2}$$

And product of roots:

$$P = (1)\left(\frac{1}{2}\right)$$

$$P = \frac{1}{2}$$

As required equation is:

$$x^2 - Sx + P = 0$$

Now

$$x^2 - \frac{3}{2}x + \frac{1}{2} = 0$$

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Ex # 2.4

Multiply all terms by 2

$2 \times x^2 - 2 \times \frac{3}{2}x + 2 \times \frac{1}{2} = 2 \times 0$

$2x^2 - 3x + 1 = 0$

Which is the required equation

(ii) $\mathbf{-3, 4}$

Solution:

$$-3,4$$

As -3 and 4 are the roots of required equation

Then sum of roots:

$S = -3 + 4$

$S = 1$

And product of roots:

$P = (-3)\,(4)$

$P = -12$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - 1.x + (-12) = 0$

$x^2 - x - 12 = 0$

Which is the required equation

(iii) $\mathbf{3 + \sqrt{2}, 3 - \sqrt{2}}$

Solution:

$$3 + \sqrt{2}, 3 - \sqrt{2}$$

As $3 + \sqrt{2}$ and $3 - \sqrt{2}$ are the roots of required equation

Then sum of roots:

$S = 3 + \sqrt{2} + 3 - \sqrt{2}$

$S = 3 + 3 + \sqrt{2} - \sqrt{2}$

$S = 6$

And product of roots:

$P = \left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right)$

$P = (3)^2 - \left(\sqrt{2}\right)^2$

$P = 9 - 2$

$P = 7$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - 6x + 7 = 0$

Which is the required equation

Ex # 2.4

(iv) $\mathbf{a, -2a}$

Solution:

$$a, -2a$$

As a and $-a$ are the roots of required equation

Then sum of roots:

$S = a + (-2a)$

$S = a - 2a$

$S = -a$

And product of roots:

$P = (a)\,(-2a)$

$P = -2a^2$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - (-a)x + (-2a^2) = 0$

$x^2 + ax - 2a^2 = 0$

Which is the required equation

Q3: Form a quadratic equation whose roots are square of the roots of the equation $\mathbf{ax^2 + bx + c = o; a \neq 0.}$

Solution:

$ax^2 + bx + c = o$

As α and β are the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a}$

And product of roots:

$\alpha \,.\, \beta = \frac{c}{a}$

As α^2 *and* β^2 are the roots of required equation

Now

$S = \alpha^2 + \beta^2$

As $\mathbf{\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2}$

Then $\mathbf{\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta}$

So

$S = (\alpha + \beta)^2 - 2\alpha\beta$

$S = \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$S = \frac{b^2}{a^2} - \frac{2c}{a}$

$S = \frac{b^2 - 2ac}{a^2}$

Ex # 2.4

Now

$P = \alpha^2 \beta^2$

$P = (\alpha \beta)^2$

$P = \left(\frac{c}{a}\right)^2$

$P = \frac{c^2}{a^2}$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - \left(\frac{b^2 - 2ac}{a^2}\right)x + \frac{c^2}{a^2} = 0$

Multiply all terms by a^2

$a^2 \times x^2 - a^2 \times \left(\frac{b^2 - 2ac}{a^2}\right)x + a^2 \times \frac{c^2}{a^2} = 0$

$a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

This is the required equation

Q4: (i) **If α, β are the roots of $2x^2 + 3x + 1 = 0$, then find the values of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$**

Solution:

$$2x^2 + 3x + 1 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = 3, c = 1$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$

And product of roots:

$\alpha \,.\, \beta = \frac{c}{a} = \frac{1}{2}$

Now

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha.\alpha + \beta.\beta}{\beta\alpha}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

Ex # 2.4

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\left(\frac{-3}{2}\right)^2 - 2\left(\frac{1}{2}\right)}{\frac{1}{2}}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\frac{9}{4} - 1}{\frac{1}{2}}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\frac{9 - 4}{4}}{\frac{1}{2}}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\frac{5}{4}}{\frac{1}{2}}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{5}{4} \div \frac{1}{2}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{5}{4} \times \frac{2}{1}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{10}{4}$

Q4: (ii) **If α, β are the roots of $2x^2 + 3x + 1 = 0$, then find the values of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$**

Solution:

$$2x^2 + 3x + 1 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = 3, c = 1$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$

And product of roots:

$\alpha \,.\, \beta = \frac{c}{a} = \frac{1}{2}$

Now

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

So

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

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Ex # 2.4

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\left(\frac{-3}{2}\right)^2-2\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\frac{9}{4}-1}{\frac{1}{4}}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\frac{5}{4}}{\frac{1}{4}}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{5}{4}\div\frac{1}{4}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{5}{4}\times\frac{4}{1}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=5$$

Q4: (iii) **If α,β are the roots of $2x^2+3x+1=0$, then find the values of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$**

Solution:

$$2x^2+3x+1=0$$

Compare it with $ax^2+bx+c=0$

Here $a=2, b=3, c=1$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha+\beta=\frac{-b}{a}=\frac{-3}{2}$$

And product of roots:

$$\alpha\,.\beta=\frac{c}{a}=\frac{1}{2}$$

Now

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha.\alpha^2+\beta.\beta^2}{\beta\alpha}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

As $\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)=(\alpha+\beta)^3$

Then $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

So

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

Ex # 2.4

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\left(\frac{-3}{2}\right)^3-3\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right)}{\frac{1}{2}}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\frac{-27}{8}-\left(\frac{-9}{4}\right)}{\frac{1}{2}}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\frac{-27}{8}+\frac{9}{4}}{\frac{1}{2}}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\frac{-27+18}{8}}{\frac{1}{2}}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\frac{-9}{8}}{\frac{1}{2}}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{-9}{8}\div\frac{1}{2}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{-9}{8}\times\frac{2}{1}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{-9}{4}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=-\frac{9}{4}$$

Q5: **If α,β are the roots of $3x^2+2x+5=0$, find the equation whose roots are $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$**

Solution:

$$3x^2+2x+5=0$$

Compare it with $ax^2+bx+c=0$

Here $a=3, b=2, c=5$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha+\beta=\frac{-b}{a}=\frac{-2}{3}$$

And product of roots:

$$\alpha\,.\beta=\frac{c}{a}=\frac{5}{3}$$

As $\frac{\alpha}{\beta}$ *and* $\frac{\beta}{\alpha}$ are the roots of required equation

Now Sum of roots

$$S=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$$

Ex # 2.4

$S = \frac{\alpha.\alpha + \beta.\beta}{\beta\alpha}$

$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

As $\boldsymbol{\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2}$

Then $\boldsymbol{\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta}$

So

$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$S = \frac{\left(\frac{-2}{3}\right)^2 - 2\left(\frac{5}{3}\right)}{\frac{5}{3}}$

$S = \frac{\frac{4}{9} - \frac{10}{3}}{\frac{5}{3}}$

$S = \frac{\frac{4 - 30}{9}}{\frac{5}{3}}$

$S = \frac{\frac{-26}{9}}{\frac{5}{3}}$

$S = \frac{-26}{9} \div \frac{5}{3}$

$S = \frac{-26}{9} \times \frac{3}{5}$

$S = \frac{-26}{3} \times \frac{1}{5}$

$S = -\frac{26}{15}$

Now Product of roots

$P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$

$P = 1$

<u>As required equation is:</u>

$x^2 - Sx + P = 0$

Now

$x^2 - \left(-\frac{26}{15}\right)x + 1 = 0$

$x^2 + \frac{26}{15}x + 1 = 0$

Multiply all terms by 15

$15 \times x^2 + 15 \times \frac{26}{15}x + 15 \times 1 = 15 \times 0$

$15x^2 + 26x + 15 = 0$

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Ex # 2.4

Q6: **If α, β are the roots of $x^2 - 4x + 2 = 0$, find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$**

Solution:

$$x^2 - 4x + 2 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -4, c = 2$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

And product of roots:

$\alpha . \beta = \frac{c}{a} = \frac{2}{1} = 2$

As $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ are the roots of required equation

Now Sum of roots

$S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta}$

$S = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta}$

$S = \alpha + \beta + \frac{\beta + \alpha}{\alpha\beta}$

$S = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$

$S = 4 + \frac{4}{2}$

$S = 4 + 2$

$S = 6$

Now Product of roots

$P = \left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$

$P = \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}$

$P = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$P = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\alpha.\alpha + \beta.\beta}{\beta\alpha}$

$P = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\alpha^2 + \beta^2}{\alpha\beta}$

As $\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$

Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Ex # 2.4

So

$P = \alpha\beta + \frac{1}{\alpha\beta} + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$P = 2 + \frac{1}{2} + \frac{(4)^2 - 2(2)}{2}$

$P = \frac{4 + 1}{2} + \frac{16 - 4}{2}$

$P = \frac{5}{2} + \frac{12}{2}$

$P = \frac{5 + 12}{2}$

$P = \frac{17}{2}$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - 6.x + \frac{17}{2} = 0$

Multiply all terms by 2

$2 \times x^2 - 2 \times 6x + 2 \times \frac{17}{2} = 2 \times 0$

$2x^2 - 12x + 17 = 0$

Ex # 2.5

Synthetic Division

Synthetic division is the process of finding the quotient and remainder with less writing and fewer calculations.

Synthetic division is the shortcut of long division method and allows one to calculate without writing variables.

Note:

Synthetic division can be used only when the divisor is a linear factor.

Must write a zero for the coefficient of each missing term in descending order.

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Ex # 2.5

Use synthetic division to find the quotient $Q(x)$ and remainder R when the polynomial $3x^3 - 2x^2 - 150$ is divided by $x - 4$

In this question

$$\text{Dividend} = 3x^3 - 2x^2 - 150$$
$$\text{Divisor} = x - 4$$

First find synthetic divisor

So we compare $x - a$ with the divisor $x - 4$

$$x - a = x - 4$$
$$-a = -4$$
$$a = 4$$

Thus 4 is the synthetic divisor

Now we can write the dividend like:

$$\text{P}(x) = 3x^3 - 2x^2 - 150$$

First write the polynomial in descending order.

Write zero with coefficient if missing in order.

$$\text{P}(x) = 3x^3 - 2x^2 + 0x - 150$$

Write the co-efficients of x from dividend polynomial in a row

Write the synthetic divisor 4 on the left side.

4	3	−2	0	−150

Bring the first number straight down

4	3	−2	0	−150
	3			

Now multiply 4 with 3 of third row and write the result under 2nd number of 2nd row.

4	3	−2	0	−150
		12		
	3			

Now add the numbers under 2nd column and write in 3rd row.

4	3	−2	0	−150
		12		
	3	10		

Now multiply 4 with 10 of third row and write the result under 3rd number of 2nd row and so on….

Ex # 2.5

And also add the numbers under 3rd column and so on….

4	3	−2	0	−150
		12	40	160
	3	10	40	10

$$\text{Thus } Q(x) = 3x^2 + 10x + 40$$
$$\text{And } R = 10$$

Example # 19

Use synthetic division to find the quotient $Q(x)$ and remainder R when the polynomial $3x^3 - 2x^2 - 150$ is divided by $x - 4$

Solution:

$$3x^3 - 2x^2 - 150$$
$$\text{Let P}(x) = 3x^3 - 2x^2 + 0x - 150$$
$$\text{Now}$$
$$-a = x - 4$$
$$-a = -4$$
$$a = 4$$

4	3	−2	0	−150
		12	40	160
	3	10	40	10

$$\text{Thus } Q(x) = 3x^2 + 10x + 40$$
$$\text{And } R = 10$$

Example # 20

Use synthetic division to find the values of k if 2 is a zero of the polynomial $2x^4 + x^3 + kx^2 - 8$

Solution:

$$2x^4 + x^3 + kx^2 - 8$$

Let $P(x) = 2x^4 + x^3 + kx^2 + 0x - 8$

As -2 is a zero of $P(x)$

So $P(2) = 0$

2	2	1	k	0	−8
		4	10	$2k + 20$	$4k + 40$
	2	5	$k + 10$	$2k + 20$	$4k + 32$

Here Remainder $= 4k + 32$

As Remainder $= 0$

$4k + 32 = 0$

$4k = -32$

$\frac{4k}{4} = \frac{-32}{4}$

$k = -8$

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Ex # 2.5

Example # 21

Use synthetic division to find the values of m amd n if $x-1$ and $x+2$ are the factors of $x^3-mx^2+nx+12$

Solution:

As $P(x)=x^3-mx^2+nx+12$

Now

$x-a=x-1$

$-a=-1$

$a=1$

And

$x-a=x+2$

$-a=2$

$a=-2$

1	1	$-m$	n	12
		1	$1-m$	$1-m+n$
−2	1	$1-m$	$1-m+n$	$13-m+n$
		-2	$2+2m$	
	1	$-1-m$	$3+m+n$	

Since $x-1$ and $x+2$ are the factors of $P(x)$

So the remainders are equal to zero.

$13-m+n=0$ ……..*equ (i)*

$3+m+n=0$ ……..*equ (ii)*

Add equ(i) and equ(ii)

$$13-m+n=0$$
$$3+m+n=0$$
$$16\quad +2n=0$$

As $16+2n=0$

$2n=-16$

$n=\frac{-16}{2}$

$n=-8$

Put $n=-8$ *equ (ii)*

$3+m+(-8)=0$

$3+m-8=0$

$m+3-8=0$

$m-5=0$

$m=5$

Thus $m=5$ and $n=-8$

Ex # 2.5

Example # 22

If -1 and 2 are roots of the quartic equation $x^4-5x^2+4=0$, use synthetic division to find other roots.

Solution:

$$x^4-5x^2+4=0$$

Let $P(x)=x^4+0x^3-5x^2+0x+4$

As -1 and 2 are the roots of $P(x)$

Now

−1	1	0	−5	0	4
		−1	1	4	−4
2	1	−1	−4	4	0
		2	2	−4	
	1	1	−2	0	

Thus $Q(x)=x^2+x-2$

Now to find the other roots

R.W	
$(x^2)(-2)=-2x^2$	
Add	**Multiply**
$-1x$	$-1x$
$+2x$	$+2x$
$+x$	$-2x^2$

$x^2-1x+2x-2=0$

$x(x-1)+2(x-1)=0$

$(x-1)(x+2)=0$

$x-1=0$ *or* $x+2=0$

$x=1$ *or* $x=-2$

Thus the other roots are 1 and -2

Ex # 2.5

Page # 42

Q1: **Use synthetic division to find the quotient $Q(x)$ and remainder R when the first polynomial is divided by the second binomial in each case:**

(i) $3x^3+2x^2-x-1;x+3$

Solution:

$$3x^3+2x^2-x-1$$

Let $P(x)=3x^3+2x^2-x-1$

Now

$x-a=x+3$

$-a=3$

$a=-3$

−3	3	2	−1	−1
		−9	21	−60
	3	−7	20	−61

Thus $Q(x)=3x^2-7x+20$

And $R=-61$

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(ii) $\mathbf{2x^3 - 7x^2 + 12x - 27; x - 3}$

Solution:

$2x^3 - 7x^2 + 12x - 27$

Let $P(x) = 2x^3 - 7x^2 + 12x - 27$

Now

$x - a = x - 3$

$-a = -3$

$a = 3$

3	2	−7	12	−27
		6	−3	27
	2	−1	9	0

Thus $Q(x) = 2x^2 - x + 9$

And $R = 0$

(iii) $\mathbf{2x^4 - 3x^2 + 5x - 7; x + 2}$

Solution:

$2x^4 - 3x^2 + 5x - 7$

Let $P(x) = 2x^4 + 0x^3 - 3x^2 + 5x - 7$

Now

$x - a = x + 2$

$-a = 2$

$a = -2$

−2	2	0	−3	5	−7
		−4	8	−10	10
	2	−4	5	−5	3

Thus $Q(x) = 2x^3 - 4x^2 + 5x - 5$

And $R = 3$

Q2: **Use synthetic division to find the value of k if -2 is zero of the polynomials $x^3 + 4x^2 + kx + 8$**

Solution:

$x^3 + 4x^2 + kx + 8$

Let $P(x) = x^3 + 4x^2 + kx + 8$

As -2 is a zero of $P(x)$

So $P(-2) = 0$

−2	1	4	k	8
		−2	−4	$-2k + 8$
	1	2	$k - 4$	$-2k + 16$

Here Remainder $= -2k + 16$

As Remainder $= 0$

$-2k + 16 = 0$

$-2k = -16$

Divide B.S by – 2

$\frac{-2k}{-2} = \frac{-16}{-2}$

$k = 8$

Q3: **Use synthetic division to find the values of p amd q if $x + 1$ and $x - 2$ are the factors of $x^3 + px^2 + qx + 6$.**

Solution:

As $P(x) = x^3 + px^2 + qx + 6$

Now $x - a = x + 1$

$-a = 1$

$a = -1$

And $x - a = x - 2$

$-a = -2$

$a = 2$

−1	1	p	q	6
		−1	$-p + 1$	$-q + p - 1$
2	1	$p - 1$	$q - p + 1$	$-q + p + 5$
		2	$2p + 2$	
	1	$p + 1$	$q + p + 3$	

Since $x + 1$ and $x - 2$ are the factors of $P(x)$

So the remainders are equal to zero.

$-q + p + 5 = 0$ ……..*equ* (*i*)

$q + p + 3 = 0$ ……..*equ* (*ii*)

Add equ(i) and equ(ii)

$-q + p + 5 = 0$

$q + p + 3 = 0$

$2p + 8 = 0$

As $2p + 8 = 0$

$2p = -8$

$p = \frac{-8}{2}$

$p = -4$

Put $p = -4$ *equ* (*ii*)

$q + (-4) + 3 = 0$

$q - 4 + 3 = 0$

$q - 1 = 0$

$q = 1$

Thus $p = -4$ and $q = 1$

Q4: **If $x + 1$ and $x - 2$ are the factors of the polynomial $x^3 + ax^2 + bx + 2$, then using synthetic division, find the values of a and b.**

Solution:

As $P(x) = x^3 + ax^2 + bx + 2$

Now

$x - a = x + 1$

$-a = 1$

$a = -1$

And

$x - a = x - 2$

$-a = -2$

$a = 2$

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$$\begin{array}{c|ccc|c}
-1 & 1 & a & b & 2 \\
 & & -1 & -a+1 & -b+a-1 \\ \hline
2 & 1 & a-1 & b-a+1 & -b+a+1 \\
 & & 2 & 2a+2 & \\ \hline
 & 1 & a+1 & b+a+3 &
\end{array}$$

Since $x+1$ and $x-2$ are the factors of $P(x)$

So the remainders are equal to zero.

$-b+a+1=0 \quad \ldots\ldots equ\ (i)$

$b+a+3=0 \quad \ldots\ldots equ\ (ii)$

Add equ(i) and equ(ii)

$$\begin{array}{r}
-b+a+1=0 \\
b+a+3=0 \\ \hline
2a+4=0
\end{array}$$

As $2a+4=0$

$2a=-4$

$a=\frac{-4}{2}$

$a=-2$

Put $a=-2\ equ\ (ii)$

$b+(-2)+3=0$

$b-2+3=0$

$b+1=0$

$b=-1$

Thus $a=-2$ and $b=-1$

Q5: **One root of the cubic equation $x^3-7x-6=0$ is 3. Use synthetic division to find the other roots.**

Solution:

$$x^3-7x-6=0$$

Let $P(x)=x^3+0x^2-7x-6=0$

As 3 is the root of P(x). So

$$\begin{array}{c|ccc|c}
3 & 1 & 0 & -7 & -6 \\
 & & 3 & 9 & 6 \\ \hline
 & 1 & 3 & 2 & 0
\end{array}$$

Thus $Q(x)=x^2+3x+2$

And $R=0$

Now to find the other roots

R.W	
$(x^2)(2)=2x^2$	
Add	**Multiply**
$+1x$	$+1x$
$+2x$	$+2x$
$+3x$	$2x^2$

$x^2+3x+2=0$

$x^2+1x+2x+2=0$

$x(x+1)+2(x+1)=0$

$(x+1)(x+2)=0$

$x+1=0 \quad or \quad x+2=0$

$x=-1 \quad or \quad x=-2$

Thus the other roots are -1 and -2

Ex # 2.5

Q6: **If -1 and 2 are roots of the quartic equation $x^4-5x^3+3x^2+7x-2=0$, use synthetic division to find other roots.**

Solution:

$$x^4-5x^3+3x^2+7x-2=0$$

Let $P(x)=x^4-5x^3+3x^2+7x-2$

As -1 and 2 are the roots of $P(x)$

Now

$$\begin{array}{c|cccc|c}
-1 & 1 & -5 & 3 & 7 & -2 \\
 & & -1 & 6 & -9 & 2 \\ \hline
2 & 1 & -6 & 9 & -2 & 0 \\
 & & 2 & -8 & 2 & \\ \hline
 & 1 & -4 & 1 & 0 &
\end{array}$$

Thus $Q(x)=x^2-4x+1$

Now to find the other roots

$x^2-4x+1=0$

Compare it with $ax^2+bx+c=0$

$Here\ a=1, b=-4, c=1$

As we have

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Put the values

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-4}}{2}$$

$$x=\frac{4\pm\sqrt{12}}{2}$$

$$x=\frac{4\pm\sqrt{4\times 3}}{2}$$

$$x=\frac{4\pm 2\sqrt{3}}{2}$$

$$x=\frac{2(2\pm\sqrt{3})}{2}$$

$x=2\pm\sqrt{3}$

$x=2+\sqrt{3} \quad or \quad x=2-\sqrt{3}$

Thus the other roots are $2+\sqrt{3}$ and $2-\sqrt{3}$

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Ex # 2.6

Simultaneous Equations

More than one equation which are satisfied by the same values of the variables involved are called simultaneous equations.

Note:

A system of Linear equation consists of two or more linear equations in the same variables.

Example # 23

Solve the system

$\mathbf{2x + y = 10}$

$\mathbf{4x^2 + y^2 = 68}$

Solution:

$2x + y = 10 \quad \ldots\ldots\ldots \text{Equ (i)}$

$4x^2 + y^2 = 68 \quad \ldots\ldots\ldots \text{Equ (ii)}$

Equ (i) $\Rightarrow$

$2x + y = 10$

$y = 10 - 2x \quad \ldots\ldots\ldots \text{Equ (iii)}$

Put the value of y in Equ (ii)

$4x^2 + (10 - 2x)^2 = 68$

$4x^2 + (10)^2 - 2(10)(2x) + (2x)^2 = 68$

$4x^2 + 100 - 40x + 4x^2 = 68$

$4x^2 + 4x^2 - 40x + 100 - 68 = 0$

$8x^2 - 40x + 32 = 0$

$8(x^2 - 5x + 4) = 0$

Divided B.S by 8, we get

$x^2 - 5x + 4 = 0$

$x^2 - 1x - 4x + 4 = 0$

$x(x-1) - 4(x-1) = 0$

$(x-1)(x-4) = 0$

$x - 1 = 0 \quad or \quad x - 4 = 0$

$x = 1 \quad or \quad x = 4$

R.W	
$(x^2)(4) = 4x^2$	
Add	Multiply
$-1x$	$-1x$
$-4x$	$-4x$
$-5x$	$4x^2$

Now put $x = 1$ in equ (iii)

$y = 10 - 2(1)$

$y = 10 - 2$

$y = 8$

Now put $x = 4$ in equ (iii)

$y = 10 - 2(4)$

$y = 10 - 8$

$y = 2$

Solution Set $= \{(\mathbf{1}, \mathbf{8}), (\mathbf{4}, \mathbf{2})\}$

Ex # 2.6

Example # 24

Solve the system

$\mathbf{x - y = 7}$

$\mathbf{x^2 + 3xy + y^2 = -1}$

Solution:

$x - y = 7 \quad \ldots\ldots\ldots \text{Equ (i)}$

$x^2 + 3xy + y^2 = -1 \quad \ldots\ldots\ldots \text{Equ (ii)}$

Equ (i) $\Rightarrow$

$x - y = 7$

$x = 7 + y \quad \ldots\ldots\ldots \text{Equ (iii)}$

Put the value of x in Equ (ii)

$(7 + y)^2 + 3(7 + y)y + y^2 = -1$

$(7)^2 + 2(7)(y) + (y)^2 + 3y(7 - y) + y^2 = -1$

$49 + 14y + y^2 + 21y + 3y^2 + y^2 = -1$

$49 + 14y + y^2 + 21y + 4y^2 = -1$

$y^2 + 4y^2 + 14y + 21y + 49 + 1 = 0$

$5y^2 + 35y + 50 = 0$

$5(y^2 + 7y + 10) = 0$

Divided B.S by 5, we get

$y^2 + 7y + 10 = 0$

$y^2 + 2y + 5y + 10 = 0$

$y(y+2) + 5(y+2) = 0$

$(y+2)(y+5) = 0$

$y + 2 = 0 \quad or \quad y + 5 = 0$

$y = -2 \quad or \quad y = -5$

R.W	
$(y^2)(10) = 10y^2$	
Add	Multiply
$+2y$	$+2y$
$+5y$	$+5y$
$+7y$	$10y^2$

Now put $y = -2$ in equ (iii)

$x = 7 + (-2)$

$x = 7 - 2$

$x = 5$

Now put $y = -5$ in equ (iii)

$x = 7 + (-5)$

$x = 7 - 5$

$x = 2$

Solution Set $= \{(\mathbf{5}, \mathbf{-2}), (\mathbf{2}, \mathbf{-5})\}$

Example # 25

$\mathbf{x^2 + y^2 = 4}$

$\mathbf{2x^2 - y^2 = 8}$

Solution:

$x^2 + y^2 = 4 \quad \ldots\ldots\ldots \text{Equ (i)}$

$x^2 = -y^2 + 45 \quad \ldots\ldots\ldots \text{Equ (ii)}$

Add equ(i) and equ(ii)

$$x^2 + y^2 = 4$$
$$2x^2 - y^2 = 8$$
$$3x^2 \quad = 12$$

Thus $3x^2 = 12$

$x^2 = \frac{12}{3}$

$x^2 = 4$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

$x = 2 \quad or \quad x = -2$

Now put $x = 2$ in equ (i)

$(2)^2 + y^2 = 4$

$4 + y^2 = 4$

$y^2 = 4 - 4$

$y^2 = 0$

$y = 0$

Now put $x = -2$ in equ (i)

$(-2)^2 + y^2 = 4$

$4 + y^2 = 4$

$y^2 = 4 - 4$

$y^2 = 0$

$y = 0$

Solution Set $= \{(2, 0), (-2, 0)\}$

Ex # 2.6

Page # 45

Q1: Solve the following system of equations.

(i) $2x - y = 3$

$x^2 + y^2 = 2$

Solution:

$2x - y = 3$ ……… Equ (i)

$x^2 + y^2 = 2$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$2x - y = 3$

$-y = 3 - 2x$

$y = -3 + 2x$

$y = 2x - 3$ ……… Equ (iii)

Put the value of y in Equ (ii)

$x^2 + (2x - 3)^2 = 2$

Ex # 2.6

$x^2 + (2x)^2 - 2(2x)(3) + (3)^2 = 2$

$x^2 + 4x^2 - 12x + 9 = 2$

$5x^2 - 12x + 9 - 2 = 0$

$5x^2 - 12x + 7 = 0$

$5x^2 - 5x - 7x + 7 = 0$

$5x(x - 1) - 7(x - 1) = 0$

$(x - 1)(5x - 7) = 0$

$x - 1 = 0 \quad or \quad 5x - 7 = 0$

$x = 1 \quad or \quad 5x = 7$

$x = 1 \quad or \quad x = \frac{7}{5}$

R.W	
$(5x^2)(7) = 35x^2$	
Add	**Multiply**
$-5x$	$-5x$
$-7x$	$-7x$
$-12x$	$35x^2$

Now put $x = 1$ in equ (iii)

$y = 2(1) - 3$

$y = 2 - 3$

$y = -1$

Now put $x = \frac{7}{5}$ in equ (iii)

$y = 2\left(\frac{7}{5}\right) - 3$

$y = \frac{14}{5} - 3$

$y = \frac{14 - 15}{5}$

$y = \frac{-1}{5}$

Solution Set $= \left\{(1, -1), \left(\frac{7}{5}, \frac{-1}{5}\right)\right\}$

(ii) $x + 2y = 0$

$x^2 + 4y^2 = 32$

Solution:

$x + 2y = 0$ ……… Equ (i)

$x^2 + 4y^2 = 32$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$x + 2y = 0$

$x = -2y$ ……… Equ (iii)

Put the value of x in Equ (ii)

$(-2y)^2 + 4y^2 = 32$

$4y^2 + 4y^2 = 32$

$8y^2 = 32$

Divide B.S by 8

$\frac{8y^2}{8} = \frac{32}{8}$

$y^2 = 4$

Ex # 2.6

Taking square root on B. S

$\sqrt{y^2} = \pm\sqrt{4}$

$y = \pm 2$

$y = 2 \quad or \quad x = -2$

Now put $y = 2$ in equ (iii)

$x = -2(2)$

$x = -4$

Now put $x = -2$ in equ (iii)

$x = -2(-2)$

$x = 4$

Solution Set $= \{(\mathbf{-4, 2}), (\mathbf{4, -2})\}$

(iii) $\mathbf{2x - y = -8}$

$\mathbf{x^2 + 4x = y}$

Solution:

$2x - y = -8$ ……… Equ (i)

$x^2 + 4x = y$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$2x - y = -8$

$-y = -8 - 2x$

$-y = -(8 + 2x)$

$y = 8 + 2x$

$y = 2x + 8$ ……… Equ (iii)

Put the value of y in Equ (ii)

$x^2 + 4x = 2x + 8$

$x^2 + 4x - 2x - 8 = 0$

$x(x + 4) - 2(x + 4) = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0 \quad or \quad x - 2 = 0$

$x = -4 \quad or \quad x = 2$

Now put $x = -4$ in equ (iii)

$y = 2(-4) + 8$

$y = -8 + 8$

$y = 0$

Now put $x = 2$ in equ (iii)

$y = 2(2) + 8$

$y = 4 + 8$

$y = 12$

Solution Set $= \{(\mathbf{-4, 0}), (\mathbf{2, 12})\}$

Ex # 2.6

(iv) $\mathbf{2x + y = 4}$

$\mathbf{x^2 - 2x + y^2 = 3}$

Solution:

$2x + y = 4$ ……… Equ (i)

$x^2 - 2x + y^2 = 3$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$2x + y = 4$

$y = 4 - 2x$ ……… Equ (iii)

Put the value of y in Equ (ii)

$x^2 - 2x + (4 - 2x)^2 = 3$

$x^2 - 2x + (4)^2 - 2(4)(2x) + (2x)^2 = 3$

$x^2 - 2x + 16 - 16x + 4x^2 = 3$

$x^2 + 4x^2 - 2x - 16x + 16 - 3 = 0$

$5x^2 - 18x + 13 = 0$

$5x^2 - 5x - 13x + 13 = 0$

$5x(x - 1) - 13(x - 1) = 0$

$(x - 1)(5x - 13) = 0$

$x - 1 = 0 \quad or \quad 5x - 13 = 0$

$x = 1 \quad or \quad 5x = 13$

$x = 1 \quad or \quad 5x = 13$

$x = 1 \quad or \quad x = \frac{13}{5}$

R.W	
$(5x^2)(13) = 65x^2$	
Add	**Multiply**
$-5x$	$-5x$
$-13x$	$-13x$
$\mathbf{-18x}$	$\mathbf{65x^2}$

Now put $x = 1$ in equ (iii)

$y = 4 - 2(1)$

$y = 4 - 2$

$y = 2$

Now put $x = \frac{13}{5}$ in equ (iii)

$y = 4 - 2\left(\frac{13}{5}\right)$

$y = 4 - \frac{26}{5}$

$y = \frac{20 - 26}{5}$

$y = \frac{-6}{5}$

Solution Set $= \left\{(\mathbf{1, 2}), \left(\mathbf{\frac{13}{5}, \frac{-6}{5}}\right)\right\}$

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Ex # 2.6

(v) $\mathbf{4x^2 + 5y^2 = 4}$

$\mathbf{3x^2 + y^2 = 3}$

Solution:

$4x^2 + 5y^2 = 4$ ……… Equ (i)

$3x^2 + y^2 = 3$ ……… Equ (ii)

Multiply equ(ii) with 5

$15x^2 + 5y^2 = 15$ ……… Equ (iii)

Subtract equ(i) from equ(iii)

$15x^2 + 5y^2 = 15$

$\pm 4x^2 \pm 5y^2 = \pm 4$

$11x^2 = 11$

Thus $11x^2 = 11$

$x^2 = \frac{11}{11}$

$x^2 = 1$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{1}$

$x = \pm 1$

$x = 1 \quad or \quad x = -1$

Now put $x = 1$ in equ (ii)

$3(1)^2 + y^2 = 3$

$3(1) + y^2 = 3$

$3 + y^2 = 3$

$y^2 = 3 - 3$

$y^2 = 0$

$y = 0$

Now put $x = -1$ in equ (ii)

$3(-1)^2 + y^2 = 3$

$3(1) + y^2 = 3$

$3 + y^2 = 3$

$y^2 = 3 - 3$

$y^2 = 0$

$y = 0$

Solution Set $= \{(\mathbf{1}, \mathbf{0}), (-\mathbf{1}, \mathbf{0})\}$

(vi) $\mathbf{5x^2 = y^2 + 9}$

$\mathbf{x^2 = -y^2 + 45}$

Solution:

$5x^2 = y^2 + 9$ ……… Equ (i)

$x^2 = -y^2 + 45$ ……… Equ (ii)

Ex # 2.6

Add equ(i) and equ(ii)

$5x^2 = y^2 + 9$

$x^2 = -y^2 + 45$

$6x^2 = 54$

Thus $6x^2 = 54$

$x^2 = \frac{54}{6}$

$x^2 = 9$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

$x = 3 \quad or \quad x = -3$

Now put $x = 3$ in equ (i)

$5(3)^2 = y^2 + 9$

$5(9) = y^2 + 9$

$45 = y^2 + 9$

$45 - 9 = y^2$

$36 = y^2$

$y^2 = 36$

Taking Square root on B.S

$\sqrt{y^2} = \pm\sqrt{36}$

$y = \pm 6$

$x = 6 \quad or \quad x = -6$

Now put $x = -3$ in equ (i)

$5(-3)^2 = y^2 + 9$

$5(9) = y^2 + 9$

$45 = y^2 + 9$

$45 - 9 = y^2$

$36 = y^2$

$y^2 = 36$

Taking Square root on B.S

$\sqrt{y^2} = \pm\sqrt{36}$

$y = \pm 6$

$x = 6 \quad or \quad x = -6$

Solution Set $= \{(\mathbf{3}, \mathbf{6}), (\mathbf{3}, -\mathbf{6}), (-\mathbf{3}, \mathbf{6}), (-\mathbf{3}, -\mathbf{6})\}$

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Ex # 2.6

(vii) $\mathbf{4x^2 + 3y^2 - 5 = 0}$

$\mathbf{2x^2 + 3y^2 - 4 = 0}$

Solution:

$4x^2 + 3y^2 - 5 = 0$ ……… Equ (i)

$2x^2 + 3y^2 - 4 = 0$ ……… Equ (ii)

Subtract equ(ii) from equ(i)

$$\begin{array}{l} 4x^2 + 3y^2 - 5 = 0 \\ \pm 2x^2 \pm 3y^2 \mp 4 = 0 \\ \hline 2x^2 \qquad\quad - 1 = 0 \end{array}$$

Thus $2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}}$ *or* $x = -\frac{1}{\sqrt{2}}$

Now put $x = \frac{1}{\sqrt{2}}$ in equ (ii)

$2\left(\frac{1}{\sqrt{2}}\right)^2 + 3y^2 - 4 = 0$

$2\left(\frac{1}{2}\right) + 3y^2 - 4 = 0$

$1 + 3y^2 - 4 = 0$

$3y^2 + 1 - 4 = 0$

$3y^2 - 3 = 0$

$3y^2 = 3$

$y^2 = \frac{3}{3}$

$y^2 = 1$

$\sqrt{y^2} = \pm\sqrt{1}$

$y = \pm 1$

$x = 1$ *or* $x = -1$

Now put $x = -\frac{1}{\sqrt{2}}$ in equ (ii)

$2\left(-\frac{1}{\sqrt{2}}\right)^2 + 3y^2 - 4 = 0$

Ex # 2.6

$2\left(\frac{1}{2}\right) + 3y^2 - 4 = 0$

$1 + 3y^2 - 4 = 0$

$3y^2 + 1 - 4 = 0$

$3y^2 - 3 = 0$

$3y^2 = 3$

$y^2 = \frac{3}{3}$

$y^2 = 1$

$\sqrt{y^2} = \pm\sqrt{1}$

$y = \pm 1$

$x = 1$ *or* $x = -1$

Solution Set

$= \left\{\left(\frac{1}{\sqrt{2}}, 1\right), \left(\frac{1}{\sqrt{2}}, -1\right), \left(-\frac{1}{\sqrt{2}}, 1\right), \left(-\frac{1}{\sqrt{2}}, -1\right)\right\}$

Challenge!

Q2: **Solve the system of equations**

(i) $\mathbf{x + y = 9}$

$\mathbf{x^2 + 3xy + 2y^2 = 3}$

Solution:

$x + y = 9$ ……… Equ (i)

$x^2 + 3xy + 2y^2 = 0$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$x + y = 9$

$x = 9 - y$ ……… Equ (iii)

Put the value of x in Equ (ii)

$(9 - y)^2 + 3(9 - y)y + 2y^2 = 0$

$(9)^2 - 2(9)(y) + (y)^2 + 3y(9 - y) + 2y^2 = 0$

$81 - 18y + y^2 + 27y - 3y^2 + 2y^2 = 0$

$81 - 18y + y^2 + 27y - y^2 = 0$

$y^2 - y^2 + 27y - 18y + 81 = 0$

$9y + 81 = 0$

$9y = -81$

Divide B.S by 9

$\frac{9y}{9} = \frac{-81}{9}$

$y = -9$

Now put $y = -9$ in equ (iii)

$x = 9 - (-9)$

$x = 9 + 9$

$x = 18$

Solution Set $= \{(\mathbf{18}, \mathbf{-92})\}$

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Ex # 2.6

(ii) $\boldsymbol{y - x = 4}$

$\boldsymbol{2x^2 + xy + y^2 = 8}$

Solution:

$y - x = 4$ ……… Equ (i)

$2x^2 + xy + y^2 = 8$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$y - x = 4$

$y = 4 + x$ ……… Equ (iii)

Put the value of y in Equ (ii)

$2x^2 + x(4 + x) + (4 + x)^2 = 8$

$2x^2 + 4x + x^2 + (4)^2 + 2(4)(x) + (x)^2 = 8$

$2x^2 + 4x + x^2 + 16 + 8x + x^2 = 8$

$2x^2 + x^2 + x^2 + 4x + 8x + 16 - 8 = 0$

$4x^2 + 12x + 8 = 0$

$4(x^2 + 3x + 2) = 0$

Divide B.S by 4, we get

$x^2 + 3x + 2 = 0$

$x^2 + 1x + 2x + 2 = 0$

$x(x + 1) + 2(x + 1) = 0$

$(x + 1)(x + 2) = 0$

$x + 1 = 0 \quad or \quad x + 2 = 0$

$x = -1 \quad or \quad x = -2$

R.W	
$(x^2)(2) = 2x^2$	
Add	Multiply
$+1x$	$+1x$
$+2x$	$+2x$
$+3x$	$2x^2$

Now put $x = -1$ in equ (iii)

$y = 4 + (-1)$

$y = 4 - 1$

$y = 3$

Now put $x = -2$ in equ (iii)

$y = 4 + (-2)$

$y = 4 - 2$

$y = 2$

Solution Set $= \{(-1, 3), (-2, 2)\}$

Ex # 2.7

Example # 26

Suppose a rectangular shed is being built that has an area of 120 square feet and is 7 feet longer than its wide. Determine its dimensions.

Solution:

Let Width $= x \ ft$

So Length $= (x + 7)ft$

As Area $= 120 \ ft^2$

Ex # 2.7

As we have

Width × Length = Area

$x(x + 7) = 120$

$x^2 + 7x = 120$

$x^2 + 7x - 120 = 0$

$x^2 - 8x + 15x - 120 = 0$

$x(x - 8) + 15(x - 8) = 0$

$(x - 8)(x + 15) = 0$

$x - 8 = 0 \quad or \quad x + 15 = 0$

$x = 8 \quad or \quad x = -15$

R.W	
$(x^2)(-120) = -120x^2$	
Add	Multiply
$-8x$	$-8x$
$+15x$	$+15x$
$+7x$	$-120x^2$

As distance will be positive

Then we take $x = 8$

So Width $= x \ ft = 8 \, m$

And Length $= (8 + 7)ft$

$= 15 \ ft$

Example # 27

A man purchased a number of shares of stock for an amount of Rs. 6000. If he had paid Rs. 20 less per share, the number of shares that could have been purchased for the same amount of money would have increased by 10. How many shares did he buy?

Solution:

Let numbers of share $= x$

And cost of per share $= y$

As he purchase share for Rs. 6000

Then $xy = 6000$ ……. Equ (i)

Now if he paid Rs. 20 per share

Then amount $= y - 20$

So he purchased 10 more shares

Then number of shares $= x + 10$

According to new condition

$(x + 10)(y - 20) = 6000$ ……. Equ (ii)

Equ (i) $\Rightarrow$

$xy = 6000$

$y = \dfrac{6000}{x}$ …… Equ (iii)

Put the value of y in Equ (ii)

$(x + 10)\left(\dfrac{6000}{x} - 20\right) = 6000$

$(x + 10)\left(\dfrac{6000 - 20x}{x}\right) = 6000$

$(x + 10)(6000 - 20x) = 6000x$

$6000x - 20x^2 + 60000 - 200x = 6000x$

$-20x^2 + 6000x - 200x - 6000x + 60000 = 0$

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Ex # 2.7

$-20x^2 - 200x + 60000 = 0$

$-200(x^2 + 10x - 3000) = 0$

Divided B.S by -20, we get

$x^2 + 10x - 3000 = 0$

$x^2 - 50x + 60x - 270 = 0$

$x(x - 50) + 60(x - 50) = 0$

$(x - 50)(x + 60) = 0$

$x - 50 = 0 \quad or \quad x + 60 = 0$

$x = 50 \quad or \quad x = -60$

As $x = -60$ is not possible

Thus the number of Shares purchased $= 15$

Ex # 2.7

Page # 47

Q1: Find the two consecutive positive integers whose product is 72.

Solution:

As there are two consecutive integers

Let first integer $= x$

And second integer $= x + 1$

According to given condition

$x(x + 1) = 72$

$x^2 + x = 72$

$x^2 + x - 72 = 0$

$x^2 + 9x - 8x - 72 = 0$

$x(x + 9) - 8(x + 9) = 0$

$(x + 9)(x - 8) = 0$

$x + 9 = 0 \quad or \quad x - 8 = 0$

$x = -9 \quad or \quad x = 8$

As there are positive integers

So first integer $= x = 8$

And second integer $= x + 1$

$= 8 + 1$

$= 9$

Q2: The sum of the squares of three consecutive integers is 50. Find the integers.

Solution:

As there are three consecutive integers

Then

First integer $= x$

Second integer $= x + 1$

Third integer $= x + 2$

Ex # 2.7

According to given condition

$x^2 + (x + 1)^2 + (x + 2)^2 = 50$

$x^2 + (x)^2 + 2(x)(1) + (1)^2 + (x)^2 + 2(x)(2) + (2)^2 = 50$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 50$

$x^2 + x^2 + x^2 + 2x + 4x + 1 + 4 - 50 = 0$

$3x^2 + 6x - 45 = 0$

$3(x^2 + 2x - 15) = 0$

Divide B.S by 3, we get

$x^2 + 2x - 15 = 0$

$x^2 - 3x + 5x - 15 = 0$

$x(x - 3) + 5(x - 3) = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0 \quad or \quad x + 5 = 0$

$x = 3 \quad or \quad x = -5$

R.W	
$(x^2)(-15) = -15x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+5x$	$+5x$
$\mathbf{+2x}$	$\mathbf{-15x^2}$

If $x = 3$

So first integer $= x = 3$

Second integer $= x + 1$

$= 3 + 1$

$= 4$

And third integer $= x + 2$

$= 3 + 2$

$= 5$

If $x = -5$

So first integer $= x = -5$

Second integer $= x + 1$

$= -5 + 1$

$= -4$

And third integer $= x + 2$

$= -5 + 2$

$= -3$

Q3: The length of a hall is 5 meters more than its width. If the area of the hall is 36sq.m. Find the length and width of the hall.

Solution:

Let Width $= x \ m$

So Length $= (x + 5)m$

As Area $= 36 \text{ m}^2$

As we have

Width $\times$ Length $=$ Area

$x(x + 5) = 36$

$x^2 + 5x = 36$

$x^2 + 5x - 36 = 0$

$x^2 - 4x + 9x - 36 = 0$

R.W	
$(x^2)(-36) = -36x^2$	
Add	**Multiply**
$-4x$	$-4x$
$+9x$	$+9x$
$\mathbf{+5x}$	$\mathbf{-36x^2}$

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Ex # 2.6

$x(x-4)+9(x-4)=0$

$(x-4)(x+9)=0$

$x-4=0 \quad or \quad x+9=0$

$x=4 \quad or \quad x=-9$

As distance will be positive

Then we take $x=4$

So Width $= x\ m = 4\ m$

And Length $= (4+5)m$

$= 9\ m$

Q4: The sum of two numbers is 11 and sum of their square is 65. Find the numbers.

Solution:

Let the number may x and y

According to first condition

$x+y=11$ ……… Equ (i)

According to second condition

$x^2+y^2=65$ ……… Equ (ii)

Equ (i) $\Rightarrow$

$x+y=11$

$x=11-y$ ……… Equ (iii)

Put the value of x in Equ (ii)

$(11-y)^2+y^2=65$

$(11)^2-2(11)(y)+(y)^2+y^2-65=0$

$121-22y+y^2+y^2-65=0$

$y^2+y^2-22y+121-65=0$

$2y^2-22y+56=0$

$2(y^2-11y+28)=0$

Divide B.S by 2, we get

$y^2-11y+28=0$

$x^2-4y-7y+28=0$

$y(y-4)-7(y-4)=0$

$(y-4)(y-7)=0$

$y-4=0 \quad or \quad y-7=0$

$y=4 \quad or \quad y=7$

R.W	
$(y^2)(28)=28y^2$	
Add	Multiply
$-4y$	$-4y$
$-7y$	$-7y$
$-11y$	$28y^2$

Now put $y=4$ in equ (iii)

$x=11-4$

$x=7$

Now put $y=7$ in equ (iii)

$x=11-7$

$x=4$

Thus the required two numbers are 4 and 7

Ex # 2.6

Q5: The sum of the squares of two numbers is 100. One number is two more than the other. Find the numbers.

Solution:

Let the first number $= x$

As the other number two more than it

So the other numbe $= x+2$

As sum of squares of two number is 100

$x^2+(x+2)^2=100$

$x^2+(x)^2+2(x)(2)+(2)^2=100$

$x^2+x^2+4x+4-100=0$

$2x^2+4x-96=0$

$2(x^2+2x-48)=0$

Divide B.S by 2, we get

$x^2+2x-48=0$

$x^2-6x+8x-48=0$

$x(x-6)+8(x-6)=0$

$(x-6)(x+8)=0$

$x-6=0 \quad or \quad x+8=0$

$x=6 \quad or \quad x=-8$

R.W	
$(x^2)(-48)=-48x^2$	
Add	Multiply
$-6x$	$-6x$
$+8x$	$+8x$
$+2x$	$-48x^2$

If $x=6$

So first integer $= x = 6$

Other integer $= x+1$

$= 6+2$

$= 8$

If $x=-8$

So first integer $= x = -8$

Second integer $= x+1$

$= -8+2$

$= -6$

Q6: The area of a rectangle field is 252 square meters. The length of its side is 9 meter longer than its width. Find its sides.

Solution:

Let Width $= x\ m$

So Length $= (x+9)m$

As Area $= 252\ m^2$

As we have

Width $\times$ Length $=$ Area

$x(x+9)=252$

$x^2+9x=252$

$x^2+9x-252=0$

R.W	
$(x^2)(-252)=-252x^2$	
Add	Multiply
$-12x$	$-12x$
$+21x$	$+21x$
$+9x$	$-252x^2$

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Ex # 2.6

$x^2 - 12x + 21x - 252 = 0$

$x(x - 12) + 21(x - 12) = 0$

$(x - 12)(x + 21) = 0$

$x - 12 = 0 \quad or \quad x + 21 = 0$

$x = 12 \quad or \quad x = -21$

As distance will be positive

Then we take $x = 4$

So Width $= x\ m = 12\ m$

And Length $= (12 + 9)m$

$= 21\ m$

Q7: One side of a rectangle is 3 centimeters less than twice the other. If the area of the rectangle is 54 square centimeters, then find the sides of the rectangle.

Solution:

Let Width $= x\ cm$

According to given condition

So Length $= (2x - 3)cm$

As Area = 54 cm^2

As we have

Width × Length = Area

$x(2x - 3) = 54$

$2x^2 - 3x = 54$

$2x^2 - 3x - 54 = 0$

R.W	
$(2x^2)(-54) = -108x^2$	
Add	Multiply
$+9x$	$+9x$
$-12x$	$-12x$
$-3x$	$-108x^2$

$2x^2 + 9x - 12x - 54 = 0$

$x(2x + 9) - 6(2x + 9) = 0$

$(2x + 9)(x - 6) = 0$

$2x + 9 = 0 \quad or \quad x - 6 = 0$

$2x = -9 \quad or \quad x = 6$

$x = \frac{-9}{2} \quad or \quad x = 6$

As distance will be positive

Then we take $x = 6$

So Width $= x\ cm = 6\ cm$

And Length $= (2x - 3)cm$

$= (2(6) - 3)cm$

$= (12 - 3)cm$

$= 9cm$

Ex # 2.6

Q8: The length of one side of right triangle exceeds the length of the other by 3 centimeters. If the hypotenuse is 15 centimeters, then find the length of the sides of the triangle.

Solution:

Let Base $= x\ cm$

Then according to condition

Perpendicular $= (x + 3)cm$

Hypotenuse $= 15\ cm$

As there is right angled triangle

Using Pythagoras theorem

$x^2 + (x + 3)^2 = (15)^2$

$x^2 + (x)^2 + 2(x)(3) + (3)^2 = 225$

$x^2 + x^2 + 6x + 9 - 225 = 0$

$2x^2 + 6x - 216 = 0$

$2(x^2 + 3x - 108) = 0$

Divided B.S by 2, we get

$x^2 + 3x - 108 = 0$

R.W	
$(x^2)(-108) = -108x^2$	
Add	Multiply
$-9x$	$-9x$
$+12x$	$+12x$
$+3x$	$-108x^2$

$x^2 - 9x + 12x - 108 = 0$

$x(x - 9) + 12(x - 9) = 0$

$(x - 9)(x + 12) = 0$

$x - 9 = 0 \quad or \quad x + 12 = 0$

$x = 9 \quad or \quad x = -12$

As distance will be positive

Then we take $x = 9$

So Base $= x\ cm = 9\ cm$

And Perpendicular $= (x + 3)cm$

$= (9 + 3)cm$

$= 12cm$

Q9: The sides of a right triangle in cm are $(x - 1), x, (x + 1)$. Find the sides of the triangle.

Solution:

Let Base $= x\ cm$

Then according to condition

Perpendicular $= (x + 3)cm$

Hypotenuse $= 15\ cm$

As there is right angled triangle

Using Pythagoras theorem

$x^2 + (x + 3)^2 = (15)^2$

$x^2 + (x)^2 + 2(x)(3) + (3)^2 = 225$

$x^2 + x^2 + 6x + 9 - 225 = 0$

Ex # 2.6

$2x^2 + 6x - 216 = 0$

$2(x^2 + 3x - 108) = 0$

Divided B.S by 2, we get

$x^2 + 3x - 108 = 0$

$x^2 - 9x + 12x - 108 = 0$

$x(x - 9) + 12(x - 9) = 0$

$(x - 9)(x + 12) = 0$

$x - 9 = 0 \quad or \quad x + 12 = 0$

$x = 9 \quad or \quad x = -12$

As distance will be positive

Then we take $x = 9$

So Base $= x\ cm = 9\ cm$

And Perpendicular $= (x + 3)cm$

$= (9 + 3)cm$

$= 12cm$

Q10: **A shepherd bought some goats for Rs.9000. If he had paid Rs. 100 less for each, he would have got 3 goats more for the same amount of money. How many goats did he buy, when the rate in each case is uniform?**

Solution:

Let numbers of goats $= x$

And cost of each goat $= y$

As he bought goats for Rs. 9000

Then $xy = 9000$ ……. Equ (i)

Now if he paid Rs. 100 for each goat

Then amount $= y - 100$

So he got 3 more goats

Then number of goats $= x + 3$

According to new condition

$(x + 3)(y - 100) = 9000$ ……. Equ (ii)

Equ (i) $\Rightarrow$

$xy = 9000$

$y = \frac{9000}{x}$ …… Equ (iii)

Put the value of y in Equ (ii)

$(x + 3)\left(\frac{9000}{x} - 100\right) = 9000$

$(x + 3)\left(\frac{9000 - 100x}{x}\right) = 9000$

$(x + 3)(9000 - 100x) = 9000x$

Ex # 2.6

$9000x - 100x^2 + 27000 - 300x = 9000x$

$-100x^2 + 9000x - 300x - 9000x + 27000 = 0$

$-100x^2 - 300x + 27000 = 0$

$-100(x^2 + 3x - 270) = 0$

Divided B.S by − 100, we get

$x^2 + 3x - 270 = 0$

$x^2 - 15x + 18x - 270 = 0$

$x(x - 15) + 18(x - 15) = 0$

$(x - 15)(x + 18) = 0$

$x - 15 = 0 \quad or \quad x + 18 = 0$

$x = 15 \quad or \quad x = -18$

As $x = -18$ is not possible

Thus the number of goats bought $= 15$

R.W	
$(x^2)(-270) = -270x^2$	
Add	**Multiply**
$-15x$	$-15x$
$+18x$	$+18x$
$+3x$	$-270x^2$

Review Exercise # 2

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Q2: **For what value of k the roots of the equations $3x^2 - 5x + k = 0$ are equal.**

Solution:

$$3x^2 - 5x + k = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = -5, c = k$

As roots are equal then

Discriminant $= b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$(-5)^2 - 4(3)(k) = 0$

$25 - 12k = 0$

$-12k = -25$

$12k = 25$

$k = \frac{25}{12}$

Q3: **Evaluate $\left(-1 + \sqrt{-3}\right)^7 + \left(-1 + \sqrt{-3}\right)^7$**

Solution:

$\left(-1 + \sqrt{-3}\right)^7 + \left(-1 + \sqrt{-3}\right)^7$

$\left(-1 + \sqrt{-1 \times 3}\right)^7 + \left(-1 + \sqrt{-1 \times 3}\right)^7$

$\left(-1 + \sqrt{-1}\sqrt{3}\right)^7 + \left(-1 + \sqrt{-1}\sqrt{3}\right)^7$

$\left(-1 + i\sqrt{3}\right)^7 + \left(-1 + i\sqrt{3}\right)^7$

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As

$$\omega = \frac{-1+i\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1-i\sqrt{3}}{2}$$

$$2\omega = -1+i\sqrt{3} \text{ and } 2\omega^2 = -1-i\sqrt{3}$$

So

$= (2\omega)^7 + (2\omega^2)^7$

$= 2^7\omega^7 + 2^7\omega^{14}$

$= 2^7(\omega^7 + \omega^{14})$

$= 128(\omega^6.\omega + \omega^{12}.\omega^2)$

$= 128(\omega^{3\times2}.\omega + \omega^{3\times4}.\omega^2)$

$= 128[(\omega^3)^2.\omega + (\omega^3)^4.\omega^2]$

$= 128[(1)^2.\omega + (1)^4.\omega^2]$

$= 128[1.\omega + 1.\omega^2]$

$= 128(\omega + \omega^2)$

As $\omega + \omega^2 = -1$

$= 128(-1)$

$= -128$

Q4: Without solving the equation, find the sum and products of the roots of the following quadratic equations.

(i) $4x^2 - 1 = 0$

Solution:

$$4x^2 - 1 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 4, b = 0, c = -1$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-0}{4} = \frac{0}{4} = 0$$

And product of roots:

$$\alpha.\beta = \frac{c}{a} = \frac{-1}{4} = -\frac{1}{4}$$

(ii) $3x^2 + 4x = 0$

Solution:

$$3x^2 + 4x = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = 4, c = 0$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-4}{3} = -\frac{4}{3}$$

Review Ex # 2

And product of roots:

$$\alpha.\beta = \frac{c}{a} = \frac{0}{3} = 0$$

Q5: Find the value of k so that the sum of the roots of the equation $3x^2 + (2k+1)x + k - 5 = 0$ is equal to the product of its roots

Solution:

$3x^2 + (2k+1)x + k - 5 = 0$

Compare it with $ax^2 + bx + c = 0$

Here $a = 3, b = 2k+1, c = k-5$

Let α and β be the roots of equation

Then sum of roots:

$$\alpha + \beta = \frac{-b}{a} = \frac{-(2k+1)}{3}$$

And product of roots:

$$\alpha.\beta = \frac{c}{a} = \frac{k-5}{3}$$

According to given condition

Sum of roots = Product of roots

$$\frac{-(2k+1)}{3} = \frac{k-5}{3}$$

Multiply B.S by 3

$$3 \times \frac{-(2k+1)}{3} = 3 \times \frac{k-5}{3}$$

$-(2k+1) = k - 5$

$-2k - 1 = k - 5$

$-2k - k = -5 + 1$

$-3k = -4$

$3k = 4$

Divide B.S by 3

$$\frac{3k}{3} = \frac{4}{3}$$

$$k = \frac{4}{3}$$

Q6: Find the value of k if the roots of $x^2 - 3x + k + 1 = 0$ differ by unity.

Solution:

$$x^2 - 3x + k + 1 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 1, b = -3, c = k+1$

Let α and $\alpha + 1$ be the roots of equation

Then sum of roots:

$\alpha + \alpha + 1 = \frac{-b}{a}$

$2\alpha + 1 = \frac{-(-3)}{1}$

$2\alpha + 1 = 3$

$2\alpha = 3 - 1$

$2\alpha = 2$

$\alpha = \frac{2}{2}$

$\alpha = 1$

And product of roots:

$\alpha\ (\alpha + 1) = \frac{c}{a}$

$\alpha^2 + \alpha = \frac{k+1}{1}$

$\alpha^2 + \alpha = k + 1$

Put the value of α

$(1)^2 + 1 = k + 1$

$1 + 1 = k + 1$

$2 = k + 1$

$2 - 1 = k$

$1 = k$

$k = 1$

Q7: Find the quadratic equation whose roots multiplicative inverse of the roots of $12x^2 - 17x + 6 = 0$

Solution:

$$12x^2 - 17x + 6 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 12, b = -17, c = 6$

Let α and β be the roots of equation

Then sum of roots:

$\alpha + \beta = \frac{-b}{a} = \frac{-(-17)}{12} = \frac{17}{12}$

And product of roots:

$\alpha\,.\,\beta = \frac{c}{a} = \frac{6}{12} = \frac{1}{2}$

As $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of required equation

Now Sum of roots

$S = \frac{1}{\alpha} + \frac{1}{\beta}$

Review Ex # 2

$S = \frac{1}{\alpha} + \frac{1}{\beta}$

$S = \frac{\beta + \alpha}{\alpha\beta}$

$S = \frac{\frac{17}{12}}{\frac{1}{2}}$

$S = \frac{17}{12} \div \frac{1}{2}$

$S = \frac{17}{12} \times \frac{2}{1}$

$S = \frac{17}{6}$

Now Product of roots

$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$

$P = \frac{1}{\alpha\beta}$

$P = \frac{1}{\frac{1}{2}}$

$P = 1 \div \frac{1}{2}$

$P = 1 \times \frac{2}{1}$

$P = 2$

As required equation is:

$x^2 - Sx + P = 0$

Now

$x^2 - \frac{17}{6}x + 2 = 0$

Multiply all terms by 6

$6 \times x^2 - 6 \times \frac{17}{6}x + 6 \times 2 = 6 \times 0$

$6x^2 - 17x + 12 = 0$

Review Ex # 2

Q8: If one of the roots of the quadratic equation $2x^2 + kx + 4 = 0$ is 2, find the other root. Also find the value of k.

Solution:

$$2x^2 + kx + 4 = 0$$

Compare it with $ax^2 + bx + c = 0$

Here $a = 2, b = k, c = 4$

Let α and β be the roots of equation

As one root is 2, then $\alpha = 2$

Then sum of roots:

$\alpha + \beta = \frac{-b}{a}$

$2 + \beta = \frac{-k}{2}$ ……. Equ (i)

And product of roots:

$\alpha . \beta = \frac{c}{a}$

$2 . \beta = \frac{4}{2}$

$2 . \beta = 2$

$\beta = \frac{2}{2}$

$\beta = 1$

So the other root = 1

Put the values of $\beta = 1$ in equ(i)

$2 + 1 = \frac{-k}{2}$

$3 = \frac{-k}{2}$

$3 \times 2 = -k$

$6 = -k$

$-k = 6$

$k = -6$

Thus $k = -6$

Q9: One root of the cubic equation $x^3 + 6x^2 + 11x + 6 = 0$ is -3. Use synthetic division to find the other roots.

Solution:

$$x^3 + 6x^2 + 11x + 6 = 0$$

Let $P(x) = x^3 + 6x^2 + 11x + 6$

As -3 is the root of P(x). So

Review Ex # 2

−3	1	6	11	6
		−3	−9	−6
	1	3	2	0

Thus $Q(x) = x^2 + 3x + 2$

And $R = 0$

Now to find other roots

$x^2 + 3x + 2 = 0$

$x^2 + 1x + 2x + 2 = 0$

$x(x + 1) + 2(x + 1) = 0$

$(x + 1)(x + 2) = 0$

$x + 1 = 0 \quad or \quad x + 2 = 0$

$x = -1 \quad or \quad x = -2$

Thus the other roots are -1 and -2

R.W	
$(x^2)(2) = 2x^2$	
Add	Multiply
$+1x$	$+1x$
$+2x$	$+2x$
$+3x$	$2x^2$

Q10: Solve the following system of equations.

(i) $x + y = 3$

$x^2 - 3xy + y^2 = 29$

Solution:

$x + y = 3$ ……… Equ (i)

$x^2 - 3xy + y^2 = 29$ ……… Equ (ii)

Equ (i) ⇒

$x + y = 3$

$x = 3 - y$ ……… Equ (iii)

Put the value of x in Equ (ii)

$(3 - y)^2 - 3(3 - y)y + y^2 = 29$

$(3)^2 - 2(3)(y) + (y)^2 - 3y(3 - y) + y^2 = 29$

$9 - 6y + y^2 - 9y + 3y^2 + y^2 = 29$

$9 - 6y + y^2 - 9y + 4y^2 - 29 = 0$

$y^2 + 4y^2 - 6y - 9y + 9 - 29 = 0$

$5y^2 - 15y - 20 = 0$

$5(y^2 - 3y - 4) = 0$

Divide B.S by 5, we get

$y^2 - 3y - 4 = 0$

$y^2 - 4y + 1y - 4 = 0$

$y(y - 4) + 1(y - 4) = 0$

$(y - 4)(y + 1) = 0$

$y - 4 = 0 \quad or \quad y + 1 = 0$

$y = 4 \quad or \quad y = -1$

R.W	
$(y^2)(-4) = -4y^2$	
Add	Multiply
$-4y$	$-4y$
$+1y$	$+1y$
$-3y$	$-4y^2$

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Review Ex # 2

Now put $y = 4$ in equ (iii)

$x = 3 - 4$

$x = -1$

Now put $y = -1$ in equ (iii)

$x = 3 - (-1)$

$x = 3 + 1$

$x = 4$

Solution Set $= \{(-1, 4), (4, -1)\}$

(ii) $\mathbf{7x^2 - 4 = 5y^2}$

$\mathbf{3x^2 + 2 = 4y^2}$

Solution:

$7x^2 - 4 = 5y^2$ ……… Equ (i)

$3x^2 + 2 = 4y^2$ ……… Equ (ii)

Multiply equ(i)with 4 and equ(ii) with 5

$28x^2 - 16 = 20y^2$ ……… Equ (iii)

$15x^2 + 10 = 20y^2$ ……… Equ (iv)

Subtract equ(iv) from equ(iii)

$$\begin{array}{rcl} 28x^2 - 16 &=& 20y^2 \\ \pm 15x^2 \pm 10 &=& \pm 20y^2 \\ \hline 13x^2 - 26 &=& 0 \end{array}$$

Thus $13x^2 - 26 = 0$

$13x^2 = 26$

$x^2 = \frac{26}{13}$

$x^2 = 2$

Taking Square root on B.S

$\sqrt{x^2} = \pm\sqrt{2}$

$x = \sqrt{2}$ *or* $x = -\sqrt{2}$

Now put $x = \sqrt{2}$ in equ (i)

$7(\sqrt{2})^2 - 4 = 5y^2$

$7(2) - 4 = 5y^2$

$14 - 4 = 5y^2$

$10 = 5y^2$

$5y^2 = 10$

$y^2 = \frac{10}{5}$

$y^2 = 2$

Taking Square root on B.S

$\sqrt{y^2} = \pm\sqrt{2}$

$y = \sqrt{2}$ *or* $y = -\sqrt{2}$

Review Ex # 2

Now put $x = -\sqrt{2}$ in equ (i)

$7(-\sqrt{2})^2 - 4 = 5y^2$

$7(2) - 4 = 5y^2$

$14 - 4 = 5y^2$

$10 = 5y^2$

$5y^2 = 10$

$y^2 = \frac{10}{5}$

$y^2 = 2$

Taking Square root on B.S

$\sqrt{y^2} = \pm\sqrt{2}$

$y = \sqrt{2}$ *or* $y = -\sqrt{2}$

Solution Set

$= \{(\sqrt{2}, \sqrt{2}), (\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})\}$

Q11: The area of a rectangle is 48 $\mathbf{cm^2}$. If length and width are each increased by 4 cm, the area of larger rectangle is 120 $\mathbf{cm^2}$. Find the length and width of the original rectangle.

Solution:

Let Width of original rectangle $= x\ cm$

And Length of original rectangle $= y\ cm$

As Area of original rectangle $= \mathbf{48\ m^2}$

As we have

Width × Length = Area

$xy = 48$ …… Equ (i)

Now

Let Width of new rectangle $= x + 4\ cm$

And Length of new rectangle $= y + 4\ cm$

As Area of new rectangle $= \mathbf{120\ m^2}$

As we have

Width × Length = Area

$(x + 4)(y + 4) = 120$

$xy + 4x + 4y + 16 = 120$ …… Equ (ii)

Now put $xy = 48$ in equ (ii)

$48 + 4x + 4y + 16 = 120$

$4x + 4y = 120 - 48 - 16$

$4(x + y) = 56$

$x + y = \frac{56}{4}$

$x + y = 14$

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Review Ex # 2

$y = 14 - x$ ……Equ (iii)

Now put $y = 14 - x$ in equ (i)

$x(14 - x) = 48$

$14x - x^2 = 48$

$-x^2 + 14x - 48 = 0$

$-1(x^2 - 14x + 48) = 0$

Divided B.S by -1, we get

$x^2 - 14x + 48 = 0$

$x^2 - 6x - 8x + 48 = 0$

$x(x - 6) - 8(x - 6) = 0$

$(x - 6)(x - 8) = 0$

$x - 6 = 0 \quad or \quad x - 8 = 0$

$x = 6 \quad or \quad x = 8$

Now put $x = 6$ in equ (iii)

$y = 14 - 6$

$y = 8$

So Width $= x\ cm = 6\ cm$

And Length $= y\ cm = 8\ cm$

R.W	
$(x^2)(48) = 48x^2$	
Add	**Multiply**
$-6x$	$-6x$
$-8x$	$-8x$
$\mathbf{-14x}$	$\mathbf{48x^2}$

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